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Optical rogue waves generation in a nonlinear metamaterial

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ABSTRACT

We investigate the behavior of electromagnetic wave which propagates in a metamaterial for negative index regime. The optical pulse propagation is described by the nonlinear Schrödinger equation with cubic-quintic nonlinearities, second- and third-order dispersion effects. The behavior obtained for negative index regime is compared to that observed for positive index regime. The characterization of electromagnetic wave uses some pulse parameters obtained analytically and called collective coordinates such as amplitude, temporal position, width, chirp, frequency shift and phase. Six frequency ranges have been pointed out where a numerical evolution of collective coordinates and their stability are studied under a typical example to verify our analysis. It appears that a robust soliton due to a perfect compensation process between second-order dispersion and cubic-nonlinearity is presented at each frequency range for both negative and positive index regimes. Thereafter, the stability of the soliton pulse and physical conditions leading to optical rogue waves generation are discussed at each frequency range for both regimes, when third-order dispersion and quintic-nonlinearity come into play. We have demonstrated that collective coordinates give much useful information on external and internal behavior of rogue events. Firstly, we determine at what distance begins the internal excitation leading to rogue waves. Secondly, what kind of internal modification and how it modifies the system in order to build-up rogue events. These results lead to a best comprehension of the mechanism of rogue waves generation. So, it clearly appears that the rogue wave behavior strongly depends on nonlinearity strength of distortion, frequency and regime considered.

1. Introduction

Veselago was the first who theoretically has investigated electromagnetic properties of metamaterials [1,2]. This kind of materials which simultaneously exhibit negative dielectric permittivity ε and magnetic permeability μ leading to a negative refraction index are also called negative index materials. These strange media will be also known later as left-handed materials corresponding to man-made artificial structures and exhibiting unusual properties [3].

Recently, we have observed an important interest for rogue waves in the field of optics and lasers [4]. This increase of investigation activities in the optics domain has firstly been triggered by the work of Jalali group [5], where the concept of optical rogue waves was introduced. Then, rogue waves have been observed in such different physical contexts as superfluid Helium [6,7], nonlinear optics [8], capillary waves [9], plasma waves [10], waveguides [11], Bose–Einstein condensates [12], deep ocean [13–17], and left-handed nonlinear transmission line [18]. Often, the common nature of these different systems can be searched in their universal description, which is based on the nonlinear Schrödinger equation [19]. In this case, rogue waves are understood as the results of modulational instability and the subsequent formation of oscillating envelope solitons. Rogue waves can be explained also in terms of inverse cascade and in some systems they arise even in the absence of nonlinearity. Furthermore, there is not yet a common definition of rogue waves. Each system displays his specificity and especially, the statistical behavior is not described by an universal distribution, even though a common

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feature of all rogue wave phenomena is the occurrence of significant deviations from the Gaussianity [20]. Another problem is the ambiguity sometimes arising from the classification of extreme events as rogue waves, the two not necessarily coinciding. More so, rogue waves require the existence of a dispersion relation and of a coherent build-up of large amplitude pulses through the collision/interference of solitons/wavepackets traveling with different group velocities [20]. Some rogue wave phenomena have been reported in the literature, where the main physical mechanisms at the origin of rogue waves are elucidated and, when possible, compared [20]. For instance, it emerges that nonlinearity and nonlocal coupling are mechanisms that play a key role in originating rogue waves. In this context, linear experiments [21], either in optics or in microwaves [22], have the role of highlighting the essential role of granularity, that is, the fragmentation of the field in fundamental grains of activity, such as solitons arising from the modulational instability or dispersive wavepackets in the linear case[20]. Once grains are activated, the spatial inhomogeneity acts as a nonlocal coupling that provides a coherent build-up of rogue waves in different space/time positions. Consequently, it must be stressed that in all the considered systems rogue waves are the result of the dispersive properties of ensembles of many waves. In these systems, the nonlinearity need not necessarily be strong, provided the spatial inhomogeneity plays its role of mixing-up the individual grains of activity [20]. Further, optical rogue waves in the form of high peak temporal pulses have also been observed in fiber Raman amplifiers [23]. A different approach to optical rogue waves is proposed [24], where the emergence of extreme events from wave turbulence regimes is demonstrated by numerical simulations of the nonlinear Schrödinger equation. Depending on the amount of nonlinearity in the system three turbulent regimes that lead to the emergence of specific rogue wave events have been identified [24]: firstly, persistent and coherent rogue quasi-solitons, secondly, intermittent-like rogue quasi-solitons that appear and disappear erratically, and thirdly, sporadic rogue waves events that emerge from turbulent fluctuations as bursts of light or intense flashes. Moreover, the main phenomenological features that are common to rogue waves can be listed as follows. Firstly, rogue wave phenomena are characterized by large deviations of the wave amplitude statistics from the Gaussian behavior, the latter being typical of fully random systems [20]. Secondly, another fundamental feature is that the emergence of high amplitude events must be the consequence of a coherent build-up that establishes itself during the propagation/interaction of many waves in an extended spatio-temporal system [20].

Otherwise, some investigations [25–28], have used the Drude model [29], for the treatment of the nonlinear Schrödinger equation [29], from collective coordinates approach [27], in order to determine the collective coordinate equations of motion and to study the dynamics of the electromagnetic wave. Various techniques in negative index materials have been proposed to analyze the impact of dispersive effects and higher-order nonlinearities, as a function of frequency [25,28,30–32], on electromagnetic wave. However, the determination of the specific frequency ranges able to counteract or not, strong perturbations coming from quintic-nonlinearity and third-order dispersion, and physical conditions leading to optical rogue waves generation remain least reported in metamaterial systems [18]. In other words, it is difficult to determine in that conditions what will exactly happen to the light pulse which propagates in a metamaterial for both negative and positive index regimes at each frequency range.

In this paper, we analyze the behavior of electromagnetic wave which propagates in a metamaterial for negative index regime at each frequency range. This behavior is compared to that obtained for positive index regime. More so, we use the mathematical model derived from Manirupa and Amarendra [32] and which we adapt to considerations made by Scalora et al. [25]. We use a lossless Drude model and the collective coordinates technique [33,34] to obtain a good characterization of the light pulse intensity profile.

The paper is organized as follows. In Section 2, we present the nonlinear Schrödinger equation model. The coefficients of this equation are calculated for negative and positive index regimes. Thereafter, we apply the collective coordinates technique in order to obtain the collective coordinate equations of motion. In Section 3, we perform numerical simulations in order to investigate numerically the collective coordinates. Some interesting results including the stability conditions of electromagnetic wave and physical conditions leading to rogue waves generation are obtained. We compare the results obtained for both regimes to analytical and numerical investigations previously published in the literature. The outcomes are summarized in Section 4.

2. Theoretical model and analytical method

2.1. Theoretical model for electromagnetic pulse propagation in a nonlinear metamaterial

The nonlinear Schrödinger equation obtained from [32] in nonlinear metamaterial and reformulated in terms of slowly varying envelope of the electric field $\psi(Z,\tau)$ as follows [35]:

$$\frac{\partial\psi}{\partial Z} = -i\frac{\Theta_2(Z)}{2}\frac{\partial^2\psi}{\partial\tau^2} + \frac{\Theta_3(Z)}{6}\frac{\partial^3\psi}{\partial\tau^3} + i\Upsilon_0(Z)|\psi|^2\psi + i\Upsilon_a(Z)|\psi|^4\psi \tag{1}$$

where the position $Z = z/\lambda_p$ and the time $\tau = ct/\lambda_p$ with $\omega_p = 2\pi\lambda_p$ where ω_p is the electric plasma frequency and λ_p its corresponding wavelength [25]. The terms Θ_2 and Θ_3, respectively, are second- and third-order dispersion coefficients. The quantities Υ_0 and Υ_a are cubic- and quintic-nonlinearities, respectively.

Following the analytical treatment similar to that developed by Manirupa and Amarendra [32] and taking into account the considerations made by Scalora et al. [25], the coefficients of the nonlinear Schrödinger equation are expressed as follows:

$$\Theta_2 = \frac{1}{n^2\beta^2}\left[\frac{1}{2}(\varepsilon\gamma' + \mu\alpha') + \frac{1}{2}\alpha\gamma\right] - \frac{1}{n\beta V_g^2} \tag{2}$$

$$\Upsilon_0 = \frac{\beta\mu\chi^{(3)}}{2n} \tag{3}$$

$$\Upsilon_a = \frac{1}{8}\frac{\beta\,\mu^2(\chi^{(3)})^2}{n^3} \tag{4}$$

$$\Theta_3 = \frac{1}{n\beta}\left[\frac{1}{2}(\varepsilon\gamma'' + \mu\alpha'') + \frac{3}{2}(\alpha\gamma' + \alpha'\gamma)\right] - \frac{3}{V_g n^2\beta^2}\left[\frac{1}{n\beta}\left(\frac{1}{4}(\varepsilon\gamma' + \mu\alpha') + \frac{1}{2}\alpha\gamma\right) - \frac{1}{V_g^2}\right] \tag{5}$$

where the quantities $\alpha = \partial[\tilde{\omega}\varepsilon(\tilde{\omega})]/\partial\tilde{\omega}$, $\alpha' = \partial^2[\tilde{\omega}\varepsilon(\tilde{\omega})]/\partial\tilde{\omega}^2$, $\alpha'' = \partial^3[\tilde{\omega}\varepsilon(\tilde{\omega})]/\partial\tilde{\omega}^3$, $\gamma = \partial[\tilde{\omega}\mu(\tilde{\omega})]/\partial\tilde{\omega}$, $\gamma' = \partial^2[\tilde{\omega}\mu(\tilde{\omega})]/\partial\tilde{\omega}^2$, $\gamma'' = \partial^3[\tilde{\omega}\mu(\tilde{\omega})]/\ \partial\tilde{\omega}^3$, $\beta = 2\pi\tilde{\omega}$, $\tilde{\omega} = \omega/\omega_p$ and $V_g = 2n/(\varepsilon\gamma + \mu\alpha)$ [25,28].

2.2. Negative index regime for electromagnetic waves

In general, a loss Drude model is defined such that

$$\varepsilon(\tilde{\omega}) = 1 - \frac{1}{\tilde{\omega}^2 + i\tilde{\gamma}\tilde{\omega}}, \quad \mu(\tilde{\omega}) = 1 - \frac{\omega_m^2/\omega_p^2}{\tilde{\omega}^2 + i\tilde{\gamma}\tilde{\omega}},$$

where $\tilde{\gamma} = 5\times10^{-4}$ is the absorption value [25,36,37].

Moreover, absorption is one of the largest problems that needs to be addressed to enable applications of metamaterials [38,39].