Optics Communications 338 (2015) 534–539

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Gouy phase and phase singularities of tightly focused, circularly polarized vortex beams

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ARTICLE INFO

Article history:
Received 10 August 2014
Received in revised form
29 October 2014
Accepted 5 November 2014
Available online 13 November 2014

Keywords:
Gouy phase
Phase anomaly
Phase singularity
Focusing
Circular polarization
Vortex beam

ABSTRACT

An investigation is made of the phase properties, especially the Gouy phase of the tightly focused, circularly polarized vortex beams. First two groups of symmetry relations of the focused field are derived, from which the effect of the topological charge on the field can be found easily. By decomposing the electric field into three specific components, the corresponding Gouy phases are defined and their properties are examined in detail. Our result shows that not only the polarization of the incident field or the numerical aperture influence the phase behavior, but also the topological charge gives much contribution to the phase structure near the focus.

1. Introduction

The tightly focused light, for its wide applications, such as in microscopy [1], in optical tweezers [2], has drawn substantial interest recently [3–10]. It was originally studied by Richards and Wolf [11] using a linearly polarized light as an incident field and is well-known that the depolarization phenomenon occurs in such systems. When tightly focusing a radially polarized beam, the strong longitudinal component and small spot will be found [12,13]. Carrying both spin angular momentum (SAM) and orbital angular momentum (OAM), when a circularly polarized vortex beam goes through a high numerical aperture (NA) lens, it can induce the interconversion between the SAM and the OAM [14] and has attracted attention in recent years [15–20].

The Gouy phase, which describes how the phase of a monochromatic, focused field differs from that of a non-diffracted wave with the same frequency, is ubiquitous in any focusing field (see [21,22] for more information) and is of great importance in many physical systems and applications, such as in optical coherence tomography [23], in Terahertz time-domain spectroscopy [24] and in optical calibration [25]. For a high NA system, it has been found that the Gouy phases of the three Cartesian components of the linearly polarized field exhibit different behaviors [8], and the rotation of the polarization state of the radially polarized field in the focal region is a manifestation of the different Gouy phases that the two (longitudinal and radial) electric field components undergo [10]. However, no detailed studies seem to have been made on the Gouy phase of a tightly focused beam carrying both SAM and OAM. So in this paper, we define and examine the Gouy phase of a tightly focused, circularly polarized vortex beam and discuss the variation of this phase anomaly on the influence of the NA angle and the topological charge. From our deduced expressions for the symmetry properties of the focused field and the plots of the Gouy phase, it is easy to find the relations between the phase structure of the field and the SAM (carried by the circular polarization) as well as OAM (carried by the vortical phase) of the beam. This investigation will provide insights in the fundamental properties of the optical focused field and help our understanding of the inter-transfer between SAM and OAM. Our result may have implications for optical trapping [26] and singular microscopy [27].

2. Focused, circularly polarized vortex field

Consider a circularly polarized vortex beam,

$$\mathbf{E}_{l/r} = P(\mathbf{r})(\mathbf{e}_x \pm \mathrm{i}\mathbf{e}_y)e^{\mathrm{i}m\phi}, \tag{1}$$

with m being the topological charge and ϕ being the azimuthal angle. $P(\mathbf{r})$ is the axially symmetric amplitude distribution, here we choose $P(\mathbf{r}) = 1$. $\pm$ denotes left-handed circular polarization (LCP) and right-handed circular polarization (RCP) respectively. A

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http://dx.doi.org/10.1016/j.optcom.2014.11.026
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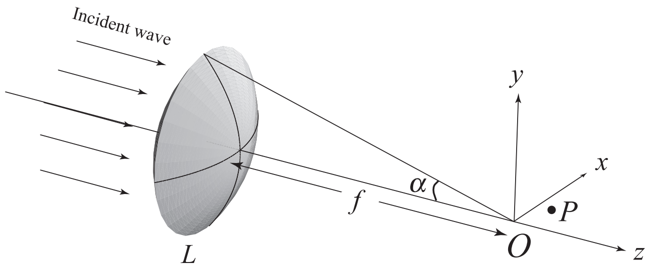

Fig. 1. Illustration of a high-numerical-aperture system. The origin O of a Cartesian coordinate system is taken at the geometrical focus.

circularly polarized beam can be generated by the superposition of two orthogonal linearly polarized beams with a retardation of $\pi/2$ between them, then a circularly polarized vortex beam can be formed by adding a spiral phase wavefront on a circularly polarized beam.

Assume an aplanatic, high numerical aperture focusing system with semi-aperture angular α, see Fig. 1. When a circularly polarized vortex beam (from now on, we only consider the incident wave of LCP) incident upon this system, the electric field in the focal region at the observation point P can be expressed using the Richards–Wolf vectorial diffraction model [11] as

$$E(u, v, \phi) = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix} = \mathrm{i}^m A \begin{bmatrix} -\mathrm{i}(I_m + I_{m+2}e^{\mathrm{i}2\phi}) \\ I_m - I_{m+2}e^{\mathrm{i}2\phi} \\ -2I_{m+1}e^{\mathrm{i}\phi} \end{bmatrix} e^{\mathrm{i}m\phi}, \tag{2}$$

here A is a constant relating to the intensity of the beam and

$$I_m(u, v) = \int_0^\alpha \cos^{1/2}\theta \sin^2\theta(1 + \cos\theta) J_m\left(\frac{v\sin\theta}{\sin\alpha}\right) e^{\mathrm{i}u\cos\theta/\sin^2\alpha}\, \mathrm{d}\theta, \tag{3}$$

$$I_{m+1}(u, v) = \int_0^\alpha \cos^{1/2}\theta \sin^2\theta J_{m+1}\left(\frac{v\sin\theta}{\sin\alpha}\right) e^{\mathrm{i}u\cos\theta/\sin^2\alpha}\, \mathrm{d}\theta, \tag{4}$$

$$I_{m+2}(u, v) = \int_0^\alpha \cos^{1/2}\theta \sin^2\theta(1 - \cos\theta) J_{m+2}\left(\frac{v\sin\theta}{\sin\alpha}\right) e^{\mathrm{i}u\cos\theta/\sin^2\alpha}\, \mathrm{d}\theta, \tag{5}$$

with J_i being the Bessel function of the first kind of order i and u, v being dimensionless Lommel variables [28], namely

$$u = kz\sin^2\alpha, \tag{6}$$

$$v = k\rho\sin\alpha, \tag{7}$$

where $k = \omega/c$, with c being the speed of light in vacuum, is the wavenumber associated with frequency ω. Using the fact

$$I_n(-u, v) = I_n^*(u, v) \quad (n = m, m + 1, m + 2) \tag{8}$$

where the asterisk denotes the complex conjugate, we can get the following relations which exist between the components of the field vectors at points $P_1(u, v, \phi)$ and $P_2(-u, v, -\phi)$, as

$$e_x(-u, v, -\phi) = (-1)^{m+1} e_x^*(u, v, \phi), \tag{9}$$

$$e_y(-u, v, -\phi) = (-1)^m e_y^*(u, v, \phi), \tag{10}$$

$$e_z(-u, v, -\phi) = (-1)^m e_z^*(u, v, \phi), \tag{11}$$

then it can be obtained that

$$|e_x(-u, v, -\phi)| = |e_x(u, v, \phi)|, \tag{12}$$

$$|e_y(-u, v, -\phi)| = |e_y(u, v, \phi)|, \tag{13}$$

$$|e_z(-u, v, -\phi)| = |e_z(u, v, \phi)|, \tag{14}$$

and

$$\psi_x(-u, v, -\phi) = (m + 1)\pi - \psi_x(u, v, \phi), \tag{15}$$

$$\psi_y(-u, v, -\phi) = m\pi - \psi_y(u, v, \phi), \tag{16}$$

$$\psi_z(-u, v, -\phi) = m\pi - \psi_z(u, v, \phi), \tag{17}$$

here ψ_j ($j = x, y, z$) denotes the phase of e_j. It should be noted that two sides of equations [Eqs. (15)–(17)] for the phase of the field are indeterminate to the extent of an additive constant $2N\pi$ (N is any integer).

In order to easily find the polarization difference between the incident field and the focused field, it is convenient to decompose the focused field into another three components: a component with the same polarization (here it is LCP) and topological charge as the incident field, a component with the opposite polarization (RCP) and a topological charge of (m+2), and a longitudinal (z-axis) polarized part with a topological charge of (m+1) [16] as

$$E(u, v, \phi) = (e_l, e_r, e_z) = -\mathrm{i}^{m+1}A \times \left(\begin{bmatrix} 1 \\ \mathrm{i} \\ 0 \end{bmatrix} I_m e^{\mathrm{i}m\phi} + \begin{bmatrix} 1 \\ -\mathrm{i} \\ 0 \end{bmatrix} I_{m+2} e^{\mathrm{i}(m+2)\phi} - 2\mathrm{i}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} I_{m+1} e^{\mathrm{i}(m+1)\phi}\right). \tag{18}$$

The relations also hold between the components of the field vectors at points $P_1(u, v, \phi)$ and $P_3(-u, v, \phi)$, which are symmetrically situated with respect to the focal plane, can be derived as

$$e_l(-u, v, \phi) = (-1)^{m+1} e_l^*(u, v, \phi) e^{\mathrm{i}2m\phi}, \tag{19}$$

$$e_r(-u, v, \phi) = (-1)^{m+1} e_r^*(u, v, \phi) e^{\mathrm{i}2m(\phi+2)}, \tag{20}$$

$$e_z(-u, v, -\phi) = (-1)^m e_z^*(u, v, \phi) e^{\mathrm{i}2m(\phi+1)}, \tag{21}$$

then we can get

$$|e_l(-u, v, \phi)| = |e_l^*(u, v, \phi)|, \tag{22}$$

$$|e_r(-u, v, \phi)| = |e_r^*(u, v, \phi)|, \tag{23}$$

$$|e_z(-u, v, \phi)| = |e_z^*(u, v, \phi)|, \tag{24}$$

and

$$\psi_l(-u, v, \phi) = (m + 1)\pi + 2m\phi - \psi_l(u, v, \phi), \tag{25}$$

$$\psi_r(-u, v, \phi) = (m + 1)\pi + 2(m + 2)\phi - \psi_r(u, v, \phi), \tag{26}$$

$$\psi_z(-u, v, \phi) = m\pi + 2(m + 1)\phi - \psi_z(u, v, \phi). \tag{27}$$

For m=0, this is the case of a circularly polarized field (without topological charge) and Eqs. (19)–(21) degenerate into

$$e_l(-u, v, \phi) = -e_l^*(u, v, \phi), \tag{28}$$

$$e_r(-u, v, \phi) = -e_r^*(u, v, \phi), \tag{29}$$

$$e_z(-u, v, -\phi) = e_z^*(u, v, \phi), \tag{30}$$

which is exactly the symmetric form for the three components (e_x, e_y, e_z) of a linearly polarized focused field [11]. From here on we will concentrate on the phase behaviors of e_l, e_r and e_z components.