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Multi-atom entanglement engineering and phase-covariant quantum cloning with a single resonant interaction assisted by external driving

Xian-Peng Zhang, Li-Tuo Shen, Zhen-Biao Yang *

Department of Physics, Fuzhou University, Fuzhou 350002, PR China

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ABSTRACT

We propose a scheme to realize multi-atom entanglement and phase-covariant quantum cloning in a short-time manner possessing the advantage of its robustness with respect to parameter fluctuations. The process is achieved by externally driving the atoms to resonantly couple to the cavity mode. Compared to other strategies, such as the adiabatic or virtual-photon techniques, it provides a method which allows the relatively fast coherent manipulation.

1. Introduction

Entanglement is a form of quantum superposition, which is a fundamental resource of quantum mechanics. It can be applied not only to test quantum mechanics against local hidden theory theoretically [1–3], but also to implement quantum information processing (QIP), such as quantum cryptography [4–6], quantum teleportation [7–9], quantum computer [10], and so on. Therefore, a flurry of researches have been stimulated to studying entangled states and its applications. On the other hand, the entangled state is of great use for quantum cloning, which produces copies of quantum information [11–13]. As is well known, the superposition principle of quantum mechanics prohibits the copies from being perfect [14,15], which is one of the most fundamental differences between classical and quantum information theory. However, it is still possible for us to copy a quantum state approximately or probabilistically [16–18].

A quantum cloning machine which achieves equal fidelity for arbitrary states is referred to as universal quantum cloning machine (UQCM) [16,18]. In some cases, when we already know partial information of the input state, it is possible for us to get a better fidelity in quantum cloning. Phase-covariant quantum cloning machine (PCQCM) makes a typical example [17]. As it produces copies for any equatorial state with a better fidelity than that achieved by UQCM. The PCQCM is of great use for the well-known BB84 quantum cryptography [17]. The experimental implementation of PCQCM has been reported in optical systems [19], nuclear magnetic resonance systems [20], and solid state systems [21].

Efforts have also been devoted to devising elegant PCQCM based on other physical systems (see Ref. [13] as a review). Typical examples can be found in Refs. [22–24]. Such schemes realize PCQCM through either adiabatic passage [22,23] or virtual-photon process [24] in cavity quantum electrodynamics (QED) setups, thus they suffer from long manipulation time. We here discuss an alternative scheme for realizing multi-atom entanglement and phase-covariant quantum cloning with cavity QED system. The scheme is based on the full resonant interaction of the multiple atoms with the cavity mode as well as with the driven fields. Compared with all such schemes [22–24] for the same purposes, the distinct feature of the present scheme is that the operation time is relatively short due to the resonant coherent atom-field coupling, which is very important in view of decoherence. We also check the stability of the scheme by the numerical simulation and show it to be robust. The scheme can also been generalized to more general systems such as ions interacting with harmonic Coulomb lattices or superconducting qubits interacting through microwave radiation.

The article is organized as follows: In Section 2, we introduce the physical model. In Section 3, we give the dynamics for N atoms, each interacts with a cavity mode as well as with an externally driven field. In Section 4, we use the reduced dynamics to realize multi-atom entanglement and phase-covariant quantum cloning. In Section 5, we discuss the influences of parameter fluctuations. In Section 6, we analyze the influences of decoherence. In Section 7, we make a conclusion.

2. The model

As shown in Fig. 1, n identical Λ-type atoms are held within an optical cavity. The atom j has an excited state $|e\rangle_j$, and two ground

* Corresponding author.
E-mail address: zbyang@fzu.edu.cn (Z.-B. Yang).

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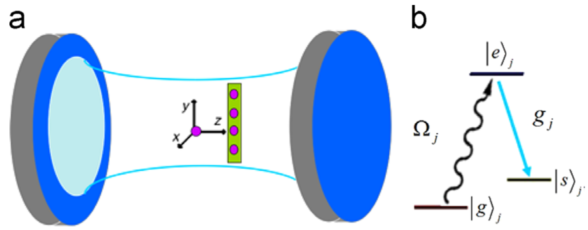

Fig. 1. (a) The schematic setup. A set of atoms (purple circles) are held within an optical cavity. An atom is trapped at the center of an optical cavity, while the other $N-1$ atoms are trapped in a one-dimension photonic crystal waveguide (green long orthogon). (b) The atom level configuration. The transitions $|e\rangle_j \leftrightarrow |g\rangle_j$ and $|e\rangle_j \leftrightarrow |s\rangle_j$ are respectively coupled with the laser and cavity fields, with the Rabi frequencies Ω_j and g_j. Both the transition channels are assumed to be completely resonant. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

states $|g\rangle_j$ and $|s\rangle_j$. The atomic configuration can be implemented with cesium atoms [25,26]. And three states $|e\rangle_j$, $|g\rangle_j$ and $|s\rangle_j$ could correspond, for instance, to the hyperfine levels $6P_{3/2}F'=5$, $6S_{1/2}F=4$ and $6S_{1/2}F=3$, respectively. The transition $|e\rangle_j \leftrightarrow |g\rangle_j$ is resonantly driven by a laser field with the Rabi frequency Ω_j ($j=1,2,\ldots,n$), while the transition $|e\rangle_j \leftrightarrow |s\rangle_j$ is resonantly coupled with the quantized cavity mode with the coupling strength g_j. In the interaction picture, the Hamiltonian of the whole system with the rotating-wave approximation can be written as ($\hbar=1$)

$$H_I = \sum_{j=1}^{n} (g_j a\sigma_{es}^j + \Omega_j e^{i\varphi_j}\sigma_{eg}^j) + H.c., \tag{1}$$

where $\sigma_{AB}^j = |A\rangle_{jj}\langle B|$ $(A,B=e,g,s)$ is the atomic jumping operator for the jth atom, a and a^+ are the annihilation and creation operators for the cavity mode respectively, and $H.c.$ represents the Hermitian conjugate. We define the excitation number operator $N_e = \sum_j (|e\rangle_{jj}\langle e| + |g\rangle_{jj}\langle g|) + a^+a$, which commutes with the Hamiltonian H_I. Thus, the global excitation number is conserved during the dynamic evolution of the Hamiltonian H_I.

3. State evolution in the single-excitation subspace

We assume that the system is initially taken to be in the state $|\Psi(0)\rangle = |g_1 s_2 s_3 \ldots s_n 0\rangle \equiv |g\rangle_1 |s\rangle_2 |s\rangle_3 \ldots |s\rangle_n |0\rangle_c$, with $|A\rangle_j$ $(A=g,s,e)$ denoting the state for the jth atom and $|n\rangle_c$ the n-photon Fock state (while the case with $n=0$ is also termed as vacuum state) for the cavity mode. Such an initial atomic state can be achieved by optical pumping with two optical laser beams focusing on atom 1, driving the transition from two ground states $|s\rangle$ and $|g\rangle$ (see typical optical QED experiments as in Refs. [27,28]). In addition, we adopt two conditions: (i) one atom couples to the cavity mode with a coupling strength g_1, while all the other $N-1$ atoms couple to the cavity mode with the same coupling strength g_2; (ii) Rabi frequencies for the laser fields driving all the atoms are equal.

In such a case, the system will evolve within the single-excitation subspace $\forall \in \{|\phi_1\rangle, |\phi_2\rangle, |\phi_3\rangle, |\phi_4\rangle, |\phi_5\rangle\}$, with

$$|\phi_1\rangle = |g_1 s_2 s_3 \ldots s_n 0\rangle, \tag{2}$$

$$|\phi_2\rangle = |e_1 s_2 s_3 \ldots s_n 0\rangle, \tag{3}$$

$$|\phi_3\rangle = |s_1 s_2 s_3 \ldots s_n 1\rangle, \tag{4}$$

$$|\phi_4\rangle = \sqrt{\frac{1}{n-1}} \sum_{i=2}^{i=n} \sigma_{es}^i |s_1 s_2 s_3 \ldots s_n 0\rangle, \tag{5}$$

$$|\phi_5\rangle = \sqrt{\frac{1}{n-1}} \sum_{i=2}^{i=n} \sigma_{gs}^j |s_1 s_2 s_3 \ldots s_n 0\rangle. \tag{6}$$

The system's state at an arbitrary time t can be expressed as (see Appendix)

$$\begin{aligned}|\Psi(t)\rangle = &\left[\frac{g_1^2}{\alpha^2}\left(\frac{\Omega^2}{\Sigma_g}\cos\,\alpha t + 1\right) + \frac{(n-1)g_2^2}{\Sigma_g}\cos\,\Omega t\right]|\phi_1\rangle \\ &-\frac{i}{\Sigma_g}\left[\frac{\Omega}{\alpha}g_1^2\,\sin\,\alpha t - (n-1)g_2^2\sin\,\Omega t\right]e^{-i\varphi_1}|\phi_2\rangle \\ &+\frac{\Omega g_1}{\alpha^2}(\cos\,\alpha t - 1)e^{-i\varphi_1}|\phi_3\rangle \\ &+\frac{i\sqrt{n-1}g_1 g_2}{\Sigma_g}\left(-\frac{\Omega}{\alpha}\sin\,\alpha t + \sin\,\Omega t\right)e^{-i\varphi_1}|\phi_4\rangle \\ &+\sqrt{n-1}g_1 g_2\left[\frac{1}{\alpha^2}\left(\frac{\Omega^2}{\Sigma_g}\cos\,\alpha t + 1\right)\right. \\ &\left.-\frac{1}{\Sigma_g}\cos\,\Omega t\right]e^{i(\varphi_2-\varphi_1)}|\phi_5\rangle,\end{aligned} \tag{7}$$

with $\Sigma_g = g_1^2 + (n-1)g_2^2$ and $\alpha = [\Sigma_g + \Omega^2]^{1/2}$. Such a coherent resonant dynamics allows us to engineer multi-atom entanglement and phase-covariant cloning.

4. Multi-atom entanglement and PCQCM via resonant interaction

By setting $g_1 t_0 = \sqrt{(3(\sqrt{n}+1)/2\sqrt{n})}\pi$, $\Omega/g_1 = \sqrt{2\sqrt{n}/3(\sqrt{n}+1)}$, and $g_2/g_1 = 1/(\sqrt{n}+1)$, we get

$$|\Psi_m\rangle = \sqrt{\frac{1}{n}} \sum_{i=1}^{i=n} \sigma_{gs}^j |s_1 s_2 s_3 \ldots s_n 0\rangle. \tag{8}$$

In this way, the atoms are prepared in the W state with the cavity left in the vacuum state.

The accurate control of the systems parameters can in principle be achieved according to the recently demonstrated cavity QED techniques in [29–32]. Using the ground state cooling [29] and photonic crystal waveguide technologies [30], the specific atomic positions and states in the cavity can be controlled. We realize the ideal cavity QED situation by trapping the single atoms in a three-dimensional optical lattice with the resonator as one of the lattice axes (z-axes). The resulting standing-wave optical lattice has a local maximum of atom–cavity coupling, corresponding to a minimum of the potential energy of optical lattice, with additional minima farther away spaced in increments of $\lambda_z/2$, where λ_α is the trap laser wavelength in α-axes direction ($\alpha = x,y,z$). A single atom is trapped at the center of an optical cavity and prepared in ground state with the maximal coupling to light [29]. $N-1$ atoms with space interval $D = m\lambda_y/2 \gg d$ (d is the distance that characterizes the interatomic forces, and m is a positive integer) are trapped in a one-dimension photonic crystal waveguide arises from the interference of an optical tweezer with its reflection from the structure [30], and located at a line: $x=0, z=D_z$, satisfying $g_2 = g_1\,\cos^2(2\pi D_z/\lambda_c), D_z = n\lambda_z/2$ (n is an integer much larger than one), where λ_c is the wavelength of the cavity model satisfying $\lambda_c \gg \lambda_z$. On the other hand, as the Rabi frequency Ω of the classical laser field can be adjusted by external control [31], thus the requirement $\Omega/g_1 = \sqrt{2\sqrt{n}/3(\sqrt{n}+1)}$ can also be fixed. And inhomogeneities may be compensated for by numerically optimizing a control pulse that implements an effective atom-locking Rabi drive of constant strength over a range of atom–cavity coupling and control field amplitudes [32]. Besides, the atom-field interaction time can be controlled by Stark tuning to and out of the cavity as well as the laser field resonance [31].

We now show how the phase-covariant quantum cloning (PCQCM) can be realized based on such a dynamic model. The PCQCM is used to clone the state of a qubit in the equatorial plane of the Bloch sphere with phase factor ν. We here assume that atom