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Discussion

Entanglement of two atomic ensembles in coupled cavities via adiabatic passage

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ABSTRACT

We propose a potentially practical scheme for creating entanglement between two atomic ensembles in two coupled cavities via adiabatic passage. The three-level Λ-type atoms in each ensemble dispersively interact with the nonresonant classical field and cavity mode. By choosing appropriate parameters of the system, the effective Hamiltonian describes two atomic ensembles interact with "the same cavity mode" and has a dark state. Consequently, the entanglement between the two ensembles is gained via adiabatic passage. Numerical calculations show that the scheme is robust against moderate fluctuations of the experimental parameters. In addition, the effect of decoherence can be suppressed effectively.

1. Introduction

As an unambiguous and quantifiable property of sufficiently simple multi-party quantum systems, entanglement has recently begun to be studied in several contexts [1–5]. Also, entanglement has potential applications in quantum information processing, such as quantum cryptography [6], quantum teleportation [7] and quantum dense coding [8]. The qubits storing information include phonons [9,10], trapped ions [11,12], atoms [12,13], nitrogen-vacancy (NV) center ensembles [14,15], etc. Among them, the atoms in cavity quantum electrodynamics (QED) [16] are well developed and regarded as ideal candidates for that the information stored in the atoms is stable. In recent years, many schemes for generating entangled states of atoms have been proposed [17–20]. Compared with those schemes that use a single particle as a qubit, the schemes proposed by Lukin [21], Xue [22], Duan [23], and Han [24] et al. use an atomic ensemble with a large number of identical atoms as the basic system. The atomic ensemble that contains a large number of identical atoms increases the light–matter coupling strength, which scales with the square-root of the number of the atoms involved in the ensemble. This greatly reduces the operation time and thus suppresses the decoherence. The advantage allows one to take a more positive view of the atomic ensemble and regard it as an essential resource for many ingenious applications such as realizing of quantum repeaters [25,26], quantum metrology [27], quantum interference [28], and generation of squeezed states for atomic ensembles [30,29].

Besides, adiabatic techniques are of interest since they feature a certain robustness, and in systems of Λ type one can avoid a transient large population in the excited state. Recently, the techniques of stimulated Raman adiabatic passage (STIRAP) [31] and fractional stimulated Raman adiabatic passage (f-STIRAP) [32] have been extensively used for realizing QIP [33–38].

In this paper, we present a new scheme to generate the maximally entangled state of two atomic ensembles in coupled cavities via adiabatic passage. By choosing appropriate parameters of the system, two atomic ensembles interact with "the same cavity mode". Under adiabatic condition, the system can be into steady state at the end of evolution. The process for gaining entanglement is insensitive to the fluctuations of atomic number in each ensemble and the coupling coefficient between the atoms and cavity modes. It does not need to control the evolution time accurately. In addition, based on the effective Hamiltonian, the atoms are always in ground states and the cavity mode in the vacuum state, so spontaneous emission and cavity decay can be efficiently suppressed.

2. Generation of entanglement of atomic ensembles

Let us first briefly describe the dynamical model of our considered system. Two atomic ensembles are separately trapped in

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two coupled cavities. Suppose that the two cavity modes are resonant with each other. In the interaction picture, the resonant coupling between the two cavity modes is given by the interaction Hamiltonian $H_0 = \nu(a^+b + ab^+)$, where a and b are the annihilation operators for the cavity modes, and ν is the coupling strength. The atomic number in the ith cavity is N_i $(i = 1, 2)$. Each atom has an excited state $|e\rangle$ and two ground states $|f\rangle$ and $|g\rangle$. The atomic transition $|e\rangle \leftrightarrow |g\rangle$ is coupled to the cavity with coupling coefficient g_i and detuning Δ_{gi}. Meanwhile, the atoms are driven by a classical laser field with the Rabi frequency Ω_i and detuning Δ_i $(i = 1, 2)$. In the interaction picture, the Hamiltonian describing atom–field interaction is $(\hbar = 1)$

$$H_I = \sum_{j=1}^{N_1} (\Omega_1 e^{i\Delta_1 t}|e_j\rangle_{11} \langle f_j| + g_1 e^{i\Delta_{g1}t} a|e_j\rangle_{11}\langle g_j|) + \sum_{j=1}^{N_2} (\Omega_2 e^{i\Delta_2 t}|e_j\rangle_{22} \langle f_j| + g_2 e^{i\Delta_{g2}t} b|e_j\rangle_{22}\langle g_j|) + H.\,c. \quad (1)$$

Introducing the new bosonic modes $c_1 = (a - b)/\sqrt{2}$, $c_2 = (a + b)/\sqrt{2}$ [39]. We can rewrite H_0 as $\nu(c_2^+c_2 - c_1^+c_1)$. In terms of the bosonic modes c_1 and c_2, the Hamiltonian H_0 is diagonal. So we can take H_0 as the "free Hamiltonian" mathematically and perform the transformation e^{iH_0t} to obtain the atom–field interaction Hamiltonian in the interaction picture. In this case, the Hamiltonian H_I can be rewritten as

$$H_I = g_1(c_1^+e^{-i\nu t} + c_2^+e^{i\nu t})e^{-i\Delta_{g1}t} \sum_{j=1}^{N_1} |g_j\rangle_{11} \langle e_j| + g_2(-c_1^+e^{-i\nu t} + c_2^+e^{i\nu t})e^{-i\Delta_{g2}t} \sum_{j=1}^{N_2} |g_j\rangle_{22}\langle e_j| + \Omega_1 e^{-i\Delta_1 t} \sum_{j=1}^{N_1} |f_j\rangle_{11} \langle e_j| + \Omega_2 e^{-i\Delta_2 t} \sum_{j=1}^{N_2} |f_j\rangle_{22}\langle e_j| + H.\,c. \quad (2)$$

In the case that Δ_1, Δ_2, $|\Delta_{g1} \pm \nu|$, $|\Delta_{g2} \pm \nu|$, $\nu \gg g_1$, g_2, Ω_1, Ω_2, the upper-level $|e\rangle$ can be adiabatically eliminated. Choose the detunings appropriately so that the dominant Raman transitions are induced by the atomic modes and normal mode c_2 while the probability of the coupling between the classical field Ω_i $(i = 1, 2)$ and normal mode c_1 is negligible due to a large detuning. Then the effective Hamiltonian is written as

$$H_{e1} = -\Bigg[\alpha_1\left(\frac{N_1}{2} - S_{1z}\right)c_1^+c_1 + \alpha_2\left(\frac{N_1}{2} - S_{1z}\right)c_2^+c_2 - \beta_1\left(\frac{N_2}{2} - S_{2z}\right)c_1^+c_1 + \beta_2\left(\frac{N_2}{2} - S_{2z}\right)c_2^+c_2 + \gamma_1\left(\frac{N_1}{2} + S_{1z}\right) + \gamma_2\left(\frac{N_2}{2} + S_{2z}\right) + (\lambda_1 S_1^- c_2^+ + \lambda_2 S_2^- c_2^+ + H.\,c.)\Bigg], \quad (3)$$

where

$$\alpha_1 = \frac{g_1^2}{\Delta_{g1} + \nu}, \quad \alpha_2 = \frac{g_1^2}{\Delta_{g1} - \nu}, \quad \beta_1 = \frac{g_2^2}{\Delta_{g2} + \nu}, \quad \beta_2 = \frac{g_2^2}{\Delta_{g2} - \nu},$$

$$\gamma_1 = \frac{\Omega_1^2}{\Delta_1}, \quad \gamma_2 = \frac{\Omega_2^2}{\Delta_2}, \quad \lambda_1 = \frac{g_1\Omega_1}{\Delta_{g1} - \nu}, \quad \lambda_2 = \frac{g_2\Omega_2}{\Delta_{g2} - \nu},$$

$$S_{1z} = \frac{1}{2}\sum_{j=1}^{N_1} (|f_j\rangle_{11} \langle f_j| - |g_j\rangle_{11}\langle g_j|), \quad S_{2z} = \frac{1}{2}\sum_{j=1}^{N_2} (|f_j\rangle_{22}\langle f_j| - |g_j\rangle_{22}\langle g_j|),$$

$$S_1^- = \sum_{j=1}^{N_1} |g_j\rangle_{11}\langle f_j|, \quad S_2^- = \sum_{j=1}^{N_2} |g_j\rangle_{22}\langle f_j|.$$

We have set $\Delta_i = \Delta_{gi} - \nu$. The first four terms of the Hamiltonian correspond to the photon-number-dependent Stark shift for the state $|g\rangle$ induced by normal modes c_1 and c_2. The next two terms correspond to the Stark shift for $|f\rangle$ induced by classical fields. The last two terms describe the coupling between atomic operator S_1^- (S_2^-) and normal mode c_2. After the Holstein–Primakoff transformation [40], the collective atomic operators $(S_1^\pm, S_2^\pm, S_{1z}, S_{2z})$ are associated with the bosonic annihilation operators (creation operators) A and B (A^+ and B^+) via

$$S_1^+ = A^+\sqrt{N_1 - A^+A}, \quad S_{1z} = A^+A - \frac{N_1}{2}$$
$$S_2^+ = B^+\sqrt{N_2 - B^+B}, \quad S_{2z} = B^+B - \frac{N_2}{2}. \quad (4)$$

When the average number of atoms in the state $|f\rangle$ is much smaller than the total number, i.e., A^+A âª¡ N_1 (B^+B âª¡ N_2), the collective atomic operators are well approximated by $S_1^+ \simeq \sqrt{N_1}A^+$, $S_2^+ \simeq \sqrt{N_2}B^+$, $S_{1z} \simeq -N_1/2$ and $S_{2z} \simeq -N_2/2$. The effective Hamiltonian reduces to

$$H_{e2} = -\Big[(N_1\alpha_1 - N_2\beta_1)c_1^+c_1 + (N_1\alpha_2 + N_2\beta_2)c_2^+c_2 + (\sqrt{N_1}\lambda_1 c_2 A^+ + \sqrt{N_2}\lambda_2 c_2 B^+ + H.\,c.)\Big]. \quad (5)$$

Experimentally, we set $N_1 = N_2 = N$, $g_1 = g_2 = g$, $\Delta_1 = \Delta_2 = \Delta$, $\Delta_{g1} = \Delta_{g2} = \Delta_g$, i.e, $N_1\alpha_1 = N_2\beta_1$. Then H_{e2} reduces to

$$H_{e3} = -\Big[\omega c_2^+c_2 + (\Upsilon_1 c_2 A^+ + \Upsilon_2 c_2 B^+ + H.\,c.)\Big], \quad (6)$$

where $\omega = 2Ng^2/\Delta$, $\Upsilon_1 = \sqrt{N}g\Omega_1/\Delta$, $\Upsilon_2 = \sqrt{N}g\Omega_2/\Delta$. In this paper, we set $N = 10^2$, $\nu = 600g$, $\Delta = 100g$, $\Delta_g = 700g$ and have the parameter values $\omega = 2g$, $\Upsilon_1 = 0.1\Omega_1$, $\Upsilon_2 = 0.1\Omega_2$. Suppose the system is initially prepared in the state $|\phi\rangle_1 = |10\rangle_{AB}|0\rangle_{c2}$, where $|10\rangle_{AB}$ denotes the one-excitation and zero-excitation states of the collective atomic modes A and B, and $|0\rangle_{c2}$ is the vacuum state of the mode c_2. The subspace under our consideration includes the other two basis states $|\phi\rangle_2 = |01\rangle_{AB}|0\rangle_{c2}$ and $|\phi\rangle_3 = |00\rangle_{AB}|1\rangle_{c2}$. Then the whole space can be decomposed into subspaces with the following dark state:

$$\psi_0 = K\left(-\frac{\Omega_2}{\Omega_1}|\phi\rangle_1 + |\phi\rangle_2\right), \quad (7)$$

where

$$K = \left(\sqrt{\frac{\Omega_2^2 + \Omega_1^2}{\Omega_1^2}}\right)^{-1}.$$

Now we proceed to show how the maximally entangled state of the atomic modes can be generated via adiabatic passage of the dark state. Based on Eq. (7), if we design the pulse shapes to satisfy the relations

$$\lim_{t\to-\infty}\frac{\Omega_1(t)}{\Omega_2(t)} = 0,$$
$$\lim_{t\to\infty}\frac{\Omega_2(t)}{\Omega_1(t)} = 1, \quad (8)$$

we can then adiabatically transfer the initial state $|\phi\rangle_1$ to $\frac{1}{\sqrt{2}}(|\phi\rangle_1 + |\phi\rangle_2)$. That is, we first prepared the initial state $|\phi\rangle_1 = |10\rangle_{AB}|0\rangle_{c2}$ and the Rabi frequency Ω_1 is initially zero while Ω_2 is larger than zero. Then, reduce Ω_2 and simultaneously increase Ω_1 adiabatically until $\Omega_2/\Omega_1 = 1$. The phase of the classical pulse Ω_1 is then adjusted so that $\Omega_1 \to -\Omega_1$, while the other parameters keep unchanged. As a result, we can obtain the target