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Vortex and dipole solitons in lattices possessing defects and dislocations

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ABSTRACT

We report the numerical existence of dipole and vortex solitons for the two-dimensional nonlinear Schrödinger (NLS) equation with external potentials that possess strong irregularities, i.e., edge dislocations and a vacancy defects. Multi-humped solitons are computed by employing a spectral fixed-point computational scheme. The nonlinear stability of these solitons is investigated using direct simulations of the NLS equation and it is observed that these multi-humped modes in the defect lattices can be stable or unstable.

1. Introduction

The soliton theory is an interdisciplinary topic, where many ideas from mathematical physics, nonlinear optics, solid state physics and quantum theory are mutually benefited from each other. Solitons are localized nonlinear waves and their properties have provided a deep and fundamental understanding of complex nonlinear systems. In recent years, there has been considerable interest in studying solitons in system with periodic potentials or lattices, in particular, those that can be generated in nonlinear optical materials [1–6]. There have been very few studies of complex-phase solutions in the presence of irregular type potentials [7]. In [8], properties of solitons supported by optical lattices featuring topological dislocations are investigated and it has been found that these solitons experience attractive and repulsive forces around the dislocations. For a recent review on lattice solitons in various types of optical lattices and potential stabilization of these structures are addressed in [9].

In periodic lattices, solitons can form when their propagation constant, or eigenvalue, is within a certain region, often called gaps, a concept that is borrowed from the Floquet–Bloch theory for linear propagation. But the external potential of complex systems can be much more general and physically richer than a periodic lattice. For example, atomic crystals can have various irregularities, such as defects and edge dislocations, and also quasicrystal structures, which have long-range orientational order but no translational symmetry [10–12]. In general, when the lattice periodicity is slightly perturbed, the band-gap structure and soliton properties also become slightly perturbed, and solitons are expected to exist in much the same as in the perfectly periodic case [13,14]. But when the perturbation is large as it is discussed in [15], very little is known. In [16] a new method is presented to create a lattice with defects by manipulating individual sites in a 2D optical lattice. The modified optical lattice is created by interference of plane waves and spiral phase waves. We note that, irregular lattice structures can be fabricated experimentally [17–20].

Vortex-type solitons in the presence of an (optically or magnetically) induced lattice have been investigated analytically and experimentally in Bose–Einstein condensates (BECs) (cf. [21,22]) and in optical Kerr media (cf. [23–28]). Such structures appear as special solutions of the focusing two-dimensional cubic nonlinear Schrödinger (NLS) equation with an external potential. Using a fixed point spectral computational scheme, multiple dipole and vortex solitons are shown to exist in (2+1)-dimensional NLS equation with an external quasicrystal lattice [29]. Stable fundamental solitons for the two-dimensional NLS equation with external potentials that possess large variations from periodicity are obtained numerically in [15], but the existence and stability properties of multiple dipole and/or vortex structures have not been studied extensively in the current literature. Our results can find applications to photonic band-gap systems.

In this study, we compute multiple dipole and vortex soliton solutions of the focusing cubic (2+1)-dimensional NLS equation with external potentials (lattices) that possess defects and dislocations. This is achieved using the spectral renormalization method which is a fixed point spectral scheme. Nonlinear stability properties of these solitons are also analyzed.

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The governing equation used in this study is the focusing (2+1)-dimensional NLS equation with an external potential,

$$iu_z+\Delta u+|u|^2u-V(x,y)u=0. \tag{1}$$

In optics, $u(x,y,z)$ corresponds to the complex-valued, slowly varying amplitude of the electric field in the xy plane propagating in the z direction, $\Delta u\equiv u_{xx}+u_{yy}$ corresponds to diffraction, the cubic term in u originates from the nonlinear (Kerr) change of the refractive index and $V(x,y)$ is an external optical potential that can be written as the intensity of a sum of N phase-modulated plane waves, i.e. (see [15]),

$$V(x,y)=\frac{V_0}{N^2}\left|\sum_{n=0}^{N-1}e^{i\vec{k}_n.\vec{x}+i\theta_n(x,y)}\right|^2. \tag{2}$$

where $V_0>0$ is constant and corresponds to the peak depth of the potential, i.e., $V_0=\max_{x,y}V(x,y)$, $\vec{x}=(x,y)$, $\vec{k}_n$ is a wave vector, $\theta_n(x,y)$ is a phase function that characterizes edge irregularities or vacancy defects.

In order to compute localized solutions (i.e., soliton solutions) to nonlinear evolution equations, various techniques have been used. For a detailed information on numerical methods for solving wave equations see [30]. Below we mention some of these methods. Shooting, relaxation techniques, and the self-consistency method have been around for decades, but they are not always efficient and/or applicable for multidimensional problems. A different method was introduced by Petviashvili [31] to construct localized solutions in the two-dimensional Korteweg–de Vries equation (usually referred to as the Kadomtsev–Petviashvili equation). The idea behind Petviashvili's method is to transform the underlying governing equation to Fourier space and determine a convergence factor based upon the degree (homogeneity) of a single nonlinear term. This method has been extensively used to find localized solutions in a wide range of nonlinear systems. This method can be successfully applied to nonlinear systems only if the degree of the nonlinearity is fixed in the associated evolution equation. In fact, in nonlinear optics, many equations involve nonlinearities with different homogeneities, such as cubic-quintic, or even lack of homogeneity, as in saturable nonlinearity.

Ablowitz and Musslimani [32] proposed a generalized numerical scheme for computing solitons in nonlinear wave guides called Spectral Renormalization. The essence of the method is to transform the governing equation into Fourier space and find a nonlinear nonlocal integral equation coupled to an algebraic equation. The coupling prevents the numerical scheme from diverging. The optical mode is then obtained from an iteration scheme, which converges rapidly. This method can efficiently be applied to a large class of problems including higher order nonlinear terms with different homogeneities.

In recent years, Lakoba and Yang proposed generalizations of Petviashvili's iteration method to scalar and vector Hamiltonian equations with an arbitrary form of nonlinearity and potential functions in [33]. Later they extended this method to eliminate from the iterations a mode that is responsible either for the divergence or the slow convergence of the iterations [34]. The conjugate gradient method is yet another iterative method for solving linear systems. Lately, the conjugate gradient method was modified for finding solitary waves of nonlinear evolution [35,36].

In this work, we use the spectral renormalization method. To do this, we seek a soliton solution of Eq. (1) in the form $u(x,y,z)=f(x,y)e^{-i\mu z}$ where $f(x,y)$ is a complex-valued function and μ is the propagation constant (frequency). Substituting this form of solution into Eq. (1), the following nonlinear eigenequation for f is obtained:

$$\Delta f+[\mu+|f|^2-V(x,y)]f=0. \tag{3}$$

Eigenequation is completed with the boundary conditions

$$u\to 0 \quad \text{as} \quad |x|\to\infty$$

After applying the Fourier transformation to Eq. (3), we add and subtract a term $r\hat{u}$, where $r>0$. This procedure leads us to the following equation:

$$\hat{f}(\nu)=\hat{R}[\hat{f}]\equiv\frac{(r+\mu)\hat{f}+\mathcal{F}\{[|f|^2-V(x,y)]f\}}{r+|\nu|^2}. \tag{4}$$

Here $\mathcal{F}$ denotes the Fourier transformation, $\nu=(\nu_x,\nu_y)$ are Fourier variables and r is used to avoid a possible singularity in the denominator. We introduce a new field variable $f(x,y)=\lambda w(x,y)$, where $\lambda\neq 0$ is a constant to be determined. The iteration method takes the form $\hat{w}_{m+1}=\lambda_m^{-1}\hat{R}[\lambda_m\hat{w}_m]$, $m=0,1,2,\ldots$, where λ_m satisfies the associated algebraic condition

$$\iint_{-\infty}^{\infty}|\hat{w}_m(\nu)|^2\,d\nu=\lambda_m^{-1}\iint_{-\infty}^{\infty}\hat{R}[\lambda_m\hat{w}_m]\hat{w}_m^*(\nu)\,d\nu \tag{5}$$

It has been found that this method often prevents the numerical scheme from diverging. Thus, the soliton is obtained from a convergent iterative scheme. In this work, in order to investigate the dipole and vortex structures, as initial condition, we use multi-humped Gaussian (two-humped for dipoles, three-humped and four-humped for vortex modes) centered at either maxima or minima on the lattice structure. The iteration continues until the relative error $\delta=|\lambda_{m+1}/\lambda_m-1|$ reaches 10^{-8}. Convergence is usually obtained quickly in the semi-infinite band gap when the mode is strongly localized. Further it is observed that, the mode becomes more extended as μ gets closer to the nonlinear band gap edge and convergence of such a mode slows down during the iterations.

2. Defect lattices

Defect lattices that are considered in this study are the lattice with an edge dislocation and the lattice with a vacancy defect.

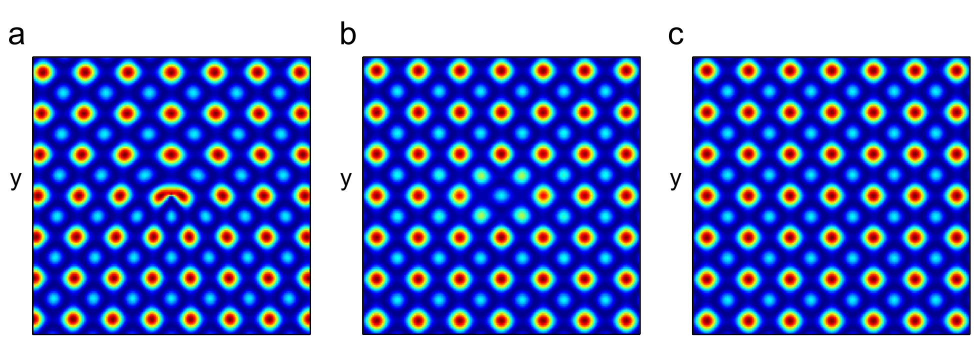

Fig. 1. Contour images of lattices: (a) lattice with an edge dislocation; (b) lattice with a vacancy defect; (c) periodic (N=4).