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Theoretical examination of the slot channel waveguide configured in a cylindrically symmetric dielectric ring profile

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ABSTRACT

It has recently been experimentally demonstrated that slot channel waveguides, configured in cylindrical space, can support high azimuthal order modes similar to whispering-gallery modes. This paper presents a mode solver based on Maxwell's vector wave equation for the electric field cast into an eigenvalue problem using a Fourier–Bessel basis function space. The modal frequencies and field profiles of the high azimuthal order slot-channel-whispering-gallery (SCWG) modes are computed for a set of nanometer spaced silicon rings supported by oxide. The computations show, that in addition to the traditionally observed, lowest order mode, the structure may support higher order SCWG modes. We complete the analysis by computing structures response as an ambient medium index of refraction sensor which achieves over 400 nm per RIU sensitivity.

1. Introduction

Silicon based photonics provides researchers with a suitable platform from which a large number of waveguide and device configurations have been successfully realized. A common guidance method is to concentrate the light fields in the silicon region and rely on index or band gap guiding [1,2]. An alternate guidance technique, known as the slot channel waveguide, exploits the continuity of the normal component of the electric flux density to provide a large electric field in a low relative dielectric medium sandwiched between thin silicon strips. Two standard slot waveguide configurations are usually encountered; A vertical integration consisting of parallel narrow silicon ridges with sub-micron inner separation resting on a low dielectric oxide [3,4]; A horizontal layer stacked configuration consisting of two thin silicon strips separated by a narrow oxide with the lower silicon resting on a support oxide [5]. Both of these waveguide geometries are generally configured into planar devices [6].

In the cylindrical domain, it has been shown that light propagation in the azimuthal direction, circulating the cylinder axis, gives rise to what has been termed whispering-gallery modes [7]. The high rotational order and strong light confinement close the dielectric air interface of these modes makes them suitable for a number of devices [8]. Whispering-gallery modes have been excited in the plane spheres [9], flattened spheroids [10], thin disks [11], hollow cylinders [12], rolled up silicon layers [13] and the bottle resonator [14], a list which indicates the most commonly encountered geometries. It has been demonstrated experimentally that the optical properties of the horizontal stacked layer slot channel waveguide are present when the waveguide structure is configured in the azimuthal plane of a disk configuration and the guided light resembles whispering-gallery modes with the field the concentrated in the low index of refraction channel region [15,16].

This paper is devoted to theoretically examining the slot waveguide configuration in cylindrical space. By using a Fourier–Bessel set of basis functions we transform the electric field vector wave equation into an eigenvalue problem. Matching the basis functions to the rotational symmetry of the dielectric and of the slot waveguide modes significantly reduces the order of the matrix to be diagonalized and renders computations possible on a desktop PC [17,18]. Computation results will show that the combined optical guidance features of the slot channel waveguide and whispering-gallery modes are easily extracted from the eigenvalues and eigenvectors returned from the matrix. Such a solver makes future research in the disk shaped slot channel waveguide more efficient as other computational techniques such as finite-difference time domain (FDTD), finite element method (FEM) and plane wave (PWM) can exhaust computer resources due to the large number of grid points required (FDTD, FEM) and large number of plane waves (PWM) needed for accurate and well converged numerical results. The computation technique presented here has been extensively tested for accuracy against results obtained from the FDTD and PWM for geometries which

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can be efficiently modelled using any of the indicated techniques [19]. Results agree to within a few percent and with the accuracy improving with higher discretization resolution (FDTD) and basis functions (PWM). The technique presented here has been used to compute the slot channel waveguide modes for the structure presented in [15]. In addition to reproducing the edge region slot waveguide states, out technique predicts the presence of other states localised at the edge of the central glass support for the geometry of [15].

The next section provides the basic dielectric structure and key equations used to populate the eigen-matrix. This is followed by an examination of the dielectric profile decomposed on the basis function space. In Section 4, several of the cylindrically symmetric slot channel waveguide properties are presented. We show the results for two sets of waveguide geometry parameters. The final section shows the sensitivity of the slot channel waveguide modes to the index of refraction of the ambient medium, a property that is often exploited in the design of whispering-gallery mode based sensors.

2. Computation engine for waveguide mode solver

The cylindrical symmetry dielectric profile for the slot channel waveguide examined is shown in Fig. 1. The two silicon ring ridges have a width D and axial length L and are separated by an air gap of width W. The silicon is supported on a central glass rod of radius S. The z-axis height of the dielectric structure T is chosen such that the slot channel waveguide modes are isolated from the boundaries at $\pm T/2$. The overall radius of the dielectric extent is R and is such that the cylindrical boundary is sufficiently far from the waveguide modes of interest. Using this structure as a guide for the cylindrically symmetric dielectric structures of interest an eigen-value matrix expression is developed in the next paragraphs.

The traditional disk type slot channel waveguide configuration consists of alternating layers of oxide and silicon patterned as stacked disks [15,16]. The inner most oxide residing between the silicon is etched inwards to produce the channel. The mode solver presented below is able to obtain the states of these structures easily. We have chosen to present the states of the structure shown in Fig. 1 as it has not yet been experimentally examined as a candidate for disk type slot channel waveguide configuration. It does present some fabrication challenges which may be overcome

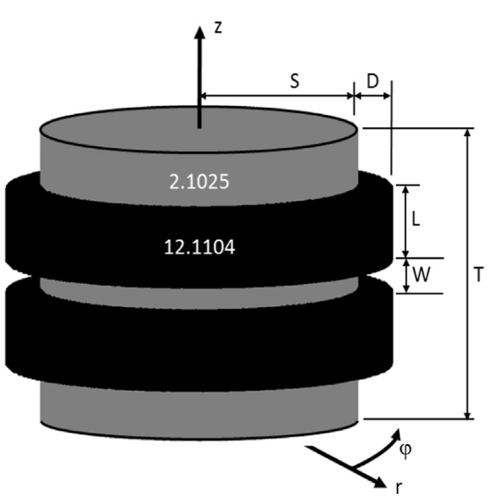

Fig. 1. 3-D representation of the dielectric profile for a slot channel waveguide on dielectric cylinder. Black – Silicon, Grey – Oxide, White – Air. The structure is axially periodic with period T chosen such that the slot waveguide modes are well contained within one period.

using a rolled up silicon membrane [20,21] or through a number of film deposition, lithography and etch steps.

The optical properties of the slot channel waveguide in cylindrical space can best be examined through the E_z field component. In the cylindrical geometry this is the field component that is discontinuous by the ratio of the relative dielectric constants when passing the air–silicon interfaces in the z direction. The analysis proceeds using the full 3-D wave equation for the electric field as the three field components are coupled when dielectric transitions are present in the axial direction:

$$\frac{1}{\varepsilon_r}\vec{\nabla}\times(\vec{\nabla}\times\vec{E})=\left(\frac{\omega}{c}\right)^2\vec{E} \tag{1}$$

In this expression it is assumed that the medium is non-magnetic, uncharged, no currents are present, a time dependence of the form $e^{j\omega t}$ (ω, frequency, c, free space speed of light) and ε_r the relative dielectric constant. Eq. (1) can be cast into an eigenvalue equation by expanding the inverse of the dielectric and three components of the electric field using a set of basis function and expansion coefficients. The cylindrical symmetry of the dielectric (and fields) dictates that Fourier–Bessel basis functions of the form:

$$F_o(i)=J_o\left(\rho_{p_i}\frac{r}{R}\right)e^{jm_i\varphi}e^{jG_{n_i}z} \tag{2}$$

are best suited for establishing the eigen-matrix. Here (i) indicates either a field component $(i\rightarrow):(r\rightarrow E_r,\varphi\rightarrow E_\varphi,z\rightarrow E_Z)$ or the inverse dielectric $(i\rightarrow):(\Omega\rightarrow 1/\varepsilon_r)$. In the radial direction only the lowest order Bessel function is required provided the zeros of the Bessel function, ρ_{pi} are used in the expansion (integer index with $p_i\geq 1$). In the angular direction the integer indices, m_i, can take on all possible values. The azimuthal direction is expanded using a set of reciprocal scalars (vector in 1-D), $G_{n_i}=n_i(2\pi/T)$, requiring that the dielectric structure be considered as period in z. The integer index, n_i, can take on all possible values. The expansion used for the inverse dielectric and field components are:

$$\begin{bmatrix}\Omega\\ E_i\end{bmatrix}=\sum_{p_im_in_i}\kappa^iJ_o\left(\rho_{p_i}\frac{r}{R}\right)e^{jm_i\varphi}e^{jG_{n_i}z}=\sum_i\kappa^iF_o(i) \tag{3}$$

In a typical application of the process to follow, the index space is restricted to a finite set which provides sufficient accuracy and convergence for determination of the optical properties of the modes of interest.

The mathematical steps require to obtain the eigen-matrix is to first expand the double curl in (1) in cylindrical coordinates followed by taking all required derivatives of the field components. This is then multiplied by the series representing the inverse dielectric. Since the dielectric is known the expansion coefficients related to the dielectric are also known. The expansion coefficients of the three field components and eigen-frequencies associated with each state are unknown. These are be collected and written as an eigen-matrix of the form:

$$\begin{bmatrix}R_r & \varphi_r & Z_r\\ R_\varphi & \varphi_\varphi & Z_\varphi\\ R_z & \varphi_z & Z_z\end{bmatrix}\begin{bmatrix}R\\ \Phi\\ Z\end{bmatrix}=\left(\frac{\omega}{c}\right)^2 I\begin{bmatrix}R\\ \Phi\\ Z\end{bmatrix} \tag{4}$$

In this matrix $(\omega/c)^2$ are the eigenvalues and can be related to the frequency and free space wavelength by:

$$\omega_{real}=\sqrt{\left(\frac{\omega}{c}\right)^2_{real}+\sqrt{\left(\frac{\omega}{c}\right)^2_{real}+\left(\frac{\omega}{c}\right)^2_{imaginary}}} \tag{5}$$

$$\omega_{imaginary}=\sqrt{-\left(\frac{\omega}{c}\right)^2_{real}+\sqrt{\left(\frac{\omega}{c}\right)^2_{real}+\left(\frac{\omega}{c}\right)^2_{imaginary}}} \tag{6}$$