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“Transfer matrix” method for direct and indirect coupling of cascaded cavities in resonator-waveguide systems

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ABSTRACT

Using formalism similar to the transfer matrix, we analyze the problem of direct and indirect coupling of cascaded cavities in resonator-waveguide systems based on the temporal coupled-mode theory. The recurrence relations between modes supported by the cascaded cavities are established by using transfer matrix technique. By setting two auxiliary cavities as the start and termination conditions, the optical properties of the cascaded cavity system can be obtained, especially convenient to get the transmission properties. We validate our method by comparing its prediction with the finite-difference time-domain method in a channel drop filter system in photonic crystals and obtaining excellent agreement between the two. The new analytical method provides a powerful tool to design and analyze complex cavity-waveguide systems.

1. Introduction

Resonator-waveguide coupling systems have received a great deal of attention recently because of the potential use in design of optical devices in photonic integrated circuits, for example, channel drop filters [1–3], power splitters [4,5] and intersections [6,7]. Both the directly coupled systems, where the nearest resonators are coupled with each other via an evanescent optical field, and the indirectly coupled systems, where the multiple resonators are coupled together by the propagating modes within the waveguide have been intensively studied for the mode coupling mechanisms and the transmission properties using various methods such as tight-binding approximation [8], scattering-theory analysis [1,9,10] and temporal coupled-mode theory [11]. Among them, tight-binding approximation, where the optical mode of the whole system are regarded as a linear combination of the optical modes within each individual cavity can be used to analyze mode properties of the direct coupling systems. Scattering-theory analysis, similar to the quantum scattering theory, is generally used to investigate the indirect coupling systems [9], as well as direct coupling systems [1,10]. Temporal coupled-mode theory, which provides an extraordinarily intuitive and accurate analytical framework for modeling resonant behavior, has been tremendously successful in a wide variety of systems [11–15], even for general free-space resonant scattering of waves [16].

In this work, we analyze direct and indirect coupling resonator-waveguide systems and investigate the transmission properties by using formalism similar to the transfer matrix based on the temporal coupled-mode theory. By introducing two auxiliary cavities as the start and termination conditions, the recurrence relations between modes are established by transfer matrix and optical properties of the resonator-waveguide system can be obtained. The method is highly effective for periodic resonator-waveguide systems and can handle both direct and indirect coupling. As a model system, a photonic crystal channel drop filter was numerically studied by the transfer matrix method and compared with the finite-difference time-domain method.

2. “Transfer matrix” method

Fig. 1 shows the structure of the coupling system, which consists of a cascade of n cavities side-coupled to m waveguides. The ith cavity can support p modes and the normalized amplitudes of the ith cavity are denoted by a_i, which is a vector described by

$$a_i = \begin{pmatrix} a_i^{(1)} \\ a_i^{(2)} \\ \vdots \\ a_i^{(p-1)} \\ a_i^{(p)} \end{pmatrix} \quad (1)$$

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$S_{\pm ijr}(i=1,2,\cdots,n,\ j=1,2,\cdots,m,\ r=1,2)$ are the amplitudes of the incoming $(+)$ and outgoing $(-)$ waves in the jth waveguide side-coupled into the ith cavity. The transmission and reflection properties of such a system can be calculated by the coupled-mode theory in time domain [11]. When the electromagnetic wave at a frequency ω is incident upon the system, the time evolution of the amplitudes of the cavities in steady state can be described as

$$\frac{da_i}{dt}=(j\Omega_i-\Gamma_i)a_i-M_i a_{i+1}-M_{i-1}a_{i-1}+K_{i1}^T|S_{+i1}\rangle+K_{i2}^T|S_{+i2}\rangle \quad i=1,2,3,\cdots,n \tag{2}$$

$$|S_{-i1}\rangle=C_i|S_{+i2}\rangle+D_i a_i, \tag{3}$$

$$|S_{-i2}\rangle=C_i|S_{+i1}\rangle+D_i a_i, \tag{4}$$

where Ω_i is a $p\times p$ matrix which represents resonant frequencies and coupling between the cavity modes, Γ_i is a $p\times p$ matrix which represents the decay rate, M_i is a $p\times p$ matrix which represents the mutual coupling between mode amplitudes a_i and a_{i+1} through a direct evanescent tunneling process, and the amplitudes of the incoming $(+)$ and outgoing waves $(-)$ are

$$|S_{\pm ir}\rangle=\begin{pmatrix} S_{\pm i1r}\\ S_{\pm i2r}\\ \vdots \\ S_{\pm i(m-1)r}\\ S_{\pm imr}\end{pmatrix}(r=1,2)\cdot \tag{5}$$

K_i^T and D_i represent the coupling matrices between the resonant modes and the incoming and the outgoing waves, respectively. C_i is the $m\times m$ scattering matrix, which describes the coupling between the incoming waves and the outgoing waves through a direct pathway. Due to the direct coupling between cavities and also the indirect coupling via waveguides, the incoming waves to the cavities should satisfy the relationships,

$$|S_{-i2}\rangle=|S_{+(i-1)1}\rangle e^{j\beta_i}, \tag{6}$$

$$|S_{+i2}\rangle=|S_{-(i-1)1}\rangle e^{-j\beta_i}, \tag{7}$$

where β_i is the phase shift incurred as the waveguide mode propagates from ith cavity to $(i-1)$th cavity. So Eqs. (2)–(4) can be rewritten in transfer matrix form as

$$\phi_i=A_i\phi_{i-1} \tag{8}$$

with

$$A_i=\begin{bmatrix} -M_i^{-1}[j(\omega I-\Omega_i)+\Gamma_i+K_{i1}^T C_i^{-1}D_i] & -M_i^{-1}M_{i-1} & M_i^{-1}K_i^T C_i^{-1}e^{j\beta_i} & M_i^{-1}K_i^T e^{-j\beta_i}\\ I & 0 & 0 & 0\\ -D_i & 0 & C_i^{-1}e^{j\beta_i} & 0\\ D_i & 0 & 0 & C_i e^{-j\beta_i}\end{bmatrix} \tag{9}$$

$$\phi_i=\begin{pmatrix} a_{i+1}\\ a_i\\ |S_{+i1}\rangle\\ |S_{-i1}\rangle\end{pmatrix}, \tag{10}$$

where I is a $p\times p$ identity matrix. Examining Eq. (8), an equation $a_i=a_i$ is added to bridge a_{i+1} and a_{i-1} in Eq. (2) and to make the transfer matrix connect the inputs and the outputs of each cavity successively. It should be noted that matrices A_i are sparse block matrices and therefore easy to evaluate them numerically or analytically. ϕ_i are the normalized amplitudes of the resonant and propagation modes. The relationships between the two cavity segments of the system can be described as

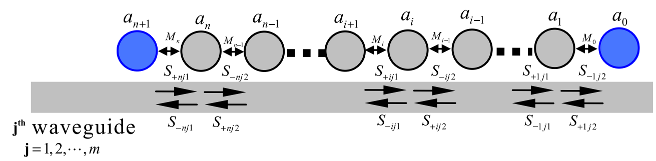

Fig. 1. Illustration of the direct and indirect coupling of n cascaded cavities into m waveguides in waveguide-resonator system. The two side cavities are the auxiliary cavities with $a_{n+1}=0$ and $a_0=0$.

$$\phi_n=A_nA_{n-1}\cdots A_1\phi_0, \tag{11}$$

where ϕ_0 and ϕ_n are the start and termination conditions described as

$$\phi_0=\begin{pmatrix} a_1\\ 0\\ |S_{-12}\rangle e^{-j\beta_0}\\ |S_{+12}\rangle e^{j\beta_0}\end{pmatrix}, \tag{12}$$

$$\phi_n=\begin{pmatrix} 0\\ a_n\\ |S_{+n1}\rangle\\ |S_{-n1}\rangle\end{pmatrix}\cdot \tag{13}$$

In the start and termination conditions, the amplitudes of the two auxiliary cavities $a_{n+1}=0$ and $a_0=0$ are used to make Eq. (11) solvable.

For simplification without sacrificing the physics, we consider that all the cavities have the same resonant frequencies, coupling coefficients, and phase shift during the indirect coupling process, i.e., $\Omega_i=\Omega$, $\Gamma_i=\Gamma$, $M_i=M$, $D_i=D$, $C_i=C$, $K_i^T=K^T$, and $\beta_i=\beta$, Eq. (9) can be written as

$$\phi_n=W\phi_0, \tag{14}$$

where

$$W=A^n\cdot \tag{15}$$

Analytically, Eq. (15) can be solved by using the Hamilton–Cayley theorem [17]. When the inputs of the system $|S_{+n1}\rangle$ and $|S_{+12}\rangle$ are known, the properties of the system can be obtained from Eqs. (14) and (8). For example, when the input light is incident upon the resonator-waveguide system from one side, i.e., $|S_{+12}\rangle=0$, Eq. (14) can be expressed as

$$\begin{pmatrix} 0\\ 0\\ |S_{+n1}\rangle\\ 0\end{pmatrix}=\begin{bmatrix} W_{11} & 0 & W_{13} & 0\\ W_{21} & -I & W_{23} & 0\\ W_{31} & 0 & W_{33} & 0\\ W_{41} & 0 & W_{43} & -I\end{bmatrix}\begin{pmatrix} a_1\\ a_n\\ |S_{-12}\rangle e^{-j\beta}\\ |S_{-n1}\rangle\end{pmatrix}\cdot \tag{16}$$