Optics Communications 329 (2014) 81–87

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Focusing properties of the vector beam with complicated polarization and helical phase

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ARTICLE INFO

Article history:
Received 25 February 2014
Received in revised form
2 May 2014
Accepted 3 May 2014
Available online 14 May 2014

Keywords:
Image formation theory
Diffractive optics
Polarization

ABSTRACT

We report a vector beam with complicated polarization and helical phase and derive the corresponding focusing model of the aplanatic system, where the 10 variables optimized provide a more powerful and flexible tool to construct various interesting focal spots. By optimizing these 10 variables, we directly construct some specific focal spots, including the single focal spot with lateral resolution beyond the Abbe diffraction limit and without axial electric field component and the focal spots with unique structures in the intensity, such as the square and triangular bright spots, the various double lines, the cross lines, the ring-shaped square, the ring-shaped hexagon, and so on. Some of those focal spots with unique structures in the intensity have high resolution, even beyond the Abbe diffraction limit. We conclude the functions of some variables affecting the structures of the focal spots, draw some regulars constructing the focal spot with high quality, and indicate how to find the new values of various variables optimized in order to rotate the focal spot distribution. Although only the focal spot structures of the intensity distribution are discussed in this paper, the ideas and the developed focusing model can be used for the investigations of the various focal spot structures of polarizations, gradient forces, and spin and orbital angular momentums.

1. Introduction

Various vector beams with special polarizations and electric field distributions have very great potential in applications for nanoscale imaging, nanolithography, trapping and manipulating particles, and so on [1–13]. In the field of nanoscale imaging and nanolithography, the primary aim is to construct the focal spot with the smallest size, namely to enhance the resolution of the imaging system. In the last few decades, the most common way to enhance resolution is to use pupil filters, which have been very difficult to be further optimized. Recently, the applications of the polarization of light to enhance resolution get much attention [7–11]. This method is based on the unique focusing properties of various polarized beams in optical system with high numerical aperture (NA). For example, a radially polarized beam can generate the focal spot with a resolution higher than that for a linearly polarized illumination [14] and hence was used for microscopy [8,9]. A Bessel beam with special polarization can give a central lobe width 9% narrower than for radially polarized illumination [10]. However, those polarized beams are usually inhomogeneous polarized, which means that they are hardly used for wide field imaging and projection lithography and are more suitable for the illuminating light in confocal imaging and laser direct writing lithography. In this case, it becomes very important to enhance the speed of the imaging or lithography. An ideal way is a multifocus imaging and lithography [15–17]. However, the generation of the multifocus with high resolution, high degree of uniformity in focal intensity, and a circularly symmetrical distribution for each focal spot is very difficult [15]. Except for the multifocus imaging and lithography, another effective method is to directly generate the focal spot of specific structure, especially for the lithography of period micro/nano pattern, which is also equivalent to the projection of the micro/nano pattern. In the field of trapping and manipulating particles, the smaller the focal spot size, due to the larger gradient forces, the stronger the ability of trapping and manipulating particles. The multifocal properties of a vortex vector beam makes it become possible for the optical tweezers with multitraps [18]. Because carrying orbital angular momentum, the vortex beam with a helical phase can directly rotate particles [5]. We expect that the focal spots with various structures will have more abundant and surprising gradient forces, spin and orbital angular momentums. Therefore, if one can directly construct the focal spot of specific structure with high resolution, it is a very interesting work in the fields of nanoscale imaging, nanolithography, trapping and manipulating particles.

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http://dx.doi.org/10.1016/j.optcom.2014.05.011
0030-4018/

Recently, some analytic inversion methods are reported [19,20], but when the polarization of the light fields, calculated by the inversion method, at the incident pupil of the focusing system is complicated, one might not know how to generate the calculated incident light fields.

As we know, in theory, a complicated polarized beam may be considered as the coherent superposition of two orthogonally polarized beams, such as vector-vortex beams [18] and cylindrical vector beams [21]. In other words, we can also utilize any two orthogonally polarized beams with various distributions to compose a more complicated vector beam that has surprising focusing properties. Recently, we present a composite vector beam composed of two orthogonally linearly polarized beams with inhomogeneous polarization modulation so that the multifocus with small size, uniform intensity, and nearly circular symmetry is obtained [15,22]. However, the too few variables optimized in the polarization modulation factor [15,22] seriously limit the abilities in constructing the focal spot of specific structure with high resolution. In the past few years, the focusing properties of various vector beams also have drawn much attention [23–28], but the same problem, i.e., the too few variables optimized, still exists.

In this paper, we not only increase several variables optimized in the polarization modulation factor of the complicated vector beam, but also introduce the helical phase, which makes the numbers of the variables optimized in this vector beam reach 10 so that it is more powerful and flexible to construct interesting focal spots than the previous literatures [15,18,22–28]. We first derive the focusing model of the aplanatic system for this vector beam with complicated polarization and helical phase. Basing on this focusing model, the various focal spot structures of polarizations, gradient forces, and spin and orbital angular momentums can be constructed and are more important for many applications, but, for simplicity, we will only focus on the focal spot structures of the intensity distribution in this paper. By optimizing these 10 variables, we directly construct some unique and interesting focal spots, including the single focal spot with lateral resolution beyond Abbe diffraction limit and without axial electric field component and the focal spots with unique structures in the intensity, such as the square and triangular bright spots, the various double lines, the cross lines, the ring-shaped square, the ring-shaped hexagon, and so on. We conclude the functions of some variables in this vector beam and draw some regulars optimizing various variables to construct the focal spot with high quality (see Section 4). Finally, we indicate how to find the new values of various variables optimized in order to rotate the focal spot distribution.

2. Theory

Here, we assume that the time dependence is $\exp(-j\omega t)$ and the incident light propagating along $+z$ axis is focused by an aplanatic focusing system L obeying the sine condition (Fig. 1). The ideal focus O is the origin of rectangular axes xyz. $\sum$ is the sphere with center at O and with radius f equal to the focal length of L. n_i is the refractive index of the image space. θ_i is the angle of the emergent ray with the $-z$ axis. We assume that the incident vector beam is the coherent superposition, in varying proportions, of two orthogonally linearly polarized beams with various distributions. Therefore, the electric field of the incident vector beam at the incident pupil may be expressed as

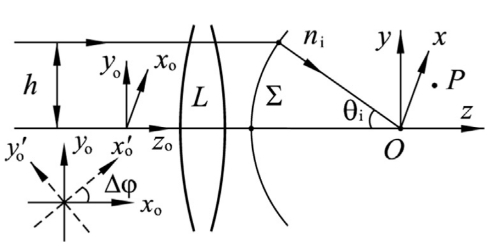

Fig. 1. Geometry of imaging of an aplanatic system.

$$\mathbf{E}_o = [\mathbf{x}e_x \cos^n(a\varphi+\varphi_{0x})+\mathbf{y}e_y \sin^m(b\varphi+\varphi_{0y})] \exp(jq\varphi)B(h)A_o(h), \tag{1}$$

where $A_o(h)$ denotes an electric field amplitude in a cylindrical coordinate system (h,φ) and $\mathbf{x}$ and $\mathbf{y}$ are the unit vector along the x and y axes, respectively. We introduce the 10 variables n, m, a, b, q, φ_{0x}, φ_{0y}, e_x, e_y and $B(h)$in Eq. (1). The first factor $\mathbf{x}e_x\cos^n(a\varphi+\varphi_{0x})+\mathbf{y}e_y\sin^m(b\varphi+\varphi_{0y})$ determines the polarization of the incident vector beam and generates very complicated polarization. Moreover, the nonnegative integers n and m determine the order of the sin&cos modulation, and the integers a and b might cause the vortex polarization of the incident beam. So, for convenience, the first factor is hereafter called the vortex polarization modulation factor. The second factor $\exp(jq\varphi)$ is a helical phase, where the integer q is the topological charge. The variables φ_{0x} and φ_{0y} denote the initial phase in the two orthogonally polarized beams and the complex numbers e_x and e_y represent the weight of the coherent superposition of two orthogonally polarized beams, respectively. As simple examples, for $n=m=0$, $\mathbf{E}_o$ is linearly polarized for $e_y/e_x=\pm 1$ and circularly polarized for $e_x/e_y=\pm j$. For $n=m=a=b=e_x=e_y=1$, it is radially polarized for $\varphi_{0x}=\varphi_{0y}=0,\pi$ and azimuthally polarized for $\varphi_{0x}=\varphi_{0y}=0.5\pi,\ 1.5\pi$. The modulation factor $B(h)$ denotes a pupil filter with cylindrical symmetry. If $B(h)$ is a real, pure imaginary, or complex number, it represents an additional amplitude, phase, or complex pupil filter, respectively.

For the aplanatic focusing system obeying the sine condition, by a procedure described in the vector diffraction theory of Richards and Wolf [29], the vector plane-wave spectrum at the focal plane can be expressed as [30]

$$\begin{aligned}\tilde{\mathbf{E}}_i = j\lambda f \cos^{-\frac{1}{2}}\theta_i \times \Big\{&\mathbf{x}[(\sin^2\varphi+\cos\theta_i\cos^2\varphi)e_{ox}\\ &+(\cos\theta_i-1)\sin\varphi\cos\varphi e_{oy}]\\ &+\mathbf{y}[(\cos\theta_i-1)\sin\varphi\cos\varphi e_{ox}+(\cos^2\varphi+\cos\theta_i\sin^2\varphi)e_{oy}]\\ &+\mathbf{z}\sin\theta_i(\cos\varphi e_{ox}+\sin\varphi e_{oy})\Big\},\end{aligned} \tag{2}$$

where e_{ox} and e_{oy} are different from the e_x and e_y shown in Eq. (1) and may represent any function in the direction of the x and y polarization of the incident vector beam. Therefore, for the complicated electric field of the incident vector beam given by Eq. (1), the corresponding vector plane-wave spectrum at the focal plane can be written as

$$\begin{aligned}\tilde{\mathbf{E}}_i = j\lambda f &\cos^{-\frac{1}{2}}\theta_i \exp(jq\varphi)B(h)A_o(h)\\ &\times\{\mathbf{x}[(\sin^2\varphi+\cos\theta_i\cos^2\varphi)\cos^n(a\varphi+\varphi_{0x})e_x\\ &+(\cos\theta_i-1)\sin\varphi\cos\varphi\sin^m(b\varphi+\varphi_{0y})e_y]\\ &+\mathbf{y}[(\cos\theta_i-1)\sin\varphi\cos\varphi\cos^n(a\varphi+\varphi_{0x})e_x\\ &+(\cos^2\varphi+\cos\theta_i\sin^2\varphi)\sin^m(b\varphi+\varphi_{0y})e_y]\\ &+\mathbf{z}\sin\theta_i[\cos\varphi\cos^n(a\varphi+\varphi_{0x})e_x+\sin\varphi\sin^m(b\varphi+\varphi_{0y})e_y]\}.\end{aligned} \tag{3}$$

For an arbitrary point P in the image space, its coordinates are referred to the spherical polar coordinates (r_p,α,β) with origin at the focus O, with the polar axis $\alpha=0$ in the $+z$ axis, and with the azimuth $\beta=0$ in the $+x$ axis, i.e., $x=r_p\sin\alpha\cos\beta$, $y=r_p\sin\alpha\sin\beta$ and $z=r_p\cos\alpha$. In term of the inverse transform formula [30], the electric field of the point P can be expressed