# Spatial optical integrator based on phase-shifted Bragg gratings 

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#### Abstract

We present a theoretical study of a new application of a phase-shifted Bragg grating (PSBG) in transmission mode as a spatial optical integrator. The PSBG consists of two identical Bragg gratings separated with a phase-shift defect layer. It is shown that PSBG enables performing the operation of spatial integration of the profile of the 2D incidence beam with a central spatial frequency close to the propagation constant of a quasiguided mode localized in the defect layer. The spatial integration is performed with an exponential weight function, the decay rate of which is determined by the quality factor of the resonance. The rigorous electromagnetic simulations demonstrate good agreement between numerical results and the given theoretical description.


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## 1. Introduction

Spatiotemporal transformations of optical signals are of great interest for a wide range of applications including optical pulse and beam shaping, analog optical computations and ultrafast alloptical data processing [1,2]. One of the promising structures to perform the said transformations is the phase-shifted Bragg grating (PSBG) consisting of two identical Bragg gratings separated with a phase-shift defect layer. Previously, it was demonstrated that a PSBG is able to perform temporal differentiation of an optical pulse envelope in reflection [2-4] and integration of the pulse envelope in transmission [5]. Spatial differentiation of 2D optical beams using PSBG was recently proposed in [6]. In [6], it was shown that a PSBG enables the optical computation of the first and second derivatives of the incident beam profile in reflection. In the present work, it is demonstrated for the first time that PSBGs allow performing the operation of spatial integration of a 2D beam profile in transmission. We believe that the proposed applications of the PSBG could be useful in various real-time all-optical image processing applications. In particular, the PSBG can be considered as an ultra-compact analog of a classic $4 f$ Fourier-transform correlator consisting of a pair of lenses with a spatial integrating filter in the Fourier plane. Another approach for spatial beam processing

[^0]is based on the use of spatial light modulators (SLM). The resolution of the SLM is determined by the size of the pixel, which is typically of the order of several micrometers. At the same time, the proposed structure can perform certain transformations of the beam with total transverse size comparable to the SLM pixel size.

## 2. Beam diffraction on resonant structure

Let us consider oblique incidence of a 2D optical beam on a multilayer structure consisting of homogeneous layers. The beam propagates in the negative direction of the $z$-axis in the coordinate system $(x, z)$ associated with the beam and rotated by an angle $\theta_{0}$ relative to the coordinate system of the multilayer structure ( $x_{\mathrm{ml}}, z_{\mathrm{ml}}$ ) (Fig. 1). In this coordinate system, the plane wave expansion of the incident beam has the following form:
$P_{\text {inc }}(x, z)=\frac{1}{2 \pi} \int G\left(k_{x}\right) \exp \left\{i k_{x} x-\mathrm{i} \sqrt{k_{0}^{2} n_{\text {sup }}^{2}-k_{x}^{2}} \cdot z\right\} \mathrm{d} k_{x}$,
where $G\left(k_{x}\right),\left|k_{x}\right| \leq g$, is the angular (spatial frequency) spectrum of the beam, $k_{0}=2 \pi / \lambda$ is the wave number, $k_{x}=k_{0} n_{\text {sup }} \sin \theta$ and $k_{z}=\sqrt{k_{0}^{2} n_{\text {sup }}^{2}-k_{x}^{2}}$ are the wave vector components of the incident waves, and $n_{\text {sup }}$ is the refractive index of the superstrate. The function $P_{\text {inc }}(x, z)$ corresponds to the $E_{y}$ component of the electric field in case of TE-polarization and to the $H_{y}$ component of the magnetic field in case of TM-polarization.


Fig. 1. Diffraction of an optical beam on a phase-shifted Bragg grating.

The transmitted field can be represented as
$P_{\mathrm{tr}}\left(x_{\mathrm{tr}}, z_{\mathrm{tr}}\right)=\frac{1}{2 \pi} \int G\left(k_{x}\right) T\left(\tilde{k}_{x}\right) \cdot \exp \left\{\mathrm{i} \mathrm{i}_{x} x_{\mathrm{tr}}-\mathrm{i} \sqrt{k_{0}^{2} n_{\text {sub }}^{2}-k_{x}^{2}} \cdot z_{\mathrm{tr}}\right\} \mathrm{d} k_{x}$,
where $T\left(\tilde{k}_{x}\right)$ is the complex transmission coefficient of the multilayer structure, and $\tilde{k}_{x}=k_{0} n_{\text {sup }} \sin \left(\theta+\theta_{0}\right)=k_{x} \cos \theta_{0}+\sqrt{k_{0}^{2} n_{\text {sup }}^{2}-k_{x}^{2}}$ $\sin \theta_{0}$ corresponds to the $x_{\mathrm{ml}}$ - component [in the coordinate system $\left.\left(x_{\mathrm{ml}}, z_{\mathrm{ml}}\right)\right]$ of the wave vector of a plane wave incident on the structure at an angle $\theta+\theta_{0}$. Let us note that Eq. (2) is written in the coordinate system associated with the transmitted beam (Fig. 1). Here we assume that the origins of the coordinate systems $(x, z)$ and ( $x_{\mathrm{tr}}, z_{\mathrm{tr}}$ ) coincide with that of the coordinate system $\left(x_{\mathrm{ml}}, z_{\mathrm{ml}}\right)$.

Assuming that the spectrum of the incident beam is sufficiently narrow ( $g \ll k_{0} n_{\text {sup }}$ ), we obtain
$\tilde{k}_{x} \approx k_{x} \cos \theta_{0}+k_{0} n_{\text {sup }} \sin \theta_{0}=k_{x} \cos \theta_{0}+k_{x, 0}$,
where $k_{x, 0}=k_{0} n_{\text {sup }} \sin \theta_{0}$ is the central spatial frequency of the incident beam. Let us consider the relation between the incident beam profile $P_{\mathrm{inc}}(x, 0)$ and the transmitted beam profile $P_{\mathrm{tr}}\left(x_{\mathrm{tr}}, 0\right)$. It follows from Eqs. (1) and (2) that the transformation of the incident beam profile can be described in terms of the signal transmission through a linear time-invariant system with the transfer function (TF) defined by the following expression [6]:
$H_{\mathrm{tr}}\left(k_{x}\right)=T\left(k_{x} \cos \theta_{0}+k_{x, 0}\right)$.
Let us show that a PSBG can be used for the spatial integration of the optical beams in transmission. Consider a PSBG consisting of two symmetric Bragg gratings separated by a defect layer. In the simplest case, one period of a Bragg grating contains two layers having equal optical thickness:
$\tilde{n}_{1} h_{1}=\tilde{n}_{2} h_{2}=\lambda_{B} / 4$,
where $\tilde{n}_{i}=\sqrt{n_{i}^{2}-\left(n_{\text {sup }} \sin \theta_{0}\right)^{2}}, i=1,2 ; n_{i}, h_{i}$ are the refractive indices and the thicknesses of the layers, and $\lambda_{B}$ is the Bragg wavelength. If the defect layer has the optical thickness $\tilde{n}_{\text {def }} h_{\text {def }}=\lambda_{B} / 2$, where $\tilde{n}_{\text {def }}=\sqrt{n_{\text {def }}^{2}-\left(n_{\text {sup }} \sin \theta_{0}\right)^{2}}, n_{\text {def }}$ being the refractive index of the defect layer, the reflection coefficient of the

Bragg grating vanishes at wavelength $\lambda_{B}$ and angle of incidence $\theta_{0}$ for both TE- and TM-polarizations $[3,5,6]$. Let us note that this reflectance zero is located at the center of the first photonic band gap of the Bragg grating.

The appearance of the reflection zero is associated with the excitation of a quasiguided mode localized in the defect layer. In the vicinity of the resonance, the transmission coefficient can be approximately represented in the following form [6-9]:
$T\left(\tilde{k}_{x}\right) \approx a+\frac{b}{\tilde{k}_{x}-k_{x, p}}-\frac{b}{\tilde{k}_{x}+k_{x, p}}$,
where $a$ is the non-resonant transmission coefficient, $b$ is the coefficient describing the resonant light scattering by the structure, and $k_{x, p}$ is the complex propagation constant of the eigenmode of the PSBG corresponding to the pole of the function $T\left(\tilde{k}_{x}\right)$. Let us note that the transmission coefficient (6) is an even function with respect to the incidence angle and therefore contains two resonant terms corresponding to the modes with the propagation constants $\pm k_{x, p}$ (these modes are excited at $\tilde{k}_{x}=k_{x, 0}= \pm \operatorname{Re} k_{x, p}$ ). We further assume that the PSBG has a sufficiently large number of layers so that the non-resonant transmission coefficient $a$ in Eq. (6) can be neglected.

## 3. Beam integration

### 3.1. Oblique incidence

Let us first study the diffraction of the obliquely incident beam. We consider the case when the central spatial frequency of the incident beam $k_{x, 0}$ is large enough, so the influence of the second pole $\left(-k_{x, p}\right)$ on the spectrum is negligible. In this case, we obtain
$T\left(\tilde{k}_{x}\right) \approx \frac{b}{\tilde{k}_{x}-k_{x, p}}$.
Thus, the TF defined by Eq. (4) with the transmission coefficient in Eq. (7) takes the form
$H\left(k_{x}\right)=\frac{b}{k_{x} \cos \theta_{0}-\left(k_{x, p}-k_{x, 0}\right)}$,
where $|b|=\left|k_{x, 0}-k_{x, p}\right|$. The given value of $|b|$ could be easily derived from the condition $\left|T\left(k_{x, 0}\right)\right|=1$.

Let us examine the transformation of the input signal that is performed by a linear system with the TF of Eq. (8). For this, we write the impulse response of the system by calculating the inverse Fourier transform of the TF in Eq. (8) as

$$
\begin{align*}
h(x) & =\frac{1}{2 \pi} \int_{-\infty}^{\infty} H\left(k_{x}\right) \exp \left\{\mathrm{i} k_{x} x\right\} \mathrm{d} k_{x} \\
& =\operatorname{sgn}\left(\operatorname{Im} k_{x, p}\right) \frac{\mathrm{i} b}{\cos \theta_{0}} \exp \left\{\mathrm{i} \frac{k_{x, p}-k_{x, 0}}{\cos \theta_{0}} x\right\} \cdot u\left(\operatorname{sgn}\left(\operatorname{Im} k_{x, p}\right) \cdot x\right), \tag{9}
\end{align*}
$$

where $\operatorname{sgn} x$ is the sign function and $u(x)$ is the Heaviside step function. The integral in Eq. (9) was calculated using Cauchy's residue theorem and Jordan's lemma. According to Eq. (9), the impulse response is nonzero at $x>0$ or at $x<0$ depending on the position of the pole $k_{x, p}$ in the upper or lower half-plane, respectively.

Using Eq. (9), let us represent the profile of the transmitted beam in the form of an integral with variable upper limit of the

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