Optics Communications 338 (2015) 40–44

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Propagation properties of the pulsed hollow Gaussian beam through a circular aperture

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ARTICLE INFO

Article history:
Received 24 July 2014
Received in revised form
5 October 2014
Accepted 12 October 2014
Available online 22 October 2014

Keywords:
Pulsed hollow Gaussian beam
Diffraction effect
Spatiotemporal characteristics

ABSTRACT

Based on the angular spectrum representation of the Maxwell equations and the method of stationary phase, this paper presents the far-field analytical expression of a pulsed hollow Gaussian beam with constant waist width diffracted by the circular aperture, and the result can be simplified for the case of the paraxial propagation of pulsed Gaussian beam in the free space. Based on the analytical result, the influences of truncation parameter on the transverse intensity distribution of the pulsed beam are analyzed. Comparisons of normalized temporal intensity between the pulsed Gaussian beam and the fourth order pulsed hollow Gaussian beam are presented. We find that the spatial mode can induce the temporal shape changing.

1. Introduction

Diffraction is one of the fundamental problems of the propagation of light. The diffraction theory of monochromatic light has been developed in great detail. Recent developments in optics resulted in the generation of ultrashort pulses with extremely broad spectra. Significant efforts have been devoted to investigate their propagation properties. However, most of the studies of pulsed beam propagation have considered only the fundamental Gaussian mode. Two different assumptions are typically made in these studies, i.e., diffraction length is assumed to be constant (constant diffraction length) [1–4], or the waist width is assumed to be constant (constant waist width) [5–8]. Because of the mathematical difficulty in dealing with the propagation of ultrashort pulsed beams with constant waist width, usually numerical results or approximate propagation expressions are found in the past studies. Beyond these, there are few studies focused on the diffraction of pulsed beam from the hard edges [9–11].

In 2003, a convenient theoretical model named hollow Gaussian beam (HGB) was introduced to describe dark-hollow beams [12]. After that, considerable attention has been focused on investigating hollow Gaussian beams [13–19]. Xu et al. [20] introduced a model of isodiffracting hollow Gaussian pulsed beams and discussed the influence of beam order on on-axis temporal profiles.

In this paper, we concentrate on the propagation of pulsed hollow Gaussian beam with constant waist width through a hard aperture. In Section 2, based on the angular spectrum representation and the method of stationary phase, an analytical propagation expression in the far-field is derived without any approximation. Numerical calculation results and comparative investigations are provided in Section 3. Finally, the results obtained in this paper are summarized in Section 4.

2. Analytical expression of pulsed HGB in the far-field

The monochromatic components of the pulse electromagnetic field in vacuum satisfy the Helmholtz equation

$$(\nabla^2 + k^2)E(\mathbf{r}, \omega) = 0, \tag{1}$$

where ∇^2 is the Laplace operator; $k = \omega/c$ is the wave number with c being the light speed in vacuum; $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are the unit vectors in the x-, y- and z-directions, respectively; $E(\mathbf{r}, \omega)$ is the Fourier transform of optical field $E(\mathbf{r}, t)$. Thus, $E(\mathbf{r}, t)$ can be obtained from $E(\mathbf{r}, \omega)$

$$E(\mathbf{r}, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(\mathbf{r}, \omega)\exp(i\omega t)d\omega. \tag{2}$$

From the angular spectrum theory, $E(\mathbf{r}, \omega)$ can be expressed as

$$E(\mathbf{r}, \omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(k_x, k_y, \omega)\exp[-i(k_x x + k_y y + \sqrt{k^2 - k_x^2 - k_y^2} z)]dk_x\, dk_y, \tag{3}$$

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http://dx.doi.org/10.1016/j.optcom.2014.10.026
0030-4018/

where $A(k_x, k_y, \omega)$ is the angular spectrum. It can be obtained from the initial optical field $E(\mathbf{r}_0, \omega)$:

$$A(k_x, k_y, \omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E(r_0, \omega)\exp(ik_x x_0 + ik_y y_0)dx_0\, dy_0. \tag{4}$$

In the Cartesian coordinate system, a pulsed HGB propagates toward the half-space $z > 0$. The z-axis is adopted to be the propagation axis. A circular aperture with radius R is assumed to be located at the source plane $z = 0$. The pulsed HGB just behind the circular aperture is expressed as

$$E(\mathbf{r}_0, \omega) = \left(\frac{\rho_0^2}{W_0^2}\right)^n \exp\left(-\frac{\rho_0^2}{W_0^2}\right) circ\left(\frac{\rho_0}{R}\right) f(\omega), \tag{5}$$

where $\rho_0 = (x_0^2 + y_0^2)^{1/2}$, n is the beam order of HGB, W_0 is the beam waist radius of the beam which is independent of frequency, $f(\omega)$ is the spectrum of the initial pulse, $circ(\cdot)$ denotes the aperture function and is given by

$$circ(\rho_0/R) = 1 \quad \text{for} \quad 0 < \rho_0 < R; \quad circ(\rho_0/R) = 0 \quad \text{for} \quad \rho_0 > R. \tag{6}$$

To obtain the analytical expression of the field, the aperture function can be expanded into the sum of complex Gaussian functions:

$$circ\left(\frac{\rho_0}{R}\right) = \sum_{m=1}^{M} B_m \exp\left(-\frac{C_m \rho_0^2}{R^2}\right), \tag{7}$$

where the expansion coefficients B_m and C_m can be determined by optimization computation. Wen et al. [21] obtained a set of coefficients, containing only ten terms of coefficients, to match the original aperture function of Eq. (6). It shows that a good agreement between a ten-term Gaussian beam solution and the results of numerical integration throughout the beam field was achieved, and the discrepancies existed only in the extreme near field. In the following calculations, we will take $M = 10$. Let us consider the propagation of an electromagnetic wave with Gaussian temporal modulation at $z = 0$

$$f(t) = \exp\left(-\frac{t^2}{T^2} + i\omega_0 t\right), \tag{8}$$

so

$$f(\omega) = \frac{T}{\sqrt{2}}\exp\left[-\frac{T^2(\omega - \omega_0)^2}{4}\right], \tag{9}$$

where ω_0 is carrier frequency, $2T$ is the width of pulse (full width at e^{-1} of the maximum intensity). $T_0 = 2\pi/\omega_0$ is an optical cycle corresponding to ω_0.

Inserting Eq. (5) into Eq. (4) and performing the integral, we can obtain

$$A(k_x, k_y, \omega) = \sum_{m}^{M} \frac{W_0^2 T n!}{2\sqrt{2}} \frac{B_m}{(1 + C_m/C_0)^{n+1}} \exp\left[-\frac{(k_x^2 + k_y^2)W_0^2}{4(1 + C_m/C_0)}\right] \times L_n\left[\frac{(k_x^2 + k_y^2)W_0^2}{4(1 + C_m/C_0)}\right] \exp\left[-\frac{T^2(\omega - \omega_0)^2}{4}\right], \tag{10}$$

where $C_0 = R^2/W_0^2$ is the truncation parameter, $L_n(\cdot)$ denotes Laguerre polynomial with the order n. In the far field, the condition $kr \to \infty$ is fulfilled. Inserting Eq. (10) into Eq. (3) and performing the integral by the method of stationary phase, it can be obtained

$$E(\mathbf{r}, \omega) = \sum_{m}^{M} \frac{ikW_0^2 Tzn!}{2\sqrt{2}r^2} \frac{B_m}{(1 + C_m/C_0)^{n+1}} e^{-ikr} \exp\left[-\frac{(x^2 + y^2)k^2 W_0^2}{4r^2(1 + C_m/C_0)}\right] \times L_n\left[\frac{(x^2 + y^2)k^2 W_0^2}{4r^2(1 + C_m/C_0)}\right] \exp\left[-\frac{T^2(\omega - \omega_0)^2}{4}\right], \tag{11}$$

where $r = (x^2 + y^2 + z^2)^{1/2}$. In order to get the analytical expression of $E(\mathbf{r}, t)$, we expand the Laguerre polynomial as follows:

$$L_n\left[\frac{(x^2 + y^2)k^2 W_0^2}{4r^2(1 + C_m/C_0)}\right] = \sum_{L=0}^{n} (-1)^L \left[\frac{(x^2 + y^2)k^2 W_0^2}{4r^2(1 + C_m/C_0)}\right]^L \frac{n!}{(L!)^2 (n - L)!}. \tag{12}$$

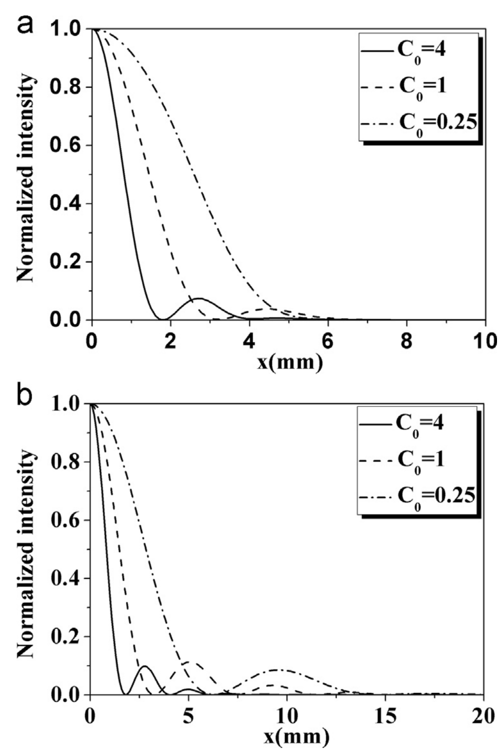

Fig. 1. Normalized transverse intensity distributions of pulsed HGB at $z = 15$ m. (a) $n = 3$, $T = 2T_0$; (b) $n = 3$, $T = 20T_0$.

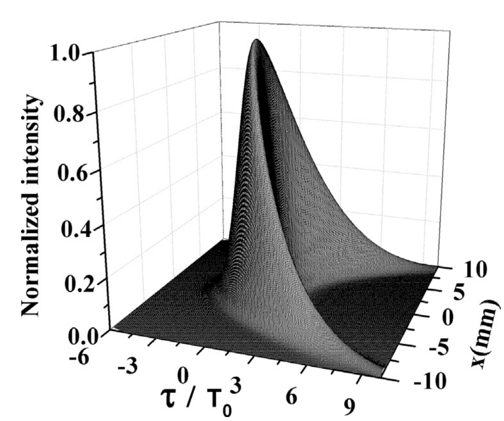

Fig. 2. Normalized space-time intensity distribution. The parameters are: $z = 10$ m, $T = 2T_0$, $n = 3$, $C_0 = 0.25$.