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Nonlinear temporal–spatial surface plasmon polaritons

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ABSTRACT

The propagation of temporal-spatial surface plasmon polaritons propagating along a flat dielectric/metal interface is investigated. The governing envelope equation for these surface plasmons is found to be the damped nonlinear Schrodinger (NLS) equation with two spatial-like dimensions and one evolution dimension. Depending on whether the dispersion is anomalous or normal the dispersion of this multidimensional nonlinear NLS equation can be elliptic or hyperbolic. In the elliptic case a localized initial mode is found to focus before damping effects begin to act. In the hyperbolic case the solution is found to be self-similar which also eventually decays due to damping.

1. Introduction

The field of plasmonics is a very active field of optics. There have been many promising advancements in using plasmonics in nanostructured materials incorporating metals. This has included a variety of applications including lasers, sensors and sub-wavelength waveguides [1]. These advances provide exciting prospects for the development of new technologies in telecommunications, computing and information processing [2].

However, it is becoming evident that the study of surface plasmon polaritons (SPPs) should take into account nonlinear effects because plasmon focusing can develop large amplitude wave phenomena. Nonlinear models can explain interesting new effects.

In recent years researchers have begun to incorporate cubic polarization of dielectrics when modeling SPPs propagating along a flat dielectric/metal interface, leading to nonlinear amplitude equations. There are both temporal and spatial waves that can exist. In [3] envelope temporal solitons were found to be able to propagate along interfacial structures composed of two dielectrics or a dielectric and metal. Here the authors employ averaging methods to establish that a one dimensional nonlinear Schrodinger equation governs the wave propagation. On the other hand it was shown in [4] that spatial solitons can also propagate along dielectric and metal interfaces. In the latter paper the authors started with the Helmholtz equation, and using the paraxial approximation, showed that the amplitude of the SPP is a solution of the following one space one evolution $(1+1)$ dimensional nonlinear Schrodinger (NLS) equation

$$iA_z+1/(2\beta)A_{yy}+\nu|A|^2A=0, \tag{1}$$

where β is the wave number in the direction (z) of propagation, and ν is a constant related to the Kerr nonlinearity. In 2009 beginning with the vector wave equation, which is derived from Maxwell's equations, and employing the paraxial approximation the damped $1+1$ dimensional NLS equation

$$-2i\beta DA_z+A_{yy}+2I|A|^2A+i\Gamma A=0 \tag{2}$$

was obtained [5]; here $D=\int E_{x0}^2\,dx/\int|\mathbf{E}_0|^2\,dx$ where $\mathbf{E}_0$ is the solution to the linear problem and $E_{x\,0}$ is its x component, I relates to the cubic nonlinearity and the damping term Γ is due to the complex permittivity of the metal. In [6] the authors considered tapered waveguides, and in [7] femtosecond pulses in the telecommunication spectrum were studied in the context of SPPs. Further studies on nonlinear SPPs have been undertaken in [8], where a cubic Ginzburg–Landau equation was derived, and [9], where general coupled-mode SPP equations in periodic media with loss and gain are derived.

In this paper we extend the analysis and modeling SPPs by performing a multiple scales analysis, and allow pulses to depend on temporal and spatial variations as they propagate along the interface. For the first time we show for a SPP propagating along a flat dielectric/metal interface in the z direction that the slowly varying amplitude of the plasmon is a solution of the following normalized $2+1$ dimensional temporal–spatial damped NLS equation:

$$iC_Z+i\eta C+C_{YY}-\mathrm{sgn}(kk'')C_{TT}+|C|^2C=0, \tag{3}$$

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where Y and T are slow spatial variables, T is retarded time, $k=k(\omega)$ is the dispersion relation ($k' \equiv \partial_\omega k$) and η is related to the damping (see Eq. (43) below). Since in this case the amplitude now depends on time, this allows for a nonzero plasmon group velocity.

This multidimensional $2+1$ dimensional NLS equation exhibits very different phenomena than its $1+1$ dimensional counterpart cf. [17]. For the above Eq. (3), depending on the sign of C_{TT}, the equation can be elliptic or hyperbolic. Both cases lead to interesting mathematical results with corresponding novel physics. In particular, in the elliptic case we show that localized modes can exhibit focusing, arrest and decay, and in the hyperbolic case localized modes tend towards a linear similarity solution for large Z. To our knowledge no one has previously found that self-similar behavior describes the long time structure of the two dimensional hyperbolic NLS equation. The outline of this paper is as follows. In Section 2 we analyze the linear problem, where we neglect the nonlinear polarization of the dielectric and also the imaginary part of the permittivity of the metal which is taken to be much smaller than the real part. The solution of the linear problem forms the basis for solving the nonlinear problem, which is done in Section 3. In Section 4 we then look at the propagation of localized modes for Eq. (3), for both the elliptic and hyperbolic cases.

2. Linear problem

We consider a SPP propagating in the z direction along a flat dielectric/metal interface at $x=0$, as shown in Fig. 1.

The governing equations for the electric field $\mathbf{E}=(E_x,E_y,E_z)$ come from Maxwell's equations, and are

$$\nabla^2\mathbf{E}-\nabla(\nabla\cdot\mathbf{E})-\frac{1}{c^2}\partial_{tt}\mathbf{D}=0 \tag{4}$$

$$\nabla\cdot\mathbf{D}=0, \tag{5}$$

where c is the speed of light in a vacuum. The displacement field $\mathbf{D}$ is given by $\mathbf{D}=\varepsilon\mathbf{E}$, where ε is the relative permittivity. For the linear problem we assume that ε is constant in both materials, and thus $\mathbf{D}$ is a linear function of $\mathbf{E}$. From the physical properties of the materials, in the dielectric we have $\varepsilon_d>0$, and in the metal $\varepsilon_m<0$. We look for a solution of the form

$$\textbf{Dielectric}: \quad \mathbf{E}=(A_d,0,C_d)e^{i\theta-r_dx}+(\star) \tag{6}$$

$$\textbf{Metal}: \quad \mathbf{E}=(A_m,0,C_m)e^{i\theta+r_mx}+(\star), \tag{7}$$

where the phase is given by $\theta=kz-\omega t$, k and ω are the wave number and frequency of the SPP respectively, $r_d>0$ and $r_m>0$ are decay constants which depend on ω (as do $\varepsilon_d,\varepsilon_m$), $(\star)$ denotes the complex conjugate of the preceding term and the amplitudes A_i and C_i are taken to be constant. With this ansatz, Eqs. (4) and (5) give in the dielectric

$$-k^2A_d+ikr_dC_d+\frac{\omega^2}{c^2}\varepsilon_dA_d=0 \tag{8}$$

$$-k^2C_d+ikr_dA_d+\frac{\omega^2}{c^2}\varepsilon_dC_d=0 \tag{9}$$

$$-r_dA_d+ikC_d=0, \tag{10}$$

and similarly in the metal. These give the relations

$$A_d=(ik/r_d)C_d, \quad A_m=-(ik/r_m)C_m, \tag{11}$$

as well as the two equivalent forms of the dispersion relation $k=k(\omega)$

$$k^2=r_d^2+\frac{\omega^2}{c^2}\varepsilon_d, \quad k^2=r_m^2+\frac{\omega^2}{c^2}\varepsilon_m. \tag{12}$$

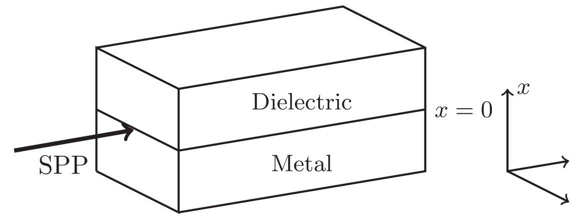

Fig. 1. Setup for propagation of SPPs along flat dielectric/metal interface at $x=0$.

We must also consider continuity across the interface $x=0$ (see e.g. [10]). Physically, the $\mathbf{D}$ field is continuous in the x direction, and the $\mathbf{E}$ field is continuous in the tangential y and z directions. This implies that k and ω are continuous, that $C_d=C_m$, and

$$\varepsilon_dA_d=\varepsilon_mA_m. \tag{13}$$

For simplicity we set $C:=C_d=C_m$, and using (13) in (11) leads to

$$\frac{\varepsilon_d}{r_d}+\frac{\varepsilon_m}{r_m}=0. \tag{14}$$

Using (14) with (12) allows us to eliminate r_d and r_m, obtaining

$$k^2=\frac{\omega^2}{c^2}\left(\frac{\varepsilon_d\varepsilon_m}{\varepsilon_d+\varepsilon_m}\right). \tag{15}$$

Eq. (15) is the SPP linear dispersion relation, and shows that k is related to the free space wavenumber $k_o=\omega/c$ by

$$k=k_o\sqrt{\frac{\varepsilon_d\varepsilon_m}{\varepsilon_d+\varepsilon_m}}. \tag{16}$$

The decay constants are then given by

$$r_d=k_o\sqrt{\frac{-\varepsilon_d^2}{\varepsilon_d+\varepsilon_m}}, \quad r_m=k_o\sqrt{\frac{-\varepsilon_m^2}{\varepsilon_d+\varepsilon_m}}. \tag{17}$$

The linear SPP solution in either the dielectric or metal is therefore given by

$$\textbf{Diel}. \quad \mathbf{E}=(ik/r_d,0,1)Ce^{i(kz-\omega t)-r_dx}+(\star) \tag{18}$$

$$\textbf{Met}. \quad \mathbf{E}=(-ik/r_m,0,1)Ce^{i(kz-\omega t)+r_mx}+(\star), \tag{19}$$

where $k(\omega)$ is given by (15) and the two decay constants $r_d(\omega)$ and $r_m(\omega)$ are given by (17).

3. Nonlinear problem

The main difference between the linear and nonlinear problems is the cubic polarization of the dielectric. Since, as is standard, we assume nonlinear effects to be small, we solve (4) and (5) by performing a multiple scales analysis where we set

$$\mathbf{E}=\epsilon\mathbf{E}^{(1)}+\epsilon^2\mathbf{E}^{(2)}+\epsilon^3\mathbf{E}^{(3)}+\cdots, \tag{20}$$

where $\epsilon\ll 1$, $\mathbf{E}$ depends on suitable fast and slow scales and then each $\mathbf{E}^{(i)}$ is independent of ϵ. Using the linear solution (18) and (19) as the basis for the nonlinear ansatz, we look for a solution in the dielectric of the form

$$E_x=\epsilon\{e^{-r_d(\omega+i\epsilon\partial_T)x}(ik/r_d)Ce^{i\theta}+(\star)\} \\ +\epsilon^2\{e^{-r_d(\omega+i\epsilon\partial_T)x}A_{2d}e^{i\theta}+(\star)\}+O(\epsilon^3) \tag{21}$$

$$E_y=\epsilon^2\{e^{-r_d(\omega+i\epsilon\partial_T)x}B_{1d}e^{i\theta}+(\star)\}+O(\epsilon^3) \tag{22}$$

$$E_z=\epsilon\{e^{-r_d(\omega+i\epsilon\partial_T)x}Ce^{i\theta}+(\star)\} \\ +\epsilon^2\{e^{-r_d(\omega+i\epsilon\partial_T)x}C_2e^{i\theta}+(\star)\}+O(\epsilon^3) \tag{23}$$