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Fiber-coupling efficiency for optical wave propagating through non-Kolmogorov turbulence

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ABSTRACT

In the past decades, both the increasing experimental evidences and some results of theoretical investigation on non-Kolmogorov turbulence have been reported. This has prompted the study of optical propagation in non-Kolmogorov atmospheric turbulence. In this paper, using a non-Kolmogorov power spectrum which owns a generalized power law instead of standard Kolmogorov power law value 11/3 and a generalized amplitude factor instead of constant value 0.033, the fiber-coupling efficiency of plane and spherical waves are derived for horizontal link in weak turbulence. The analytic expressions are obtained and then used to analyze the effect of spectral power-law variations on the fiber-coupling efficiency. It is anticipated that this work is helpful to the investigations of atmospheric turbulence and optical wave propagation in the atmospheric turbulence.

1. Introduction

It is well-known that atmospheric turbulence severely degrades the performance of imaging and laser systems [1–3]. For a long time, the Kolmogorov model for atmospheric turbulence has been widely applied to estimate the performance of imaging and laser systems operating in the atmosphere, which has been confirmed by numerous experimental evidences.

Despite the success of the Kolmogorov model, recently both the experimental data [4–8] and the theoretical investigations [9–12] have shown that it is not the only possible turbulent one in the atmosphere. This has prompted the scientist community to research optical propagation in non-Kolmogorov atmospheric turbulence. Beland developed the expressions of log-amplitude variance and the coherence length for optical wave propagating through weak isotropic non-Kolmogorov turbulence [13]. Stribling et al. analyzed the wave structure function and the Strehl ratio as the refractive-index fluctuations deviated from Kolmogorov statistics [14]. Boreman and Dainty studied the expressions of non-Kolmogorov turbulence in the light of Zernike polynomials [15]. Gurvich and Belen'kii introduced a model for the power spectrum of stratospheric non-Kolmogorov turbulence and investigated the stratospheric turbulence on the scintillation and the coherence of starlight and on the degradation of star image [16]. Belen'kii studied the influence of the stratosphere on star image motion again based on the model for the power spectrum of stratosphere [17]. Toselli et al. introduced a non-Kolmogorov theoretical power spectrum model and analyzed long term beam spread, scintillation index, probability of fade, mean SNR, and mean BER as variations of the spectrum exponent for horizontal link [18]. And then they analyzed the angle-of-arrival fluctuations for free space laser beam again [19]. Baykal et al. found the equivalence of the structure constants in non-Kolmogorov and Kolmogorov spectra in a turbulent atmosphere [20]. Chen et al. developed the expressions of temporal averaged pulse intensity for optical pulses propagating through non-Kolmogorov turbulence under the strong fluctuation conditions and the narrow-band assumption [21]. Recently, with some components developed for fiber-optic communication systems, such as transmitter and receiver modules, erbium-doped fiber amplifiers (EDFAs), used in high-speed free space optical communication links, the fiber-coupling efficiency becomes more and more important. Ruilier derived an analytical expression of the coupling efficiency for the monochromatic case, and the effect of purely static aberrations was considered [22]. Dikmelik et al. numerically evaluated the fiber-coupling efficiency for laser light distorted by horizontal Kolmogorov turbulence [23]. Hideki et al. measured the fiber-coupling efficiency for satellite-to-ground atmospheric laser downlinks in their experiment [24]. However, the fiber-coupling efficiency for plane and spherical waves through non-Kolmogorov turbulence were not discussed in their paper.

In this paper, we consider a non-Kolmogorov theoretical power spectrum for the refractive-index fluctuations [19], which obeys a more general power law that takes all the values between the range 3 to 4. When the power law is set to the standard Kolmogorov value

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11/3, the spectrum reduces to the conventional Kolmogorov one. Using this spectrum, the fiber-coupling efficiency of plane and spherical waves have been developed for horizontal link in weak turbulence, and then the effect of spectral power-law variations on the fiber-coupling efficiency has been analyzed.

2. Non-Kolmogorov spectrum

For the purpose of this paper, a theoretical power spectrum model that describes non-Kolmogorov optical turbulence is considered [19], which obeys a more general power law and in which the power-law exponents can take all the values ranging from 3 to 4,

$$\Phi_n(\kappa,\alpha)=A(\alpha)\tilde{C}_n^2\frac{\exp(-\kappa^2/\kappa_m^2)}{(\kappa^2+\kappa_0^2)^{\alpha/2}},\quad 0\le\kappa<\infty,\quad 3<\alpha<4, \tag{1}$$

where κ is the magnitude of three dimensional wave number vector (in units of rad/m), α is the spectrum power-law exponent, $\tilde{C}_n^2$ is a generalized refractive-index structure parameter (in units of $m^{3-\alpha}$) that describes the strength of the turbulence along the path, and $A(\alpha)$ is a function defined by

$$A(\alpha)=\frac{1}{4\pi^2}\Gamma(\alpha-1)\cos\left(\frac{\alpha\pi}{2}\right), \tag{2}$$

where the symbol $\Gamma(x)$ represents the gamma function, and $\kappa_0=2\pi/L_0$, L_0 is the outer scale parameter, $\kappa_m=c(\alpha)/l_0$, l_0 is the inner scale parameter and $c(\alpha)=[\Gamma((5-\alpha)/2)A(\alpha)\ (2/3)\pi]^{1/\alpha-5}$. When the power law α is equal to 11/3, $A(11/3)=0.033$, $\tilde{C}_n^2=C_n^2$, and the spectrum reduces to the conventional von Kármán spectrum for Kolmogorov turbulence [25],

$$\Phi_n(\kappa)=0.033C_n^2\frac{\exp(-\kappa^2/\kappa_m^2)}{(\kappa^2+\kappa_0^2)^{11/6}}, \tag{3}$$

where C_n^2 represents the conventional refractive-index structure parameter and has units of $m^{-2/3}$. In addition, as $\alpha\to3$, $A(\alpha)\to0$. As a result, the power spectrum for refractive-index fluctuations disappears in the limiting case $\alpha=3$.

3. Fiber-coupling efficiency

The fiber-coupling efficiency for an optical wave is defined as the ratio of the average power coupled into the fiber, $\langle P_c\rangle$, to the average power in the receiver aperture plane, $\langle P_a\rangle$, and is given by [23,26]

$$\eta=\frac{\langle P_c\rangle}{\langle P_a\rangle}=\frac{\left\langle\left|\int_A U_i(\mathbf{r})U_m^*(\mathbf{r})d\mathbf{r}\right|^2\right\rangle}{\left\langle\int_A|U_i(\mathbf{r})|^2d\mathbf{r}\right\rangle}, \tag{4}$$

where $U_i(\mathbf{r})$ is the incident optical field in the receiver aperture plane and $U_m(\mathbf{r})$ is the normalized fiber-mode profile. The overlap integral in the numerator of Eq. (4) is evaluated in the receiver aperture plane A, because it is more convenient to do so. The numerator of Eq. (4) can be rearranged by expanding the squared integration to write the coupling efficiency as

$$\eta=\frac{1}{\langle P_a\rangle}\iint_A\Gamma_i(\mathbf{r}_1,\mathbf{r}_2)U_m^*(\mathbf{r}_1)U_m(\mathbf{r}_2)d\mathbf{r}_1d\mathbf{r}_2, \tag{5}$$

where the mutual coherence function of the incident field is given by

$$\Gamma_i(\mathbf{r}_1,\mathbf{r}_2)=\langle U_i(\mathbf{r}_1)U_i^*(\mathbf{r}_2)\rangle. \tag{6}$$

Assuming that the fiber end face is positioned in the focal plane of the receiver lens and centered on the optical axis to maximize the coupling efficiency, the fiber-mode profile propagated to the front surface of the lens can be given by [23]

$$U_m(\mathbf{r})=\frac{kW_m}{\sqrt{2\pi}f}\exp\left[-\left(\frac{kW_m}{2f}\right)^2r^2\right], \tag{7}$$

where $k=2\pi/\lambda$ and λ is the optical wavelength, W_m is the fiber-mode field radius at the fiber end face, and f is the focal length of the receiver lens. The amplitude factor in Eq. (7) is included to normalize the power carried by the mode to unity. The fiber-mode profile was approximated by a Gaussian function in the derivation of Eq. (7). This approximation is commonly used in calculations of fiber-coupling efficiency and does not lead to an appreciable loss of accuracy [27].

3.1. Fiber-coupling efficiency of plane wave

For a plane wave, the mutual coherence function under weak fluctuation conditions is given by [25]

$$\Gamma_{i(pl)}(\mathbf{r}_1,\mathbf{r}_2)=\exp\{-4\pi^2k^2L\int_0^\infty\kappa\Phi_n(\kappa)[1-J_0(\kappa|\mathbf{r}_1-\mathbf{r}_2|)]d\kappa\}, \tag{8}$$

where $\Phi_n(\kappa)$ is the power spectrum for the refractive-index fluctuations, L is the length of the optical path, and $J_0(x)$ is a Bessel function of the first kind.

For non-Kolmogorov turbulence, Eq. (8) can be written as

$$\Gamma_{i(pl)}(\mathbf{r}_1,\mathbf{r}_2,\alpha)=\exp\left\{-4\pi^2k^2L\int_0^\infty\kappa\Phi_n(\kappa,\alpha)[1-J_0(\kappa|\mathbf{r}_1-\mathbf{r}_2|)]d\kappa\right\}. \tag{9}$$

Substituting Eq. (1) into Eq. (9) yields the mutual coherence function under weak fluctuation conditions for a plane wave

$$\begin{aligned}\Gamma_{i(pl)}(\mathbf{r}_1,\mathbf{r}_2,\alpha)=\exp\Big\{&-4A(\alpha)\tilde{C}_n^2\pi^2k^2L\int_0^\infty\kappa(\kappa^2+\kappa_0^2)^{-\alpha/2}\exp(-\kappa^2/\kappa_m^2)d\kappa\\&+4A(\alpha)\tilde{C}_n^2\pi^2k^2L\int_0^\infty\kappa(\kappa^2+\kappa_0^2)^{-\alpha/2}\exp(-\kappa^2/\kappa_m^2)\\&\times J_0(\kappa|\mathbf{r}_1-\mathbf{r}_2|)d\kappa\Big\}.\end{aligned} \tag{10}$$

Using the integral relation,

$$U(a;c;z)=\frac{1}{\Gamma(\alpha)}\int_0^\infty e^{-zt}t^{a-1}(1+t)^{c-a-1}dt,\quad a>0,\quad \mathrm{Re}(z)>0, \tag{11}$$

and the series representation of the Bessel function of the first kind [25],

$$J_p(x)=\sum_{n=0}^\infty\frac{(-1)^n(x/2)^{2n+p}}{n!\Gamma(n+p+1)},\quad |x|<\infty, \tag{12}$$

where $U(a;c;z)$ is the confluent hypergeometric function of the second kind and p denotes the order of the Bessel function of the first kind, Eq. (10) can be expressed as

$$\begin{aligned}\Gamma_{i(pl)}(\mathbf{r}_1,\mathbf{r}_2,\alpha)=\exp\Big\{&-2A(\alpha)\tilde{C}_n^2\pi^2k^2L\kappa_0^{2-\alpha}U\left(1;2-\frac{\alpha}{2};\frac{\kappa_0^2}{\kappa_m^2}\right)\\&+2A(\alpha)\tilde{C}_n^2\pi^2k^2L\kappa_0^{2-\alpha}\sum_{n=0}^\infty\frac{(-1)^n(|\mathbf{r}_1-\mathbf{r}_2|^2\kappa_0^2/4)^n}{n!}\\&\times U\left(n+1;n+2-\frac{\alpha}{2};\frac{\kappa_0^2}{\kappa_m^2}\right)\Big\}.\end{aligned} \tag{13}$$

For non-Kolmogorov turbulence, the condition $\kappa_0^2/\kappa_m^2\ll1$, roughly the same as $(l_0/L_0)^2\ll1$, is always satisfied. Then using the asymptotic formula [25],

$$U(a;c;z)\sim\frac{\Gamma(1-c)}{\Gamma(1+a-c)}+\frac{\Gamma(c-1)}{\Gamma(\alpha)}z^{1-c},\quad |z|\ll1, \tag{14}$$