Optics Communications 339 (2015) 108–114

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

A hybrid filter of Bragg grating and ring resonator

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ARTICLE INFO

Article history:
Received 9 June 2014
Received in revised form
22 November 2014
Accepted 24 November 2014
Available online 26 November 2014

Keywords:
Ring resonator
Bragg grating
Transfer matrix
Transmission spectrum
Stop band

ABSTRACT

A theoretical model is presented for a hybrid resonator of Bragg grating on a ring, coupled to a linear waveguide. An explicit expression of the transfer matrix is obtained for the Bragg grating with a loss in each repeat unit of the refractive index variation. Then, the transmission and reflection spectra through the coupled linear waveguide are calculated. When the ring has an even number of repeat units, the Bragg reflection occurs at one of ring resonances, capable of producing a deep and wide stop band. We identify the relationship between the grating parameter and the coupling coefficient that optimizes the filter performance. The circumference of the hybrid resonator can be as short as 1/100 of the length of the linear Bragg grating that gives similar stop-band characteristics.

1. Introduction

Fiber Bragg gratings (FBG) are widely being used to descramble superimposed light wave signals in wavelength division multiplexing (WDM) [1,2]. FBG consists of a periodic structure of high and low refractive indices (RI). The advantages of FBG include a wide spacing between adjacent stop bands and flat, near-unity transmittance of the pass band. The center wavelength of the stop band depends on the grating period and the mean RI, while the width is determined mostly by the RI contrast of the Bragg grating (BG).

To meet the demand of a decreasing channel spacing in WDM, there have been efforts to narrow the stop band [3]. The latter was achieved by employing a longer grating that consists of a smaller RI contrast. Further narrowing the reflection band without increasing the physical dimension may be made possible by forming the BG in a ring resonator and coupling it to a linear plain waveguide, as illustrated in Fig. 1. If the coupling is weak, the high Q resonance may be able to sustain an effectively long path length for traveling waves in the compact body, thus enabling a narrow band. The present report analyzes the reflection and transmission spectra through a linear waveguide coupled to the hybrid resonator.

In recent years, there has been an increasing interest in the hybrid resonator. Several groups presented experimental and theoretical works on a ring resonator with at least a part of it being a BG [4–10]. For example, a ring of 14.3 μm diameter with 100 air holes was fabricated on a silicon-on-insulator (SOI) platform [7]. In another attempt, a circular waveguide of silicon nitride was formed on a silica substrate and 200 indentations were inscribed into the waveguide [8]. Theoretical treatment for the BG on a ring resonator has been hampered by the lack of an appropriate model for lossy linear BG; without loss, the amplitude of the ring resonance skyrockets at weak coupling. To circumvent this problem, a ring that consists of two parts was considered — one is a linear lossless BG, and the rest is a plain linear waveguide with a loss [6,10]. The interplay between the ring resonance condition and BG's reflection condition led to interesting transmission and reflection spectra that either the ring or BG alone cannot produce.

The present paper is constructed as follows. First, an explicit expression is obtained for a linear BG that has a loss in each period of the RI variation. Then, the BG is formed into a ring and coupled to a linear waveguide. General formulas are derived for the transmission and reflection spectra through the linear waveguide as the RI contrast and number of periods of the grating and the coupling coefficient as parameters. The formulas are then applied to different values of the parameters to compare the performance of the hybrid device as a wavelength filter and that of the linear BG. Finally, a condition is obtained for the parameters that optimize the filter performance of the device.

2. Theoretical model

2.1. Ring resonance and Bragg reflection mode

In the absence of the grating, the ring of circumference L accommodates a resonance mode at a wavevector k (in vacuum) that

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http://dx.doi.org/10.1016/j.optcom.2014.11.077
0030-4018/

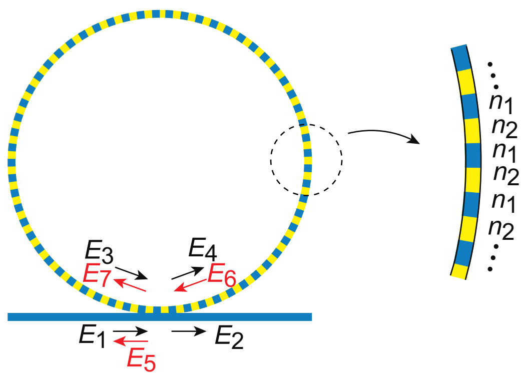

Fig. 1. Ring-Bragg grating hybrid resonator coupled to a plain linear waveguide. The ring resonator has a periodic refractive index variation of n_1 and n_2. The electric fields E_1 through E_7 are defined at the point of the waveguide–resonator contact for a pair of waves traveling in the opposite directions. The incoming wave is E_1.

satisfies

$$n_{\mathrm{eff}} kL = 2\pi l, \tag{1}$$

where n_{eff} is the effective RI of the mode and l is an integer. In a FBG with a grating period Λ, the first strong reflection occurs when the wavelength in the resonator, $2\pi/(n_{\mathrm{eff}}k)$, is equal to 2Λ; the second reflection mode appears when the wavelength is equal to $2\Lambda/2$, and so on. The wavevector at the center of the first reflection band, k_{B}, is given as

$$k_B = \pi/(\Lambda n_{\mathrm{eff}}), \tag{2}$$

and is related to the Bragg frequency ν_{B} as $k_{\mathrm{B}}=2\pi\nu_{\mathrm{B}}/c$, where c is the velocity of light in vacuum. If the whole circular orbit of the ring resonator consists of N periods of the grating elements, $n_{\mathrm{eff}}k_{\mathrm{B}}L=\pi N$. An even N makes the wave at k_{B} satisfy the ring resonance condition.

2.2. Transfer matrix of a lossy Bragg filter

Usually, lossless elements are considered for FBG [11]. We cannot apply that convention to the BG-on-a-ring resonator. Without loss, the intensity of the resonance mode, in the limit of weak coupling, would be infinite as a result of constructive interference with itself after every revolution. In this subsection, we first obtain the transfer matrix for a linear BG that consists of N lossy elements. Then in Section 2.3, we move to the hybrid filter to analyze transmission and reflection spectra through a linear waveguide coupled to the resonator.

We consider a linear BG that consists of alternating sections of RI n_i and length d_i ($i=1, 2$). When the wave travels in section i, the phase changes by $\varphi_i=n_ikd_i$, and the field amplitude experiences the attenuation of α_i. For simplicity, we assume that two sections are symmetric — $\varphi_1=\varphi_2=\varphi$ and $\alpha_1=\alpha_2=\alpha$. The transfer matrix $\mathbf{M}_0$ of the BG element that consists of a section of 1 and a section of 2 is obtained from the expression of $\mathbf{M}_0$ for a lossless element [10] by substituting φ_i with $\varphi_i-i\alpha_i$. The result is

$$\mathbf{M}_0 = \begin{bmatrix} A^2f^{-1} - B^2 & AB(f-1) \\ AB(f^{-1}-1) & A^2f - B^2 \end{bmatrix}, \tag{3}$$

where $A \equiv (4n_1n_2)^{-1/2}(n_1+n_2)$, $B \equiv (4n_1n_2)^{-1/2}(n_1-n_2)$, and $f \equiv \exp(i2\varphi+2\alpha)$. Extending Eq. (3) to an asymmetric element ($\varphi_1\neq\varphi_2$, $\alpha_1\neq\alpha_2$) is straightforward. It is easy to confirm that det $\mathbf{M}_0=1$. Squaring $\mathbf{M}_0$ gives $\mathbf{M}_0{}^2=2q\mathbf{M}_0-\mathbf{I}$, where

$$q \equiv A^2(f+f^{-1})/2 - B^2, \tag{4}$$

is complex in general, and $\mathbf{I}$ is the unit matrix. Let the transfer matrix $\mathbf{M}=\mathbf{M}_0{}^N$ for N elements of the BG be expressed as $\mathbf{M}_0{}^N=P_N\mathbf{M}_0-Q_N\mathbf{I}$, where $P_1=1$, $Q_1=0$. Then, we obtain recurrence relationships: $P_{N+1}=2qP_N-Q_N$ and $Q_{N+1}=P_N$. They lead to $P_N=(\beta_+{}^N-\beta_-{}^N)/(\beta_+-\beta_-)$, where $\beta_\pm=q\pm i(1-q^2)^{1/2}$. Let

$$q \equiv \cos \Phi. \tag{5}$$

Then, $\beta_\pm=\exp(\pm i\Phi)$ and

$$P_N = \sin N\Phi / \sin \Phi. \tag{6}$$

In general, Φ is complex: $\Phi=\Phi_{\mathrm{R}}+i\Phi_{\mathrm{I}}$. From Eqs. (4) and (5), we obtain

$$\cos \Phi_{\mathrm{R}} \cosh \Phi_{\mathrm{I}} \equiv A^2 \cos 2\varphi \cosh 2\alpha - B^2, \tag{7}$$

$$-\sin \Phi_{\mathrm{R}} \sinh \Phi_{\mathrm{I}} \equiv A^2 \sin 2\varphi \sinh 2\alpha, \tag{8}$$

which give Φ_{R} and Φ_{I} for given A, B, φ and α. Thus obtained $\mathbf{M}$ satisfies det $\mathbf{M}=1$, and therefore the diagonal elements of the scattering matrix for the N-element BG are identical. The transmittance T and reflectance R are respectively calculated as $T=1/|M_{22}|^2$ and $R=|M_{12}|^2/|M_{22}|^2$, where M_{ij} is the i, j element of $\mathbf{M}$. We can confirm that the above results reproduce the textbook results for $\alpha=0$ [11].

2.3. Transmittance and reflectance of the hybrid filter

Now we close the BG to form a ring and run a plain linear waveguide adjacent to the ring. The field amplitudes at the point of the waveguide–resonator coupling are defined in Fig. 1 for possible waves. The fields E_3, E_4, E_6, and E_7 of the circulating waves are related to each other as

$$\begin{bmatrix} E_3 \\ E_7 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} E_4 \\ E_6 \end{bmatrix}. \tag{9}$$

The field E_1 of the incident wave through the waveguide, E_2 of the transmitting wave, and E_5 of the reflected wave are related to E_3, E_4, E_6, and E_7 as [12,13]

$$E_4 = r_c E_3 - it_c E_1, \tag{10}$$

$$E_7 = r_c E_6, \tag{11}$$

$$E_2 = r_c E_1 - it_c E_3, \tag{12}$$

$$E_5 = -it_c E_6, \tag{13}$$

where the reflection and transfer coefficients, r_c and t_c, are related to each other by $r_c{}^2+t_c{}^2=1$. The sign for it_c was changed from the one in Ref. [12], as the definitions of the phase are different. From Eqs. (9)–(13), we obtain

$$\frac{E_2}{E_1} = \frac{-1 - r_c^2 + r_c(M_{11}+M_{22})}{M_{22} - 2r_c + r_c^2 M_{11}}, \tag{14}$$

$$\frac{E_5}{E_1} = \frac{(1 - r_c^2)M_{21}}{M_{22} - 2r_c + r_c^2 M_{11}}, \tag{15}$$

where det $\mathbf{M}=1$ was used. Below we discuss the transmittance $T=|E_2/E_1|^2$ and reflectance $R=|E_5/E_1|^2$ through the coupled waveguide. We focus on small RI contrasts, $|n_1-n_2|\leq 10^{-4}$, unless otherwise specified.

3. Results and discussion

Since we assume $\varphi_1=\varphi_2$, we specify the grating by n_1, n_2, d_1, α, and N. Below, we examine the characteristics of the hybrid filter.