Optics Communications 331 (2014) 32–38

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Characteristic study of anomalous vortex beam through a paraxial optical system

Yonggen Xu*, Shijian Wang

Research Center for Advanced Computation, School of Physics and Chemistry, Xihua University, Chengdu 610039, China

ARTICLE INFO

Article history:
Received 14 April 2014
Received in revised form
17 May 2014
Accepted 24 May 2014
Available online 5 June 2014

Keywords:
AVB
Paraxial optical system
Analytical formula
M^2 factor

ABSTRACT

Characteristics of an anomalous vortex beam (AVB) through the paraxial optical systems are studied. The analytical formulas for the AVBs through the aligned and misaligned optical systems are derived by using the Collins integral equation. As the numerical examples, the intensity distributions for the AVBs in the free space and the convergent optical system are given. It is shown that the far fields for AVBs will be determined by the main parameters, such as width of beam waist w_0, n, m (n is the order of the AVB, m is the topological charge). Furthermore, the analytical formula of M^2 factor of AVB is derived by using the second-order moments associated with the intensity distributions. The study results can provide references for the application of AVBs.

1. Introduction

The phase singularity is known to form an optical vortex in the light wave, the light intensity at the center of the optical vortex is zero and the phase is undetermined [1]. The optical vortex beams have been used for transferring the free space information and communications [2], quantum information and cryptography and optical manipulation [3] due to the helical wave-front and spatial propagation invariance [2–4]. Recently, Yang et al. proposed and studied the anomalous vortex beam (AVB), there are the same results approximately between experimentally and theoretically [1].

However, the convergent property, beam quality factor (M^2 factor) and propagation properties of AVB through the misalignment optical system have been not still reported in former works [1–4]. Therefore, it is useful for studying the propagation for the AVB through the aligned and misaligned optical systems in the free space and the convergent optical system. Propagation properties of many beams through the aligned optical system [5–7] and misaligned optical system [8–13] have been studied in detail. Therefore, in this paper, we will study the propagation properties of AVB by using the above methods [5–13]. Furthermore, so far, the beam quality of AVB has been not reported. It is well known that M^2 factor proposed by Siegman [14–16] is the basic method for describing the beam quality of the various beams, such as, hollow Gaussian beam (HGB) [17], Bessel–Gauss beam (BGB) [18], elegant Laguerre–Gauss beam (eLGB) [19], rectangular array beam (RAB) [20], and so on. Therefore, in this paper, we will use the M^2 factor proposed by A. E. Siegman to study the beam quality of AVB, the calculated results are important to study further the characteristics of AVB.

2. Propagation formulas of AVB through the paraxial optical system

2.1. Misaligned optical system

To describe the AVBs, we define the electric field of AVB [1] at $z=0$ as follows:

$$E_{nm}(r_0,\theta_0)=E_0\left(\frac{r_0}{w_0}\right)^{2n+|m|}\exp\left(-\frac{{r_0}^2}{{w_0}^2}\right)\exp(-im\theta_0). \quad (1)$$

where, E_0 is the constant, w_0 is the radius of beam waist, n is the order of the AVB, m is the topological charge, r_0 and θ_0 are radial and azimuthal coordinates, respectively. Eq. (1) will become the light field of HGB when $m=0$ and $n\neq0$. Eq. (1) will become the light field of the ordinary vortex Gaussian beam when $m\neq0$ and $n=0$. First, we study the propagation for the AVB through the misaligned optical systems. The geometrical optical pathway diagram of the misaligned optical system [8–13] is shown in Fig. 1, where, z and z_m are the corresponding axes, respectively. l is the distance between RP_1 and RP_2, ε and ε' are the misaligned line deviation and angle deviation, respectively. A, B, C and D are the propagation matrix elements of the beam in the aligned optical system. Therefore, the propagation formula for AVB through ABCD

* Corresponding author.
E-mail addresses: xuyonggen06@126.com, xuyg@mail.xhu.edu.cn (Y. Xu).

http://dx.doi.org/10.1016/j.optcom.2014.05.054

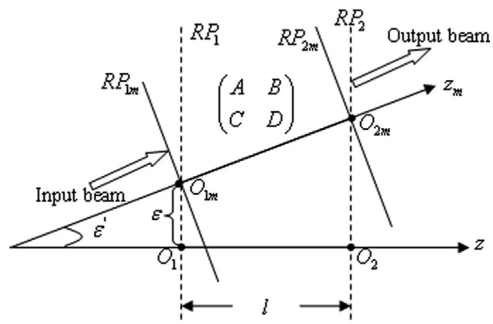

Fig. 1. Diagram of geometrical optical pathway of the misaligned optical system.

misaligned optical system can be given by

$$E_{nm}(r,\theta,z)=\frac{ik\exp(-ikz)}{2\pi B}\iint E_{nm}(r_0,\theta_0)\times\exp\left\{-\frac{ik}{2B}\left[A{r_0}^2-2rr_0\cos(\theta-\theta_0)+Dr^2+Er_0\cos\theta_0+Fr_0\sin\theta_0+Gr\cos\theta+Hr\sin\theta\right]\right\}r_0dr_0d\theta_0. \tag{2}$$

where, $k=2\pi/\lambda$ is the wave number, and parameters E, F, G and H are given by

$$E=2[(1-A)\varepsilon\cos\varphi+(l-B)\arctan(\cos\varphi\tan\varepsilon')]. \tag{3}$$

$$F=2[(1-A)\varepsilon\sin\varphi+(l-B)\arctan(\sin\varphi\tan\varepsilon')]. \tag{4}$$

$$G=2[-BC-D(1-A)]\varepsilon\cos\varphi+2[B(1-D)-D(l-B)]\arctan(\cos\varphi\tan\varepsilon'). \tag{5}$$

$$H=2[-BC-D(1-A)]\varepsilon\sin\varphi+2[B(1-D)-D(l-B)]\arctan(\sin\varphi\tan\varepsilon'). \tag{6}$$

where, φ is the misaligned azimuth angle.

Substituting Eqs. (1), (3)–(6) into Eq. (2), and using the following formulas:

$$\int_0^{2\pi}\exp[ix\cos(\xi-\theta_0)]\exp(-im\theta_0)d\theta_0=2\pi i^m J_m(x)\exp(-im\xi). \tag{7}$$

$$\int_0^{\infty}x^{n+\frac{\nu}{2}}\exp(-\alpha x)J_\nu(2\beta\sqrt{x})dx=n!\beta^\nu\exp\left(-\frac{\beta^2}{\alpha}\right)\alpha^{-n-\nu-1}L_n^\nu\left(\frac{\beta^2}{\alpha}\right). \tag{8}$$

where, $J_m(\cdot)$ is the mth-order Bessel function.

Eq. (2) is rewritten as follows:

$$E_{nm}(r,\theta,z)=\frac{c_1 n!}{2}\left(\frac{c_4}{2}\right)^{|m|}c_3^{-(c_2-n)}\exp\left(-\frac{{c_4}^2}{4c_3}\right)L_n^{|m|}\left(\frac{{c_4}^2}{4c_3}\right). \tag{9}$$

where, $L_n^{|m|}(\cdot)$ is the Laguerre polynomials, c_1, c_2, c_3, c_4 are given by

$$c_1=\frac{i^{m+1}kE_0}{B{w_0}^{2n+|m|}}\exp[-i(m\xi+kz)]\exp\left[-\frac{ik}{2B}(Dr^2+Gr\cos\theta+Hr\sin\theta)\right]. \tag{10}$$

$$c_2=2n+|m|+1. \tag{11}$$

$$c_3=\frac{2B+ikA{w_0}^2}{2B{w_0}^2}. \tag{12}$$

$$c_4=\frac{k}{B}\sqrt{(r\cos\theta-\tfrac{E}{2})^2+(r\sin\theta-\tfrac{F}{2})^2} \tag{13}$$

respectively.

Where, ξ satisfies the following expression:

$$\cos\xi=\frac{r\cos\theta-\frac{E}{2}}{\sqrt{(r\cos\theta-\frac{E}{2})^2+(r\sin\theta-\frac{F}{2})^2}}. \tag{14}$$

$$\sin\xi=\frac{r\sin\theta-\frac{F}{2}}{\sqrt{(r\cos\theta-\frac{E}{2})^2+(r\sin\theta-\frac{F}{2})^2}}. \tag{15}$$

Setting the wavelength of laser beam is λ, $z_R=\pi{w_0}^2/\lambda$ is the Rayleigh lengths, $k=2\pi/\lambda$ is the wave number. Eq. (9) is the complex and important propagation formula for AVB through the misaligned paraxial optical systems. It can provide the transformation rule and be a powerful tool to study propagation properties of the AVBs through the arbitrary misalignment ABCD optical system.

2.2. Aligned optical system

Setting the misaligned line deviation $\varepsilon=0$ and angle deviation $\varepsilon'=0$ in Eqs. (3)–(6), i.e., $E=F=G=H=0$. Therefore, the misaligned optical system in Section 2.1 reduces to the aligned optical system. Eq. (9) is rewritten as the following expressions:

$$E_{nm}(r,\theta,z)=\frac{c_1' n!}{2}\left(\frac{c_4'}{2}\right)^{|m|}{c_3'}^{-(c_2'-n)}\exp\left(\frac{{c_4'}^2}{4c_3'}\right)L_n^{|m|}\left(\frac{{c_4'}^2}{4c_3'}\right). \tag{16}$$

c_1', c_2', c_3', c_4' are given by

$$c_1'=\frac{i^{m+1}kE_0}{B{w_0}^{2n+|m|}}\exp[-i(m\xi'+kz)]\exp\left(-\frac{ik}{2B}Dr^2\right). \tag{17}$$

$$c_2'=c_2,\quad c_3'=c_3,\quad c_4'=\frac{kr}{B}\quad \xi'=\theta. \tag{18}$$

In comparison with Eqs. (9) and (16) is the important propagation formula for AVB through the aligned paraxial optical systems. It can provide the transformation rule and be a powerful tool to study propagation properties of the AVBs through the arbitrary aligned ABCD optical system.

3. Numerical examples and results discussion

3.1. Numerical examples

As the numerical examples, we will study the propagation of AVB through the aligned and misaligned optical system in the free space, and the propagation matrix for the laser beam through the ABCD optical system is given by

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}=\begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix}. \tag{19}$$

where, z is the propagation distance. Substituting Eq. (19) into Eq. (16), the intensity distributions for AVB through the aligned optical system in the free space can be given by $I=[E_{nm}(r,\theta,z)][E_{nm}(r,\theta,z)]^*$, and the intensity distributions are shown in Fig. 2. Where, it takes $z=15$ cm, $m=2$, $n=5$ as an example of studying the influence of beam waist w_0 on the light intensity.

Similarly, we take the propagation of AVB through a thin lens with focus f followed by a free space z as the illustrative examples, and the propagation matrix is rewritten as

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}=\begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}. \tag{20}$$

where, the focus $f=10$ cm. Substituting Eq. (20) into Eq. (16), the intensity distributions for AVB through the convergent ABCD optical system are shown in Fig. 3. Where, it takes $w_0=2$ mm, $m=2$, $n=5$ as an example of studying the influence of propagation distance z on the light intensity.