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Optical soliton control in inhomogeneous nonlinear media with the parity-time symmetric potentials

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ABSTRACT

In this paper, in the framework of (1+1)-dimensional inhomogeneous nonlinear Schrödinger (NLS) equation, the relevant problem of optical soliton control in inhomogeneous Kerr-type nonlinear media with PT-symmetric external potentials is investigated numerically. Particularly, the stable propagation of one, two, and three solitons in this optical setting is displayed. Moreover, the controllable parameter for optical soliton control is found. Finally, the stability of soliton control with respect to random perturbations is also studied.

Optical solitons have been widely studied in the field of nonlinear optics for many years. Especially, spatial solitons in optical nonlinear media have received intensive attention because of extensive applications [1,2]. At the same time, optical nonlinear media can support various solitons such as lattice solitons. Recently, optical lattice solitons were analyzed theoretically and demonstrated experimentally [3–6].

Concept of soliton control is a new and significant development in the application of solitons [7]. A variety of studies of soliton control have been widespread. In fact, the variation of inhomogeneous nonlinear systems in the longitudinal and transverse dimensions opens a wealth of opportunities for soliton control. For example, varying diffraction [8] and inhomogeneous nonlinearity [9] in nonlinear waveguide systems can lead to a lot of practical applications of soliton control. So far, many authors have analyzed the situations from different points of view and some interesting results have also been obtained [10–20].

On the other hand, since Bender and Boettcher found that non-Hermitian Hamiltonian can exhibit entirely real eigenvalue spectra provided that they obey the parity-time (PT) symmetry in 1998 [21], much attention has been paid to the important concept in physics. If the complex potential of a Hamiltonian has symmetric real part and antisymmetric imaginary part, one can say that the Hamiltonian holds PT symmetry. PT symmetry has practical applications in many areas, such as nonlinear optics [22]. PT-symmetric optical structures can be constructed by inclusion of gain or loss regions into waveguides [23] in the field. Recently, many topics in the aspect have been considered [24–31].

However, to our knowledge, less investigated was soliton control in optical inhomogeneous nonlinear systems with the PT-symmetric potentials. Indeed, in the case, a subtle competition between varying diffraction, inhomogeneous nonlinearity and PT-symmetric potentials can offer exciting soliton control. Hence the study about optical soliton control in inhomogeneous nonlinear media with the PT-symmetric potentials could have important consequences for further development of the theory and application of optical soliton and for the development of other branches of nonlinear optics.

In this paper we study optical soliton control in inhomogeneous nonlinear media with the PT-symmetric potentials. Interesting properties of optical soliton control in the (1+1)-dimensional inhomogeneous nonlinear system are presented by numerical methods.

We begin our discussion with electromagnetic waves propagating along the z-direction in inhomogeneous Kerr-type nonlinear media with complex PT-symmetric potentials. The light beam is only allowed to diffract along the transverse x axis and here we consider varying diffraction. To the best of our knowledge, studies for the problem are still few. The problem is governed by the (1+1)-dimensional inhomogeneous nonlinear Schrödinger

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(NLS) equation written as

$$iU_z + D(z)U_{xx} + \sigma(z, x)|U|^2U + [V(x) + iW(x)]U = 0, \quad (1)$$

where $U(x, z)$is the slowly varying complex envelope of the field. The real function $D(z)$ accounts for varying diffraction along the longitudinal$z$axes and $\sigma(x, z)$ describes inhomogeneous Kerr-type nonlinear profile along both the transverse x and longitudinal z axes. $V(x)$and $W(x)$ represent the real part and imaginary part of PT-symmetric potentials respectively. Here we consider complex PT-symmetric Scarff potentials. The potentials read [30]

$$\begin{cases} V(x) = V_0\text{sech}^2(x) \\ W(x) = W_0\,\text{sech}(x)\tanh(x) \end{cases}, \quad (2)$$

with V_0 and W_0 representing the depth of the real and imaginary parts of PT potentials, respectively. It should be noted that even if more values of V_0 and W_0 have been investigated in nonlinear systems [30], here we keep using a set of parameters as an example. In this paper we take the parameters [30] $V_0 = 8$ and $W_0 = 1.6$. Further, Eq. (1) includes two arbitrary distributed functions $D(z)$ and $\sigma(x, z)$and thus one can obtain various soliton controls by choosing the different form in them. If setting

$$D(z) = \exp(dz) \quad (3)$$

with d being the constant related to variable diffraction, and

$$\sigma(x, z) = (ax^2 + c)\exp(rz) \quad (4)$$

with a, c and r being the constants related to the variation of Kerr-type nonlinearity, we can obtain the soliton control based on the exponential modulation of diffraction parameter as well as the exponential longitudinal and quadratic transverse modulations [32] of Kerr-type nonlinearity parameter. In particular, the modulation corresponds to the quadratic transverse modulation of Kerr-type nonlinearity when$r = 0$. It should be said that the nonlinear system given by Eqs. (3) and (4) may be realized since the exponential modulation and the quadratic modulation have been considered [11,32].

We consider the Gaussian-type input optical waveform in the inhomogeneous nonlinear system as follows

$$U(x, 0) = \exp(-x^2)\exp(if\ x + i\phi_0), \quad (5)$$

where f and ϕ_0 are real constants related to incident angle and initial phase, respectively. The propagation of Gaussian-type optical beam is illustrated in Fig. 1 when$f = \phi_0 = 0$ and $r = 0$ in the system. From the plot we can find that Gaussian-type optical beam keeps its main shape as the localized mode. Taking into account

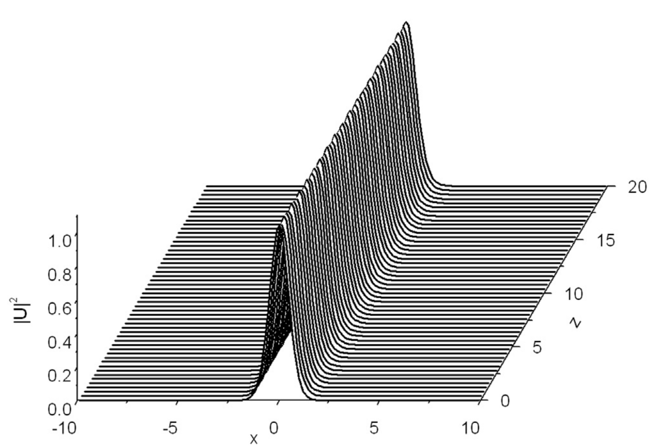

Fig. 1. The propagation plot of Gaussian beam with the input form (5) in nonlinear system given by Eqs. (3) and (4), here the parameters adopted are:$f = \phi_0 = 0$, $d = 0.01$, $a = 1$, $c = 0.2$, $r = 0$.

the sense of soliton, this situation could be called soliton control in inhomogeneous nonlinear structure with the PT-symmetric potentials. This also means that PT-symmetric Scarff potentials can support stable solitons in the inhomogeneous nonlinear system.

To comprehend the properties of optical soliton control in the inhomogeneous nonlinear media, we further consider the case of two Gaussian-type input optical waveforms as follows

$$U(x, 0) = \begin{matrix} \exp[-(x + x_0)^2]\exp[if_1(x + x_0) + i\phi_1] + \\ \exp[-(x - x_0)^2]\exp[if_2(x - x_0) + i\phi_2] \end{matrix}, \quad (6)$$

where x_0 is related to the initial position of two Gaussian-type input optical waveforms andf_1,f_2,ϕ_1, and ϕ_2 are real constants related to incident angle and initial phase respectively. $2x_0$ denotes the waveform separation. Fig. 2(a) shows the propagation of Gaussian beams in homogeneous nonlinear media [i.e. $d = a = r = 0$and $c = 1$in Eqs. (3) and (4)] with the PT-symmetric potentials when we consider the case of in-antiphase injection. From it one can observe drastic collision and interaction of two Gaussian beams. It is well known that spatial soliton interaction could be the most fascinating features of soliton phenomena. In the last years optical spatial solitons have become the main arena for investigating soliton interaction due to the ease with which exquisite experiments can be conducted with precise control [33]. In fact, as the information carriers, optical solitons face with a problem about the way to how to restrict interaction of optical solitons. In other words, it is necessary to restrict the interaction as shown in Fig. 2(a). As a matter of fact, the problem can be solved in the inhomogeneous nonlinear system given by Eqs. (3) and (4) with $d \neq 0$ and $a > 0$. Fig. 2(b) depicts the propagation of two Gaussian beams when $d = 0.01$ and $a = 1$ ($c = 0.2$). From the plot we can clearly see that the interaction between two neighboring solitons is restricted compared to Fig. 2(a). Namely, except for some oscillation in the initial stage, two optical solitons can propagate separately with the walk-off effect due to weak repulsive interaction. In addition, the width of optical solitons is reduced in the process as illustrated in Fig. 2(c). This implies the compression of optical solitons which is an important concept in the physical explanation. In fact, the optical soliton control still exists when we consider the case of in-phase and arbitrary-phase Gaussian inputs as shown in Fig. 3(a) and (b). From the plots one can find that different phase conditions for Gaussian-type input optical waveforms could not influence the main character of the soliton control compared to Fig. 2(b). It should be said that the optical soliton control obtained is due to the combined effects of controlling both the distribution of inhomogeneous nonlinear structure and the PT-symmetric potentials. In particular, the choice for the depth of the real and imaginary parts of PT-symmetric Scarff potentials is crucial in the optical soliton control. In other words, PT-symmetric Scarff potentials play an important role in the existence of the optical soliton control.

Furthermore, we consider the case of three Gaussian-type input optical waveforms as follows

$$U(x, 0) = \begin{matrix} \exp[-(x + x_0)^2]\exp[if\ (x + x_0) + i\phi_0] + \\ \exp(-x^2) + \\ \exp[-(x - x_0)^2]\exp[-if\ (x - x_0) + i\phi_0] \end{matrix}. \quad (7)$$

In this situation three optical solitons can also propagate separately in a way similar to the one shown in Fig. 2(b) except for some oscillation of middle soliton (see Fig. 4). Also, the process is generated when the interaction among three solitons is restricted.

Moreover, it is worth noting that the constant r in Eq. (4) is the controllable parameter for soliton control. Fig. 5 presents the propagation plot of two Gaussian beams given by Eq. (6) with