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Increased time resolution with multiphoton interference beating

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ABSTRACT

I investigate a variation of Hong–Ou–Mandel interference where two interference filters with different central frequencies are placed in the two output-ports of a beam splitter. Taking photons as wavepackets in the time domain, we get a general analytic formula for the probability that N photons emerge in each output-port after interference. The probability is shown to oscillate as a cosine function modulated by a dip and the oscillation period is inversely proportional to N which indicates a better time resolution with multiphoton beating.

1. Introduction

Although originally studied as a phenomenon arising from the superposition of classical waves, the interference effect demonstrated the quantum nature of light in some few-photon interference experiments [1–3]. In particular, a fourth-order interference technique, well known as Hong–Ou–Mandel (HOM) interference [3], demonstrates photon bunching from destructive interference and has been used to measure the time separation between two photons on a femtosecond time scale. In fact the precision of this kind of measurement for the time separation of two wavepackets is proportional to $1/\sqrt{N}$, where N denotes the number of photons each of the two incident wavepackets contains. This is just the standard quantum limit [4] for a phase measurement. There exists a more fundamental limit for measurement, i.e., the Heisenberg limit $(1/N)$ [5], which may be achieved with two-mode maximally entangled N-photon superposition state $((|N,0\rangle+|0,N\rangle)/\sqrt{2}$, known as the NOON state) [6–8]. However, a NOON state is difficult to generate in experiments with large N.

Subsequently, Ou and Mandel [9] constructed a variant of the HOM interference where two interference filters with different central frequencies are placed in front of detectors, in a setup that is actually an interference experiment between two photons with different frequencies. They observe quantum beating and get better time resolution compared with the original HOM interference. Additionally, by a similar setup, Kim et al. performed a kind of two-particle interference experiment with frequency-entangled photon pairs [10] and a quantum-eraser experiment [11], where the visibility of the interference fringe can be modulated by changing the arrangement of a half wave-plate to determine the degree of indistinguishability of two incident photons, thus showing the complementarity of wave-like and particle-like behaviors of photons. The beating effect was also used to confirm other source of frequency-entangled photon pairs [12]. Furthermore, Legero et al. [13] observed quantum beating with photons of different frequencies emitted from an atom-cavity system. A temporal filter, which accepts only time intervals between photo-detections shorter than the mutual coherence time, was found to be a way to obtain nearly perfect two-photon interference, which made linear optical quantum computing [14,15] more practical.

In Ref. [9], they analyzed experimental results to suggest that the period of beating is inversely proportional to the magnitude of the difference between the central frequencies of the two filters. It should be pointed out that their analysis is based on measurement operators and the method would be too difficult to apply in an analysis of beating in multiphoton interference.

In this paper, we first reanalyze this beating phenomenon with a new method [16–18], and then apply it to general situation where each incident wavepacket contains N photons. We obtain an analytic formula for multiphoton beating. In fact, we make an approximation in which the photons are considered as a single-frequency mode while existing as wavepackets in the time domain. The interference can be cast into two parts under a full quantum treatment: one corresponding to indistinguishability case and the other to distinguishability case. These two parts are mixed with a certain probability distribution. Our result shows that the oscillation period of the multiphoton beating is inversely proportional to N, which indicates that we can get better time resolution with larger N.

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2. Scheme and analysis method

We first analyze beating for two-photon interference. The setup is schematically shown in Fig. 1 which is cited from Ref. [16]. For simplicity, a beam splitter (BS) with reflectivity–transmission ratio 50/50 is used; the BS has two input-ports A and B and two output-ports C and D each having an interference filter which we assume has Gaussian-shaped transmission functions $G_{1,2}(\omega)=1/(\sqrt{\pi}\sigma)^{1/2}\exp(-(\omega-\omega_{1,2})^2/2\sigma^2)$ where $\omega_{1,2}$ is the central frequency of the filter $If_{1,2}$ and σ is the width.

Two incident photons can be produced from spontaneous parametric down conversion (SPDC) pumped with a pulsed laser [19]. The one-order down-converted field can be written as

$$|\Psi_1\rangle=\int d\omega\, d\omega'\Phi(\omega,\omega')a^\dagger(\omega)b^\dagger(\omega')|0\rangle. \tag{1}$$

It comprises signal photon ($a^\dagger(\omega)$) and idle photon ($b^\dagger(\omega')$), which in the setup will be guided to input-ports A and B, respectively. $\Phi(\omega,\omega')$ denotes the spectral function satisfying $\int d\omega\, d\omega'|\Phi\ (\omega,\omega')|^2=1$. Because of the post-selection of interference filters, we have

$$\begin{aligned}|\Psi_1\rangle=\frac{1}{\sqrt{2}}\int d\omega\, d\omega'[&G_1(\omega)G_2(\omega')a^\dagger(\omega-\omega_1)b^\dagger(\omega'-\omega_2)\\&+G_1(\omega')G_2(\omega)a^\dagger(\omega-\omega_2)b^\dagger(\omega'-\omega_1)]|0\rangle,\end{aligned} \tag{2}$$

where $G_1(\omega)G_2(\omega')a^\dagger(\omega-\omega_1)b^\dagger(\omega'-\omega_2)$ describes the state that the photon in side A is a wavepacket with spectral distribution $G_1(\omega)$ and central frequency ω_1, while the photon in side B is a wavepacket with spectral distribution $G_2(\omega')$ and central frequency ω_2; $G_1(\omega')G_2(\omega)a^\dagger(\omega-\omega_2)b^\dagger(\omega'-\omega_1)$ describes just the opposite state. Here we assume that $\omega_1+\omega_2=\omega_0$, where ω_0 is the central frequency of the pump field. Moreover, the width of $\Phi(\omega,\omega')$ is usually very large compared to those of the interference filters [19]; therefore, $\Phi(\omega,\omega')$ is neglected in Eq. (2).

In the above, the one-order down-converted field can be analyzed by the multimode theory in the frequency domain. In practice, the pump field is usually a coherent pulse with a spectrum of frequencies. The down-converted field is indeed multimode in both frequency (time) [19] and space domains. However, SPDC was shown [20] to be well described by a single-mode theory when single spatial mode filters and narrow-band interference filters are used to filter the down-converted field. This requirement can be met in our scheme. By the method used in [16], the incident photons can be considered as single-frequency modes while existing as wavepackets in the time domain. Thus the incident field can be written in the time domain as

$$\begin{aligned}|\Psi_1\rangle=\frac{1}{\sqrt{2}}\int dt\phi_a(t)\phi_b(t)[&a^\dagger(\omega_1)e^{i\omega_1 t}b^\dagger(\omega_2)e^{i\omega_2(t+\tau)}\\&+a^\dagger(\omega_2)e^{i\omega_2 t}b^\dagger(\omega_1)e^{i\omega_1(t+\tau)}]|0\rangle,\end{aligned} \tag{3}$$

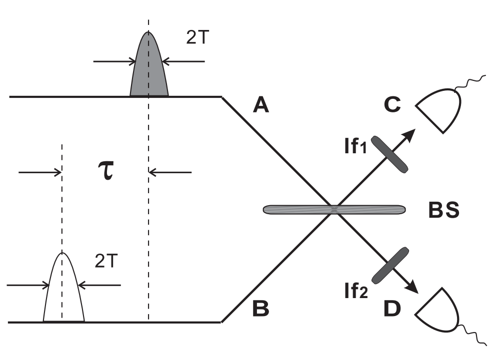

Fig. 1. Two photon wavepackets with a relative delay τ arrive at a beam splitter (50/50) from two input-ports A and B. If1 and If2 denote two interference filters with central frequencies ω_1 and ω_2, respectively. Each of the wavepackets contains N photons, and the interference at the beam splitter involves multi-photon pairs.

where $\phi_a(t)$ and $\phi_b(t)$ denote the wave functions of the incident photons in the time domain. $\phi_{a,b}(t)$ can be considered as a Fourier transformation of $G_{1,2}(\omega)$. $\phi_a(t)=1/(\sqrt{\pi}T_0)^{1/2}\exp(-t^2/2T_0^2)$, $\phi_b(t)=1/(\sqrt{\pi}T_0)^{1/2}\exp(-(t+\tau)^2/2T_0^2)$, $T_0=1/\sigma$ and $\int_{-\infty}^{+\infty}|\phi_{a,b}(t)|^2\,dt=1$ is the normalization condition. Moreover, the photon in side B is delayed for time τ to that in side A.

There exists an overlap between the two wave functions, which can be described as $|\alpha|^2=|\int_{-\infty}^{+\infty}dt\phi_a(t)\phi_b(t)|^2=\exp(-1/2(\tau/T)^2)$ [16]. Thus, when the two photons arrive at the BS, they are indistinguishable with probability $|\alpha|^2$ and distinguishable with probability $1-|\alpha|^2$.

2.1. Indistinguishable inputs

If indistinguishable, the incident field can be described as

$$|\psi_1\rangle=\frac{1}{\sqrt{2}}[a^\dagger(\omega_1)e^{i\omega_1 t}b^\dagger(\omega_2)e^{i\omega_2(t+\tau)}+a^\dagger(\omega_2)e^{i\omega_2 t}b^\dagger(\omega_1)e^{i\omega_1(t+\tau)}]|0\rangle, \tag{4}$$

The transformation of the BS is $a^\dagger(\omega_i)=(c^\dagger(\omega_i)+d^\dagger(\omega_i))/\sqrt{2}$, $b^\dagger(\omega_i)=(c^\dagger(\omega_i)-d^\dagger(\omega_i))/\sqrt{2}$, where $c^\dagger(\omega_i)$ and $d^\dagger(\omega_i)$ are the corresponding photon creation operators in sides C and D, respectively, which commute with each other. Thus the interference at BS can be described as

$$\begin{aligned}&a^\dagger(\omega_1)e^{i\omega_1 t}b^\dagger(\omega_2)e^{i\omega_2(t+\tau)}|0\rangle\\&\quad=\frac{1}{2}e^{i\omega_1 t+i\omega_2(t+\tau)}[c^\dagger(\omega_1)+d^\dagger(\omega_1)][c^\dagger(\omega_2)-d^\dagger(\omega_2)]\\&\quad=\frac{1}{2}e^{i\omega_1 t+i\omega_2(t+\tau)}[c^\dagger(\omega_1)c^\dagger(\omega_2)-c^\dagger(\omega_1)\,d^\dagger(\omega_2)+c^\dagger(\omega_2)\,d^\dagger(\omega_1)\\&\qquad-d^\dagger(\omega_1)\,d^\dagger(\omega_2)]|0\rangle,\end{aligned} \tag{5}$$

$$\begin{aligned}&a^\dagger(\omega_2)e^{i\omega_2 t}b^\dagger(\omega_1)e^{i\omega_1(t+\tau)}|0\rangle\\&\quad=\frac{1}{2}e^{i\omega_2 t+i\omega_1(t+\tau)}[c^\dagger(\omega_2)+d^\dagger(\omega_2)][c^\dagger(\omega_1)-d^\dagger(\omega_1)]\\&\quad=\frac{1}{2}e^{i\omega_2 t+i\omega_1(t+\tau)}[c^\dagger(\omega_1)c^\dagger(\omega_2)-c^\dagger(\omega_2)\,d^\dagger(\omega_1)+c^\dagger(\omega_1)\,d^\dagger(\omega_2)\\&\qquad-d^\dagger(\omega_1)\,d^\dagger(\omega_2)]|0\rangle.\end{aligned} \tag{6}$$

Because of post-selection, only the terms of $-c^\dagger(\omega_1)\,d^\dagger(\omega_2)|0\rangle$ in Eq. (5) and $c^\dagger(\omega_1)\,d^\dagger(\omega_2)|0\rangle$ in Eq. (6) can induce coincidence detection. Therefore, we can get the probability of coincidence detection in the indistinguishable case by first making sum of the two probability amplitudes and then calculating the module. The result is

$$\begin{aligned}\mathcal{P}_1(1,1)&=\left|-\frac{1}{2\sqrt{2}}e^{i\omega_1 t+i\omega_2(t+\tau)}+\frac{1}{2\sqrt{2}}e^{i\omega_2 t+i\omega_1(t+\tau)}\right|^2\\&=\frac{1}{4}[1-\cos(\omega_1-\omega_2)\tau].\end{aligned} \tag{7}$$

2.2. Distinguishable inputs

With distinguishability, the incident field can be also described by Eq. (4), but the transformation of the BS becomes $a^\dagger(\omega_i)=(c^\dagger(\omega_i)+d^\dagger(\omega_i))/\sqrt{2}$, $b^\dagger(\omega_i)=(c'^\dagger(\omega_i)-d'^\dagger(\omega_i))/\sqrt{2}$. The interference at BS should be described as

$$\begin{aligned}&a^\dagger(\omega_1)e^{i\omega_1 t}b^\dagger(\omega_2)e^{i\omega_2(t+\tau)}|0\rangle\\&\quad=\frac{1}{2}e^{i\omega_1 t+i\omega_2(t+\tau)}[c^\dagger(\omega_1)+d^\dagger(\omega_1)][c'^\dagger(\omega_2)-d'^\dagger(\omega_2)]\\&\quad=\frac{1}{2}e^{i\omega_1 t+i\omega_2(t+\tau)}[c^\dagger(\omega_1)c'^\dagger(\omega_2)-c^\dagger(\omega_1)\,d'^\dagger(\omega_2)+c'^\dagger(\omega_2)\,d^\dagger(\omega_1)\\&\qquad-d^\dagger(\omega_1)\,d'^\dagger(\omega_2)]|0\rangle,\end{aligned} \tag{8}$$