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Efficient single-photon-assisted entanglement concentration for an arbitrary entangled photon pair with the diamond nitrogen-vacancy center insides cavity

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ABSTRACT

In this paper, a protocol for single-photon-assisted entanglement concentration is proposed. Resorting to the nonlinear optics of a nitrogen-vacancy (NV) center in a diamond embedded in a photonic crystal cavity coupled to a waveguide, remote parties can share the maximally entangled photon pair with a certain probability. Compared with other entanglement concentration protocols (ECPs), the current one does not need to know the accurate coefficients of the initial state and can be repeated to get a higher success probability. Meanwhile, this protocol is more suitable for multiphoton system concentration. All these advantages make the protocol useful in long-distance quantum communication.

1. Introduction

Entanglement plays a crucial role in quantum information processing [1–10]. In order to optimally complete the quantum information processing, maximally entangled states are usually required. The entanglement, to our knowledge, is first produced locally and distributed to different distant locations. Therefore, the particles will inevitably contact with the environment during the transmission, and the channel noise will degrade the maximally entanglement into a less-entangled pure state or even make the entanglement into a mixed state, which will decrease the fidelity and the security of long-distance quantum communication. For example, the noise will make the fidelity of quantum teleportation degrade, quantum dense coding fail, and the quantum cryptography protocol insecure, etc.

To overcome this flaw, one can exploit entanglement purification [11–17] or entanglement concentration [18–33,40] to improve the entanglement of the quantum system first, and then achieve the goals of the above applications with the maximally entangled states. The methods of converting an ensemble of less-entangled mixed state into a maximally entangled state is called entanglement purification [11–17]. Since Bennett et al. proposed the first entanglement purification protocol in 1996 [11], the entanglement purification has been studied widely [12–17]. Another way to convert the partially entangled state into the maximally entangled state is called entanglement concentration [18–33,40], which operates in an ensemble of less-entangled pure state. Bennett et al. firstly proposed an entanglement concentration protocol (ECP) in 1996 [18], which is the so-called Schmidt projection protocol. In their protocol, they used collective measurements which are difficult to manipulate experimentally. Bose et al. also proposed an ECP based on entanglement swapping [19], which needs to know the coefficients of the entangled states. Ren et al. investigated the possibility of concentrating the two-photon four-qubit systems in partially hyperentangled states in both the spatial mode and the polarization DOFs with linear optics [29]. In this paper, they first introduced the parameter-splitting method to concentrate the systems in the partially hyperentangled states with known parameters, including partially hyperentangled Bell states and cluster states. Then, they presented another two nonlocal hyperentanglement concentration protocols (hyper-ECPs) for the systems in partially hyperentangled unknown states, resorting to the Schmidt projection method. The results show that the parameter-splitting method is very efficient for the concentration of the quantum systems in partially entangled states with known parameters, resorting to linear-optical elements only. Till now, many interesting ECPs have been proposed [18–33,40]. And among these protocols, the methods of introducing ancillary photons to complete the entanglement concentration have been proposed in Refs. [26–28,31–33]. For instance, Sheng et al. presented an ECP to concentrate partially entangled photon pair to a maximally entangled pair with only one ancillary single photon [26]. They also presented a two-step practical entanglement concentration protocol for concentrating an arbitrary three-particle less-entangled W state into a maximally entangled W state

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assisted with two single photons [28]. Almost at the same time, Deng proposed an optimally nonlocal ECP for multiphoton systems in a partially entangled pure state by resorting to the projection measurement on an additional photon [27]. Du et al. [31] and Gu et al. [32] proposed two different ECPs for the special W state resorted to ancillary photon. Zhou et al. [33] proposed an efficient ECP for arbitrary less-entangled NOON state assisted with a single photon. Most of the above ECPs are accomplished with the help of the cross-Kerr nonlinearity. Although a lot of works have been studied on cross-Kerr nonlinearity, a clean cross-Kerr nonlinearity in the optical single-photon regime is still quite a controversial assumption, especially the strong cross-Kerr nonlinearity, which is still a big challenge in the experiment at present.

The nitrogen vacancy (NV) center is an attractive spin qubit since it exhibits a unique combination of robust room-temperature spin coherence and efficient optical addressability, controllability, and readout [34,35]. Therefore, the NV magnetometry is very potential for future practical applications. Till now, many theoretical and experimental protocols have been proposed on quantum information processing based on the NV center. For example, Togan et al. [36] proposed a protocol for the quantum entangled state generation between a photon and an NV center. Yang et al. [37] proposed a protocol to generate quantum entangled state between electrons associated with the NV centers. Ren et al. [38] proposed a protocol to implement the hyperentanglement purification of two-photon systems in nonlocal hyperentangled Bell states with the help of the diamond NV centers inside photonic crystal cavities. Wei et al. [39] proposed some compact quantum circuits for a deterministic solid-state quantum computing, including the CNOT, Toffoli, and Fredkin gates on the diamond NV centers confined inside cavities, achieved by some input–output processes of a single photon. In Ref. [39], the quantum circuits for these universal quantum gates are simple and economic. Additional electron qubits are not employed, but only a single-photon medium. The authors have discussed the feasibility of these universal solid-state quantum gates, concluding that they are feasible with current technology.

In this paper, we propose a single-photon-assisted ECP with the assistance of the NV center. In the protocol, only one less-entangled state and one single photon are required. Due to long electronic spin lifetime, fast initialization, good qubit readout, and coherent manipulation at room temperature, the diamond NV center embedded in a photonic crystal cavity coupled to a waveguide is considered as a promising candidate for constructing the parit-check quantum nondemolition detectors (QNDs). Comparing with conventional ECPs, the single-photon-assisted ECPs are more economical, and with the help of the diamond NV center insides photonic crystal cavity, it can be repeated to get a higher success probability. In addition, we can extend the present ECP to concentrate the arbitrary N-photon Greenberger–Horne–Zeilinger (GHZ) state $|\Phi\rangle_{a_1b_1\cdots z_1} = \alpha|RR\cdots R\rangle_{a_1b_1\cdots z_1} + \beta|LL\cdots L\rangle_{a_1b_1\cdots z_1}$, the arbitrary spacial W state $|\Psi\rangle_{a_1b_1\cdots z_1} = \gamma|HHV\rangle + \delta(|HVH\rangle + |VHH\rangle)$ and the arbitrarily less-entangled standard W state $|\Psi\rangle_{a_1b_1c_1} = \zeta|RRL\rangle_{a_1b_1c_1} + \varsigma|RLR\rangle_{a_1b_1c_1} + \xi|LRR\rangle_{a_1b_1c_1}$. These advantages make the protocol more meaningful in practical applications.

The paper is organized as follows: In Section 2, we will explain the principle of the single-photon-assisted ECP with the diamond NV center insides cavity, and calculate the success probability. In Section 3, we will extend the protocol to complete the entanglement concentration for the GHZ state and the W-class state. And in Section 4, we will present the discussion and summary.

2. Single-photon-assisted entanglement concentration of less-entangled bell-state with the diamond NV center insides cavity

To discuss the principle of concentration in detail, we first briefly consider the diamond NV center insides cavity. The schematic diagram for an NV center in a diamond embedded in photonic crystal cavity coupled to a waveguide is shown in Fig. 1. The negatively charged NV center consisted of a substitutional nitrogen atom and an adjacent vacancy with six electrons from the nitrogen and three carbons surrounding the vacancy. As described in Refs. [41,42], the $|A_2\rangle$ state is robust with the stable symmetries, and decays to the ground state sublevels $|-1\rangle$ and $|+1\rangle$ with radiation of left (L) and right (R) polarizations, respectively. Therefore the zero phonon line (ZPL) is observed after the optical resonant excitation at 637 nm. In our protocol, the Λ-type three-level system is realized by employing one of the specific excited states $|A_2\rangle$ as an ancillary state. Moreover, when a Λ-type three-level diamond NV center is confined into a cavity, the Heisenberg equations of the motion for the annihilation operator of the cavity mode $\hat{a}$ and the lowing operator of the NV center operation σ_- and the input–output relation for the cavity can be given by [43]

$$\frac{d\hat{a}}{dt} = -\left[i(\omega_c - \omega) + \frac{\eta}{2} + \frac{\kappa}{2}\right]\hat{a} - g\hat{\sigma}_- - \sqrt{\eta}\,\hat{a}_{in},$$
$$\frac{d\hat{\sigma}_-}{dt} = -\left[i(\omega_e - \omega) + \frac{\gamma}{2}\right]\hat{\sigma}_- - g\hat{\sigma}_z\hat{a},$$
$$\hat{a}_{out} = \hat{a}_{in} + \sqrt{\eta}\,\hat{a}, \qquad (1)$$

where ω_c, ω, and ω_e are the frequencies of the cavity, the single photon, and the NV center, respectively. Ω is the coupling strength between the cavity and the NV center. γ, η, and κ are the decay rates of the NV center, the cavity field, and the cavity side leakage mode, respectively. If the NV center is dominantly in the ground state as well as in a weak excitation, i.e., taking $\langle\sigma_z\rangle = -1$, the form of the reflection coefficient $r(\omega)$ can be described as

$$r(\omega_p) = \frac{\left[i(\omega_c - \omega_p) - \frac{\kappa}{2}\right]\left[i(\omega_0 - \omega_p) + \frac{\gamma}{2}\right] + \Omega^2}{\left[i(\omega_c - \omega_p) + \frac{\kappa}{2}\right]\left[i(\omega_0 - \omega_p) + \frac{\gamma}{2}\right] + \Omega^2}, \qquad (2)$$

where ω_0 is the transition frequency between energy levels $|-1\rangle$ and $|A_2\rangle$. As shown in Ref. [44], Chen et al. showed that when $\Omega \geq 5\sqrt{\gamma\kappa}$ with $\omega_c = \omega_0 + \omega_p$, $r(\omega) \simeq 1$ and $r_0(\omega) = -1$. Therefore, the change of the input photon is summarized as [44,45]

$$|R, -1\rangle \to |R, -1\rangle, |R, +1\rangle \to -|R, +1\rangle,$$

$$|L, -1\rangle \to -|L, -1\rangle, |L, +1\rangle \to |L, +1\rangle. \qquad (3)$$

And as described in Ref. [38], a polarizing beam splitter in the circular basis (CPBS) and the NV center can be used to construct a

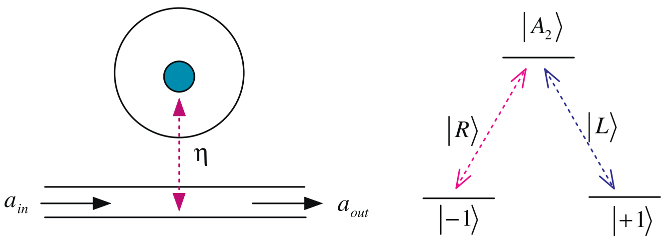

Fig. 1. Schematic diagram of a diamond NV center embedded in a photonic crystal cavity with the circularly polarized photon, and the possible Λ-type optical transition in an NV center.