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Large phase shift of (1+1)-dimensional nonlocal spatial solitons in lead glass

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The large phase shift of strongly nonlocal spatial optical soliton (SNSOS) in the (1+1)-dimensional [(1+1)D] lead glass is investigated using the perturbation method. The fundamental soliton solution of the nonlocal nonlinear Schödinger equation (NNLSE) under the 2nd order approximation in strongly nonlocal case is obtained. It is found that the phase shift rate along the propagation direction of such soliton is proportional to the degree of nonlocality, which indicates that one can realize π-phase-shift within one Rayleigh distance in (1+1)D lead glass. A full comprehension of the nonlocality enhancement to the phase shift rate of SNSOS is reached via quantitative comparisons of phase shift rates in different nonlocal systems. This can help us to conclude that, compared with SNSOSs in other nonlocal systems, SNSOS in (1+1)D lead glass is a most promising candidate which can experience large phase shift within the shortest propagation distance.

1. Introduction

Nonlocal spatial solitons have been the subject of intensive experimental and theoretical work [1–5] since the pioneering work done by Snyder and Mitchell [6]. The most prominent innovation in their work is that they transform the complex nonlocal nonlinear Schödinger equation (NNLSE) into a simple case of linear propagation of light in a quadratic self-induced index well [6]. Nonlocal nonlinearity is typically the result of certain transport processes, such as the charge drift in photorefractive crystals [7] and the heat transfer in thermal nonlinear media [3] or, long-range interaction, such as the molecular reorientations in liquid crystals [8]. There is also a huge class of parametric nonlinearities involving nonlocality, such as quadratic nonlinear materials [9], which has, e.g., enabled the prediction of good regimes for quadratic soliton pulse compression [10,11]. Another important class of materials, called liquid infiltrated photonic crystal fibers, displays an engineerable spatial nonlocality. They allow to engineer the dispersion, the nonlinearity, and the nonlocality [12] and thereby enabled the first observation of a (2+1)D nonlocal gap soliton [13]. Due to the nature of the nonlocality, solitons in nonlocal nonlinear media exhibit several distinct properties that are not possible in local settings. This includes, on one hand, resulting from the spatial 'averaging' character of the nonlocality, the arrest of catastrophic collapse [4], the ability to support the formation of complex optical spatial solitons, such as higher-order solitons [14,15] and vortex solitons [3,16]. On the other hand, out-of-phase solitons attraction between bright [17,18] and dark solitons [19,20], long-range interactions between solitons [5,21] as well as the solitons and the boundaries [22–25] in strongly nonlocal media have also been carried out or predicted due to the fact that the interactions are mediated by the light-induced refractive index which is 'enlarged' by the nonlocal response.

Except for the 'averaging' and the 'enlarging' features of the nonlocality, there exists an 'enhancing' effect of the nonlocality on the phase shift of the SNSOS. Although very large in fact, the phase shift of SNSOS is considered a trivial term, for a long time, and is neglected by the Snyder–Mitchell (SM) model [6]. The first work focused on the phase shift of SNSOS was done by Guo et al. [26,27]. They predicted a large phase shift rate of SNSOS, which is ρ^2 times (ρ is the degree of nonlocality defined as $\rho = w_m/\mu$ where w_m is the characteristic length of the response function and μ is the beam width), explicitly 100 times for the lower limit of the strongly nonlocality, larger than that of the local counterpart. Guo's conclusion results from a strongly nonlocal (SN) model in which the large phase shift is included having a dominating term proportional to the soliton critical power.

SN model can rigorously transform to SM model with a function transformation involving large phase shift term [28]. Both of them are derived from a phenomenological and regular (or at least twice-differentiable at $\mathbf{r}=0$) response function $R(\mathbf{r})$. In the nematic liquid crystal (NLC) and lead glass (LG), the two media in which SNSOSs can form, the response functions are singular at every source point (irregular) and therefore one cannot obtain accurate

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solution of NNLSE based on SN model and SM model even in the strongly nonlocal case [28]. Ouyang et al. took the higher order (the fourth and the sixth) terms of the light induced refractive index as the perturbation to the quadratic index well and obtained considerably accurate analytical soliton solutions in (1+1)D [29] and (1+2)D [30] NLC. The perturbation solution are different from Gaussian-type solution given by SN model [28], but still indicated nonlocality-enhanced large phase shifts of SNSOSs. The first theoretical and experimental study focused on the SNSOS phase shift was carried out in (1+2)D cylindrical LG by Shou et al. [31]. They retained the terms of the Taylor expansion of the light-induced refractive index up to the 2nd order whose coefficient is the on-axis light intensity. The phase shift rate in (1+2)D LG was predicted to be much smaller than the result based on SN model, but is still more than one order larger than that in the local media. More meaningfully, Shou et al. observed a linear modulation of the soliton power on the phase shift of the SNSOS [31], which coincides with Guo's prediction, indicating that the nonlocality enhancement to the phase shift of SNSOS stems from the fact that the light-induced refractive index, which directly contributes to the phase shift, is induced not by the light intensity but by the power of the whole beam.

In this paper, we investigate the phase shift of SNSOS in (1+1)D LG in the formalism of perturbation theory. The perturbation solution of the fundamental soliton is obtained under the 2nd order approximation. The result indicates that the phase shift of SNSOS in (1+1)D LG is proportional to the degree of nonlocality which is at least one order larger than the result for the local solitons. It will also be shown how the degree of nonlocality affects, or explicitly speaking, enhances the phase shift rate in different nonlocal systems.

2. The fundamental strongly nonlocal soliton solution under the 2nd order approximation

We consider a (1+1)D LG with thermal nonlocal nonlinear response occupying the region $-L \le x \le L$. The propagation behavior of a light beam u propagating along the z axis is governed by the NNLSE, coupled to the Poisson equation describing the light-induced nonlinear refractive index variation N:

$$i\frac{\partial u}{\partial z} + \frac{1}{2}\frac{\partial^2 u}{\partial x^2} + Nu = 0, \tag{1}$$

$$\frac{d^2 N}{dx^2} = -|u|^2. \tag{2}$$

The nonlocal response function in (1+1)D LG under the first-kind boundary condition $N(\pm L) = 0$ can be given as [32]

$$G(x,\xi) = \begin{cases} \dfrac{(x+L)(\xi-L)}{2L}, & (x \le \xi) \\ \dfrac{(\xi+L)(x-L)}{2L}, & (\xi \le x) \end{cases} \tag{3}$$

where x and ξ are, respectively, the field point and the source point. Green's function $G(x, \xi)$ can be normalized through an integral over x, which is $(\xi - L)(\xi + L)/2$. ξ generally takes the value around the center of the cross section of the glass but cannot take the value of $\pm L$. This is required by the boundary condition of the refractive index N making the integral nonzero. According to the Green function method, the nonlinear refractive index in LG can be written in the form of

$$N(x) = -\int_{-L}^{L} G(x,\xi)|u(\xi,z)|^2\, d\xi. \tag{4}$$

It is obvious that the response function in Eq. (3) is not differentiable at the source point $x = \xi$ (irregular) and therefore cannot be dealt with SN model [26]. We use the perturbation method, previously extended to solve the NNLSE by Ouyang et al. [29,30], to calculate the fundamental soliton solution of the NNLSE. For the soliton state $u(x, z)$, we have $|u(-x, z)|^2 = |u(x, z)|^2$ and $u(x, z) = u(x, 0)$. On the analogy of the potential in quantum mechanics which determines the state of the particle movement, we define the nonlinearity-induced trapping 'potential', explicitly the light-induced refractive index, which can determine the beam propagation behavior:

$$V(x) = -N(x) = \int_{-L}^{L} G(x,\xi)|u(\xi,z)|^2\, d\xi. \tag{5}$$

The minus sign indicates that a refractive index distribution with a maximum in its center corresponds to a light induced potential distribution with a minimum in its center. Then Eq. (1) can be reduced to

$$i\frac{\partial u}{\partial z} + \frac{1}{2}\frac{\partial^2 u}{\partial x^2} - V(x)u = 0. \tag{6}$$

Taking Taylor's expansion of $V(x)$ at $x=0$, we obtain

$$V(x) = V_0 + \frac{1}{2\mu^4}x^2 + \alpha x^4 + \beta x^6 + \cdots, \tag{7}$$

where

$$V_0 = V(0), \tag{8a}$$

$$\frac{1}{\mu^4} = V^{(2)}(0), \tag{8b}$$

$$\alpha = \frac{1}{4!}V^{(4)}(0), \tag{8c}$$

$$\beta = \frac{1}{6!}V^{(6)}(0). \tag{8d}$$

For Taylor's expansion to be valid, the variable x in Taylor's expansion of $V(x)$ must take the value in proximity of the light field ($\sim \mu$) where $V(x)$ contributes to the field phase shift. Consequently the terms αx^4 and βx^6 are, respectively, one and two orders of magnitude smaller than the term $x^2/(2\mu^4)$ [29] and then can be viewed as the perturbations. By substituting Eq. (7) into Eq. (6) and neglecting the higher-order terms, we obtain

$$i\frac{\partial u}{\partial z} = \left[-\frac{1}{2}\frac{\partial^2}{\partial x^2} + V_0 + \frac{1}{2\mu^4}x^2 + \alpha x^4 + \beta x^6\right]u. \tag{9}$$

Taking a transformation

$$u(x,z) = \phi_n(x)\exp[-i(\varepsilon_n + V_0)z], \tag{10}$$

we arrive at

$$\left[-\frac{1}{2}\frac{d^2}{dx^2} + \frac{1}{2\mu^4}x^2 + \alpha x^4 + \beta x^6\right]\phi_n = \varepsilon_n\phi_n. \tag{11}$$

here $n = 0, 1, 2, \ldots$ is the order of the soliton solution, in particular $n=0$ corresponding to the fundamental soliton solution and $n=1$ corresponding to the 2nd order soliton solution and so on. If $\alpha=0$ and $\beta=0$, Eq. (11) reduces to the well-known stationary Schrödinger equation for a harmonic oscillator. Following the