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Large phase difference of soliton-like mutually-trapped beam pairs in strong nonlocal media

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ABSTRACT

We discuss the propagation of two orthogonally polarized beams in nonlocal planar waveguides by variational approach as well as a numerical method. The evolution equations for parameters and the critical powers for soliton-like mutually-trapped propagation of the two beams are obtained. Moreover, we analyze the influence of coupling coefficient, initial power, birefringence and the degree of nonlocal on mutually-trapped propagation. In addition, we find that the two beams will have large phase difference since phase shifts of the two beams are different under certain conditions. This theoretical result may have potential applications in the light-control-light technology.

1. Introduction

As is widely known, nonlocal and local spatial optical solitons have different characteristics. For instance, $(1+2)$-dimension local spatial optical soliton is unstable, while $(1+2)$-dimension nonlocal spatial optical soliton is stable [1,2]. Therefore, the nonlocal spatial optical solitons which are modeled by the nonlocal nonlinear Schrodinger equation (NNLSE) have been extensively studied [3–7] in recent years. According to the degree of nonlocality determined by the ratio of the characteristic length of the material to the beam width, nonlocality can be classified into four categories: strongly nonlocal, generally nonlocal, weakly nonlocal and local [3]. Snyder and Mitchell [4] simplified the NNLSE to a linear model (S–M model) in a strongly nonlocal case, and obtained an exact stationary analytical solution to the model . In 2004, Guo et al. [5] expanded the response function in Taylor series to the second order twice, and obtained a strong nonlocal model and proved that the phase shift of such a nonlocal spatial optical soliton is large comparable to its local counterpart . In general nonlocal media, the analytical solution can be obtained by expanding the response function in Taylor series to the fourth-order [6]. Since the characteristic length of the weakly nonlocal medium is much narrower than the beam width, the beam width can be expanded in Taylor series to the second order, and a Sech-shaped stationary solution is obtained [7]. So far, many kinds of novel nonlocal solitons have been studied, such as nonlocal Bragg solitons [8], nonlocal vortex solitons [9], nonlocal dark solitons [10–12], spiraling and multipole solitons [13], rotating dipole solitons [14], nonlocal gap solitons [15], quadratic solitons [16–18] and nonlocal description of X waves in quadratic nonlinear materials [19]. Furthermore, more different classes of nonlocal materials have been found, for example, nematic liquid crystal [20], lead glass [21], Bose–Einstein condensates [22] and liquid infiltrated photonic crystal fibers [23]. In recent years, the nonlocal vector solitons have attracted much attention among optical community. For instance, Kartashov [24] discussed multipole vector soliton in nonlocal media , Zhi [25] analyzed the existence and stability of two-component vector soliton in nematic liquid crystals for which one of the components carries angular momentum and describes a vortex beam , Wang and coworkers [26] investigated the incoherently coupled two-color Manakov vector solitons and incoherently coupled vector dipole soliton pairs ([27]) in nonlocal media.

Rigorous conditions for realizing Manakov solitons (vector solitons) are as follows: (1) the ratio (coupling coefficient) of the self-phase modulation (SPM) to the cross-phase modulation (XPM) should be equal to unity; (2) the SPM coefficients need to be equal for the two polarizations; (3) the energy change terms, sometimes known as the four wave mixing (FWM) terms, must be zero [28,29]. Thus the coupled nonlinear Schrodinger equations which describe the propagation of optical beam with two orthogonal polarizations are integrable. However, the case of these conditions cannot be completely satisfied [29,30]. For example, the

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ratio of SPM/XPM is 3/2 in fused quartz [29]. In addition, media with linear or nonlinear anisotropy has been found in the available materials such as nematic liquid crystal [20] and lead glass [21]. Herein, the study which analyzes the impact of the coupling coefficient, initial power and birefringence on the propagation of two orthogonally polarized beams in planar nonlocal waveguides is necessary and has actual significance.

2. Theoretical model

The propagation of two orthogonally polarized beams in nonlocal planar waveguide could be well described by the following coupled Eqs. [17–20]:

$$i\frac{\partial\psi_1}{\partial z}+\mu_1\frac{\partial^2\psi_1}{\partial x^2}+\rho_1\phi_1\int_{-\infty}^{+\infty}R(x-x')\left[|\psi_1(x',z)|^2+m_1|\psi_2(x',z)|^2\right]dx'=0, \tag{1a}$$

$$i\frac{\partial\psi_2}{\partial z}+\mu_2\frac{\partial^2\psi_2}{\partial x^2}+\rho_2\psi_2\int_{-\infty}^{+\infty}R(x-x')\left[|\psi_2(x',z)|^2+m_2|\psi_1(x',z)|^2\right]dx'=0, \tag{1b}$$

The propagation of soliton in nonlocal media with different response function, such as square [10,31], exponential [20] or Gaussian-shaped response function [5], all have been studied. In this paper, we adopt the Gaussian-shaped response function. This is because the analytic solution of NNLSE can be obtained conveniently by using it and the physical properties do not depend strongly on the concrete shape of the response function [32].

$$R(x-x')=\frac{1}{\sqrt{\pi}\sigma}\exp\left[-\frac{(x-x')^2}{\sigma^2}\right], \tag{2}$$

where $\psi_j(j=1,\ 2)$ is the slowly varying amplitude for TE and TM modes (subscript 1 or 2 stands for TE or TM modes), respectively. $\mu_j=1/2k_j$, $k_j=k_0n_{0j}$, k_j is the propagation constant in the media, k_0 is the propagation constant in the vacuum and n_{0j} is the linear refractive index of the media, thus $\rho_j=k_j\eta_j=k_jn_2/n_{0j}=k_0n_2$ [5,29,30]. m_j represents coupling coefficient, σ is the characteristic length of the response function. We assume that the coupling coefficients are equal as references [29,30], namely $m_1=m_2=m$.

In this paper, we adopt the variational approach which has been exploited in studying the two-component bright solitons in nonlinear media [33,34]. The Lagrange density equation corresponding to Eqs. (1a) and (1b) is as follows:

$$\begin{aligned}L=&\frac{i}{2}\left(\psi_1^*\frac{\partial\psi_1}{\partial z}-\psi_1\frac{\partial\psi_1^*}{\partial z}\right)-\mu_1\left|\frac{\partial\psi_1}{\partial x}\right|^2\\&+\frac{1}{2}k_0n_2|\psi_1|^2\int_{-\infty}^{+\infty}R(x-x')[|\psi_1(x',z)|^2+m|\psi_2(x',z)|^2]dx'\\&+\frac{i}{2}\left(\psi_2^*\frac{\partial\psi_2}{\partial z}-\psi_2\frac{\partial\psi_2^*}{\partial z}\right)-\mu_2\left|\frac{\partial\psi_2}{\partial x}\right|^2\\&+\frac{1}{2}k_0n_2|\psi_2|^2\int_{-\infty}^{+\infty}R(x-x')[|\psi_2(x',z)|^2+m|\psi_1(x',z)|^2]dx',\end{aligned} \tag{3}$$

We assume that the two orthogonally polarized beams are Gaussian-shaped.

$$\psi_1(x,z)=A_1(z)\exp\left[ic_1(z)x^2+i\theta_1(z)-\frac{x^2}{2a_1(z)^2}\right], \tag{4a}$$

$$\psi_2(x,z)=A_2(z)\exp\left[ic_2(z)x^2+i\theta_2(z)-\frac{x^2}{2a_2(z)^2}\right], \tag{4b}$$

where $A_i(z)$ $(i=1,\ 2)$ is the amplitude, $\theta_i(z)$ is the phase of complex amplitude, $c_i(z)$ is the phase-front curvature of the beam and $a_i(z)$ represents the beam width.

Inserting the trial function Eq. (4a) and (4b) into Eq. (3) and then integrating over x, we obtained the following:

$$\begin{aligned}L_r=&-\frac{\mu_1A_1^2\sqrt{\pi}}{2a_1}-2\sqrt{\pi}\mu_1A_1^2c_1^2a_1^3-A_1^2\sqrt{\pi}\left[a_1\frac{d\theta}{dz}+a_1^3\frac{dc_1}{2dz}\right]\\&+\frac{k_0n_2\sqrt{\pi}A_1^4a_1^2}{2\sqrt{\delta^2+2a_1^2}}\\&-\frac{\mu_2A_2^2\sqrt{\pi}}{2a_2}-2\sqrt{\pi}\mu_2A_2^2c_2^2a_2^3-A_2^2\sqrt{\pi}\left[a_2\frac{d\theta}{dz}+a_2^3\frac{dc_2}{2dz}\right]\\&+\frac{k_0n_2\sqrt{\pi}A_2^4a_2^2}{2\sqrt{\delta^2+2a_2^2}}\\&+\frac{\sqrt{\pi}mk_0n_2A_1^2A_2^2a_1a_2}{\sqrt{\delta^2+a_2^2+a_1^2}},\end{aligned} \tag{5}$$

By variational principle, evolution equations for the parameters of the two beams can be obtained.

$$\frac{da_1}{dz}-4\mu_1c_1a_1=0, \tag{6a}$$

$$\frac{dc_1}{dz}=\frac{\mu_1}{a_1^4}-4\mu_1c_1^2-\frac{k_0n_2A_1^2a_1}{(\sigma^2+2a_1^2)^{3/2}}-\frac{mk_0n_2A_2^2a_2}{(\sigma^2+a_1^2+a_2^2)^{3/2}}, \tag{6b}$$

$$\begin{aligned}\frac{d\theta_1}{dz}=&-\frac{\mu_1}{a_1^2}+\frac{k_0n_2A_1^2a_1^3}{2(\sigma^2+2a_1^2)^{3/2}}+\frac{k_0n_2A_1^2a_1}{\sqrt{\sigma^2+2a_1^2}}\\&+\frac{mk_0n_2A_2^2a_1^2a_2}{2(\sigma^2+a_2^2+a_1^2)^{3/2}}+\frac{mk_0n_2A_2^2a_2}{\sqrt{\delta^2+a_2^2+a_1^2}},\end{aligned} \tag{6c}$$

$$\frac{da_2}{dz}-4\mu_2c_2a_2=0, \tag{6d}$$

$$\frac{dc_2}{dz}=\frac{\mu_2}{a_2^4}-4\mu_2c_2^2-\frac{k_0n_2A_2^2a_2}{(\sigma^2+2a_2^2)^{3/2}}-\frac{mk_0n_2A_1^2a_1}{(\sigma^2+a_2^2+a_1^2)^{3/2}}, \tag{6e}$$

$$\begin{aligned}\frac{d\theta_2}{dz}=&-\frac{\mu_2}{a_2^2}+\frac{k_0n_2A_2^2a_2^3}{2(\sigma^2+2a_2^2)^{3/2}}+\frac{k_0n_2A_2^2a_2}{\sqrt{\sigma^2+2a_2^2}}\\&+\frac{mk_0n_2A_1^2a_2^2a_1}{2(\sigma^2+a_2^2+a_1^2)^{3/2}}+\frac{mk_0n_2A_1^2a_1}{\sqrt{\delta^2+a_2^2+a_1^2}},\end{aligned} \tag{6f}$$

The incident powers of the two orthogonally polarized beams are as follows:

$$P_1=\int|\phi_1(x,z)|^2dx=A_1^2a_1\sqrt{\pi}, \tag{7a}$$

$$P_2=\int|\phi_2(x,z)|^2dx=A_2^2a_2\sqrt{\pi}, \tag{7b}$$

Combining Eqs. (6a), (6b), (6d), and (6e), we obtain the evolution equations of the beams width as follows:

$$\frac{d^2a_1}{dz^2}=\frac{4\mu_1^2}{a_1^3}-\frac{4\mu_1k_0n_2P_1a_1}{\sqrt{\pi}(\sigma^2+2a_1^2)^{3/2}}-\frac{4\mu_1a_1mk_0n_2P_2}{\sqrt{\pi}(\sigma^2+a_2^2+a_1^2)^{3/2}}, \tag{8a}$$

$$\frac{d^2a_2}{dz^2}=\frac{4\mu_2^2}{a_2^3}-\frac{4\mu_2k_0n_2P_2a_2}{\sqrt{\pi}(\sigma^2+2a_2^2)^{3/2}}-\frac{4\mu_2a_2mk_0n_2P_1}{\sqrt{\pi}(\sigma^2+a_2^2+a_1^2)^{3/2}}, \tag{8b}$$

We will discuss the evolution law of the beam which is described by Eqs. (8a) and (8b). Assuming that the two beam widths are equal and unchanged, $a_0=a_1=a_2$, $d^2a_j/dz_j^2|_{z=0}=0$, we obtain the critical powers of the mutually-trapped propagation.

$$P_{c1}=\frac{[\mu_2m-\mu_1]\sqrt{\pi}(\sigma^2+2a_0^2)^{3/2}}{a_0^4(m^2k_0n_2-k_0n_2)}, \tag{9a}$$