Optics Communications 329 (2014) 135–139

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Dynamics and trajectory of nonautonomous rogue wave in a graded-index planar waveguide with oscillating refractive index

Li Wang [a], Xiao-Qiang Feng [a], Li-Chen Zhao [b,*]

[a] Institute of Photonics Photon-Technology, Northwest University, Xi'an 710069, China
[b] Department of Physics, Northwest University, Xi'an 710069, China

ARTICLE INFO

Article history:
Received 17 December 2013
Received in revised form
4 May 2014
Accepted 5 May 2014
Available online 17 May 2014

Keywords:
Rogue wave
Oscillating refractive index
Nonlinear waveguide

ABSTRACT

We study rogue waves in a graded-index planar waveguide with oscillating refractive index. We find that an additional refractive index can be used to manipulate the trajectory of the rogue wave without changing its shape evolution characters. The density distribution profile of rogue wave with the highest peak can be kept well through manipulating the graded-index term and nonlinear coefficient. Furthermore, the trajectories of these nonautonomous rogue waves still look like an "X" shape. These results provide possibilities to manipulate rogue wave in nonautonomous nonlinear systems.

1. Introduction

Recently, rogue wave (RW) were found to exist in many differen physical systems, such as ocean [1–6], water wave tank [7], nonlinear fiber [8,9], in plasma [10], and even in microwave cavities [11]. Many RW experiments in nonlinear systems suggest that the rational solution of the simplified nonlinear Schrodinger (NLS) equation can be used to describe RW behavior well [7,10,12]. However, the simplified NLS cannot describe the dynamics of RW in nonlinear system with variable coefficients. Moreover, the nonlinear coefficient and other parameters are usually variable in real systems. Can the perturbation signals for RW still evolve to be RW in the nonlinear system with variable coefficients? If they can, can the dynamical characters of RW be manipulated through controlling some physical coefficients?

For nonlinear waveguide, there are many system coefficients which can be manipulated conveniently, such as external refractive index and Kerr-type coefficient, etc. These characters make the nonlinear waveguide be convenient to discuss the above two questions [13]. The position and direction control of optical rogue waves were proposed in nonlinear graded-index waveguide amplifiers [14]. Furthermore, the refractive index in the waveguide can be varied to be oscillating with propagation distance conveniently. Can RW still exist on plane wave background even with an oscillating refractive index? On the other hand, RW just seems to appear from nowhere and disappear without a trace. In fact it has its trajectory characters, for example, the trajectory of Peregrine RW's valleys is an "X" shape [20]. The highest peak is found to emerge when the distance between two valleys is the smallest. Then, what about the trajectories of RWs in these nonautonomous cases?

In this paper, we study on rogue waves in a nonlinear planar waveguide with oscillating refractive index. The explicit trajectory and structural properties are investigated through defining certain property functions. The results indicate that RW can still exist in the planar waveguide with oscillating refractive index. The oscillating term has no effects on the structure properties of RW, and it just affect the trajectory of the rogue wave. The trajectory and width evolution of the "catched" RW are compared with the standard Peregrine RW explicitly. The trajectory of RW valleys still look like an "X" type with some deviations. But the changing rate of width between the two valleys can be managed well through varying the graded-index and nonlinear coefficient. These results are helpful to find ways to manage them experimentally in a planar waveguide system.

2. The model and nonautonomous rogue waves solutions

For nonlinear waveguide, there are many variable physical coefficients, such as nonlinear coefficient, graded-refractive index, gain or loss effects and long-period grating. Therefore, we begin from the following generalized NLS with varying coefficients:

$$i\frac{\partial u(x,z)}{\partial z}+\frac{\partial^2 u(x,z)}{\partial x^2}+R(z)|u(x,z)|^2u(x,z)$$

* Corresponding author.
E-mail addresses: xqfeng@nwu.edu.cn (X.-Q. Feng), zhaolichen3@163.com (L.-C. Zhao).

http://dx.doi.org/10.1016/j.optcom.2014.05.017
0030-4018/

$$+[F(z)x^2+f(z)x]u(x,z)+iG(z)u(x,z)=0. \quad (1)$$

Here, x and z are the spatial coordinate and propagation distance, respectively. $R(z)$, $F(z)$, and $G(z)$ are functions of the normalized distance z. $R(z)$ represents Kerr nonlinearity, which can be negative or positive, corresponding to the graded-index medium acts as self-focusing or self-defocusing Kerr nonlinearities. $F(z)x^2$ denotes the graded-refractive index of the waveguide, $G(z)$ stands for gain or loss in the waveguide and $f(z)x$ represents an additional variable refractive index. The nonautonomous soliton in this system has been studied in [15]. Here, we focus on the dynamics of RW in this system. Considering the following form to simplify Eq. (1):

$$u(x,z)=Q(X,Z)\exp\left[ia(x,z)+C(z)-\int G(z)\,dz\right]. \quad (2)$$

With constrain conditions that

$$F(z)=\left(\frac{dC(z)}{dz}\right)^2-\frac{d^2C(z)}{2\,dz^2},$$

$$R(z)=2\exp\left[2C(z)+2\int G(z)\,dz\right],$$

and

$$a(x,z)=-\frac{dC(z)}{2\,dz}x^2+xe^{2C(z)}\int f(z)e^{-2C(z)}\,dz-\int e^{4C(z)}\left(\int f(z)e^{-2C(z)}\,dz\right)^2 dz,$$

we can transform Eq. (1) to be the simplified NLS [19]

$$i\frac{\partial Q(X,Z)}{\partial Z}+\frac{\partial^2 Q(X,Z)}{\partial X^2}+2|Q(X,Z)|^2Q(X,Z)=0, \quad (3)$$

where

$$X=xe^{2C(z)}-2\int e^{4C(z)}\left(\int f(z)e^{-2C(z)}\,dz\right)dz,$$

$$Z=\int e^{4C(z)}\,dz.$$

Then we derive an exact RW solution of the nonautonomous system from the well-known Peregrine RW as follows:

$$u(x,z)=\left[1-\frac{4(1+4iZ)}{1+4X^2+16Z^2}\right]\exp[2iZ+ia(x,z)]\times\exp\left[C(z)-\int G(z)\,dz\right]. \quad (4)$$

The functions $G(z)$ and $C(z)$ can be chosen arbitrary, as long as the function can be integrated. We emphasize that RW solution obtained here could be used to study dynamics of nonautonomous RW in many different cases conveniently. Especially, when $C(z)=0$, and $G(z)=0$, the nonautonomous RW solution will become the well known Peregrine RW [9,10,12]. Especially, when $C(z)=\int G(z)\,dz$, the amplitude of background on which RW exists will be unchanged. To observe dynamics of RW conveniently, all following discussions are made with the condition. Under this condition, we can manipulate RW through varying the parameters $C(z)$ and $f(z)$. These manipulations could be helpful to understand the fundamental character or even mechanism of RW.

When $f(z)=l\cos(wz)$, a similar nonautonomous RW solution has been derived in [16]. Until now, most studies demonstrate the whole evolution characters of nonautonomous RW [13,16–18]. However, the explicit physical properties have not been shown explicitly. In this paper, we define some related property functions to describe its explicit characters. For example, the trajectory of RW can be described by the trajectories of its hump and two valleys's centers [20]. For the generalized nonautonomous RW, the trajectory can be described by x_h and x_v which denote the trajectory of the hump and valleys of RW respectively. The explicit expressions are

$$x_h=2e^{-2C(z)}\int e^{4C(z)}\left(\int f(z)e^{-2C(z)}\,dz\right)dz, \quad (5)$$

$$x_v=2e^{-2C(z)}\int e^{4C(z)}\left(\int f(z)e^{-2C(z)}\,dz\right)dz\pm\frac{\sqrt{3}}{2}e^{-2C(z)}\sqrt{1+16\left(\int e^{4C(z)}\,dz\right)^2}. \quad (6)$$

Moreover, the RW's structure evolution can be described by hump's peak and the valleys' depth. In [21], the width of RW was defined as the distance between two valleys' centers. Here, to observe the

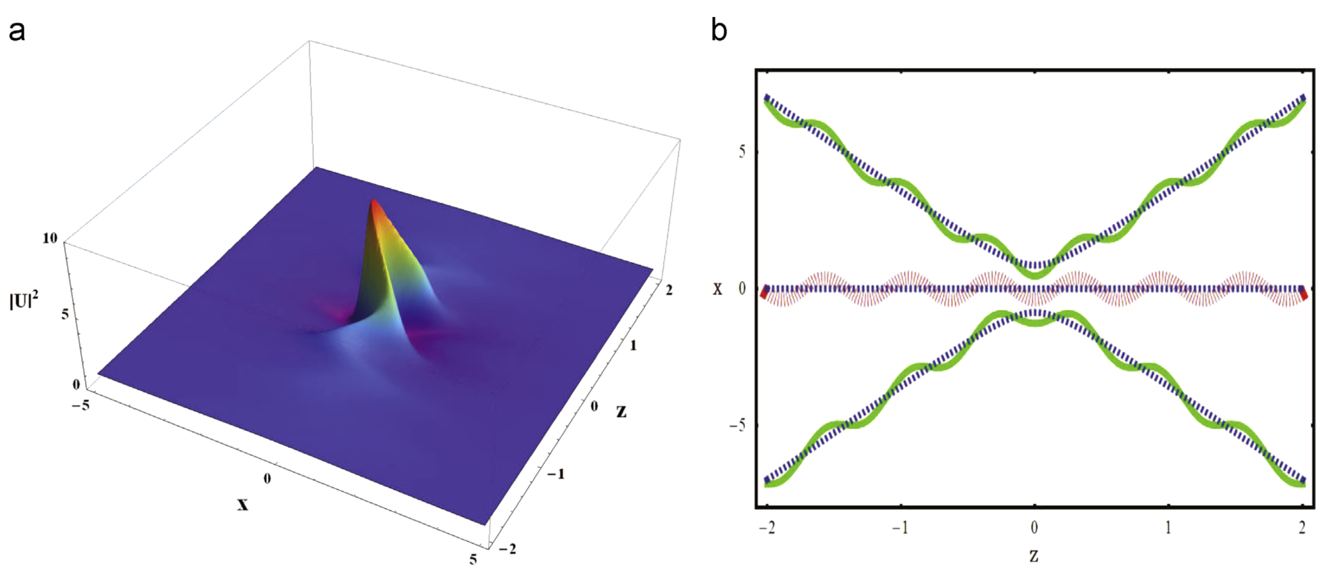

Fig. 1. (a) The evolution of RW which appears in planar waveguide with oscillating refractive index and without graded-index and gain. It is seen the RW oscillates with propagation evolution. (b) The trajectories of the oscillating RW and Peregrine RW. Red dashed line denotes the trajectory of oscillating RW's hump's center, green solid lines correspond to the oscillating RW's two valleys. The blue dashed lines are the trajectory of Peregrine RW. The parameters are $l=10$, $w=20$, $G(z)=0$, and $C(z)=0$. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)