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Thermal expansion and stress and measurement using high-order diffraction: Possibilities and theoretical limits

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ABSTRACT

We investigate the theoretical limits of using diffraction of a single laser beam by a patterned surface as a way to measure surface deformation caused by stress or thermal expansion. Applying Gaussian beam optics to a lens-grating system, we identify the relevant parameters of the grating, the laser beam and the diffraction order and other conditions to the sensitivity limits. Theory suggests that sensitivity increases linearly with the diffraction order, regardless of the diffracted angle, and that relative deformation of the order of 10^{-6} should be detectable with common optical components and laser beams. This corresponds to thermal expansion coefficients as low as $10^{-8}\,^{\circ}\mathrm{C}^{-1}$ measured over 100 °C and stresses as low as 10^{-6}. The proposed technique would be applicable to transparent or opaque samples and very small sample volumes.

1. Introduction

Optical diffraction techniques are useful for measuring material deformation, particularly in the case of solids modified by strain or by heat [1–13]. A plane grating deflects light beam by an angle which depends on the grating period, so changes in this period is detected by monitoring the diffracted beam direction. This method is effectively a strain gauge that requires no physical contact with the sample, and it can be done remotely and in a variety of harsh environments. To give an example, unlike push-rod techniques and optical methods using interferometry to measure sample displacements [8–10,14–16], the beam deflection technique is relatively insensitive to thermal refractive index changes in the vicinity of the sample. While some variations of the diffraction technique have used incoherent light sources [1,7], highest accuracies are obtained with laser beams because of their spectral purity and spatial coherence. These properties are desirable to measure diffraction angle changes accurately, as can be done by monitoring the position of a laser beam focused on a camera.

In some of the previous works [2,6,9,10], some aspects of instrumental resolution were considered. However, a more detailed approach to the problem is needed, in particular to take into account the effect of higher diffraction orders and the properties and size of diffracted beams and the various grating parameters. In this paper, we apply the diffraction theory of Gaussian beams to the measurement of thermal expansion and strain deformation. We investigate the theoretical detection limits and discuss the importance of relevant beam, grating beam focusing parameters. We then reports measurements of thermal expansion and verify certain key features of the theory.

2. Diffracted Gaussian beam theory

Let's consider the problem outlined in Fig. 1. A Gaussian laser beam with width w and wavelength λ undergoes diffraction to m-th order on a grating with a spatial period Γ. To achieve measurements with high angular accuracies, the beam is focused by a lens with focal length f located just before the grating. Assuming the Gaussian beam to have a nearly flat phase front at the lens, the beam reaches a minimum beam waist w_o on a detector (a camera) located at a distance f from the grating (the distance from the lens to the grating is negligible compared to f). Angles θ_i and θ_m represent the incident angle and diffracted angles relative to the normal of the grating. Both the incident and diffracted beams lie in the plane normal to the ruling of the grating.

According to the Bragg's condition for diffraction

$$m\lambda = \Gamma(\sin\,\theta_m - \sin\,\theta_i) \quad (1)$$

If the period of the grating changes as a result of thermal expansion (or stress), the diffracted angle θ_m is expected to change accordingly. We will first tackle the problem as it applies to the

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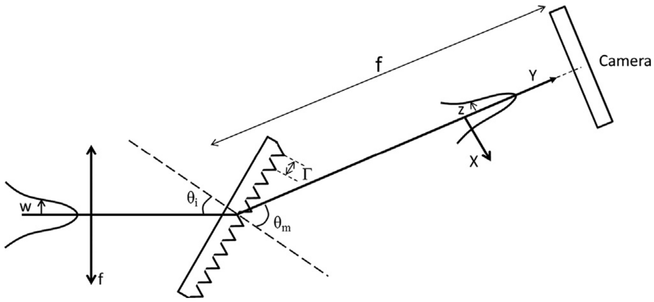

Fig. 1. A Gaussian beam of width w is focused by a lens of focal length f onto a camera. The beam is diffracted to m-th order by a grating.

thermal expansion, then, in Section 3, we will convert the key results for the case of mechanical strain measurement.

In principle, from Eq. (1), it is possible to obtain the thermal expansion coefficient of the grating by measuring the beam's angular deflection per unit of temperature. To this end, there are two factors that are relevant to the sensitivity (accuracy) of the method (1) the sweep rate, or the diffracted beam angle change per unit of temperature, and (2) the angular resolution on the measured diffracted beam. The following sections analyze both factors.

2.1. Angular sweep rate

By differentiating both sides of Eq. (1), we obtain the following rate of change for the diffraction angle:

$$\frac{d\theta_m}{d\Gamma} = -\frac{m\lambda}{\Gamma^2 \cos\theta_m} \tag{2}$$

This result is obtained by taking m, λ and θ_i to be constant. If the grating is made of a material with linear expansion coefficient α, the sweep rate is calculated in the following way. Starting with

$$\Gamma(T) = \Gamma_o(1+\alpha T) \tag{3}$$

where Γ_o is the grating period at some reference temperature and T is the temperature difference from this reference temperature, we obtain by differentiation

$$\frac{d\Gamma}{dT} = \alpha\Gamma_o \tag{4}$$

and from the identity

$$\frac{d\theta_m}{dT} = \frac{d\theta_m}{d\Gamma}\frac{d\Gamma}{dT} \tag{5}$$

we obtain the sweep rate

$$\frac{d\theta_m}{dT} = -\frac{\lambda\alpha}{\Gamma_o}M \tag{6}$$

The quantity

$$M = \frac{m}{\cos\theta_m} \tag{7}$$

Effectively acts as a magnification factor.

We expect the sensitivity to thermal expansion to be highest when the sweep rate is largest. Therefore, it is desirable to uses highest possible diffraction orders m and the largest possible θ_m. Note that θ_m can be made arbitrarily close to 90° by appropriately choosing Γ_o and λ. Indeed, from Eq. (1) we see that $\theta_m = 90°$ for $\Gamma_o = m\lambda$ if $m \neq 0$.

Note that the sweep rate does not depend on the incidence angle θ_i. For simplicity and convenience, we can therefore assume normal incidence.

2.2. Angular resolution

The other important consideration is the angular resolution in measuring the diffracted beam angle with the lens-grating-camera system. The accuracy on measuring beam angle change is limited by the beam position uncertainty δx on the camera, which is some fraction of the size of the beam w_m on the camera, so we can write $\delta x = Aw_m$. The dimensionless constant A depends on factors such as noise, the smoothness of the laser beam profile, the camera resolution, the averaging and the fitting algorithm used. From experience, we consider values ranging from 10^{-4} to 10^{-1} to be typical.

Assuming $\delta x \ll f$, the minimum detectable change in diffraction angle on the camera is

$$\delta\theta_m = \frac{\delta x}{f} = \frac{Aw_m}{f} \tag{8}$$

We now develop this expression further by finding the minimum spot size of the beam on the camera given the incident laser beam parameters. Ideally, the camera would be located at the focal spot of the beam. From Gaussian beam optics [17], a laser beam with a flat phase front that is focused by a lens with focal length f yields a minimum beam waist given by

$$w_o = \frac{\lambda f}{\pi w} \tag{9}$$

where w is the beam waist at the lens. However, this applies only to the non-diffracted beam, or the $m=0$ case. To calculate the minimum beam waist of focused beams diffracted to higher orders, we need to consider the problem shown in Fig. 2. Here a beam diffracted at an angle θ_m is focused to a point M located at a distance f_m away.

Bragg's law imposes that the phase front of a diffracted beam be uniform in the plane perpendicular to the direction of propagation. The phases at points O and I are therefore equal to within a multiple of 2π. We can therefore write $\varphi_O = \varphi_I$ so that $\varphi_A - \varphi_O = \varphi_A - \varphi_I$. In addition, all light rays arriving at M have the same phase, which is also true for all rays arriving at N. From this we have the condition that NA − NO = MA − MI, so that

$$\sqrt{f^2 + OA^2} - f = \sqrt{f_m^2 + (OA\cos\theta_m)^2} - f_m \tag{10}$$

$$f\sqrt{1+\frac{OA^2}{f^2}} - f = f_m\sqrt{1+\frac{(OA\cos\theta_m)^2}{f_m^2}} - f_m \tag{11}$$

and since $OA \ll f$

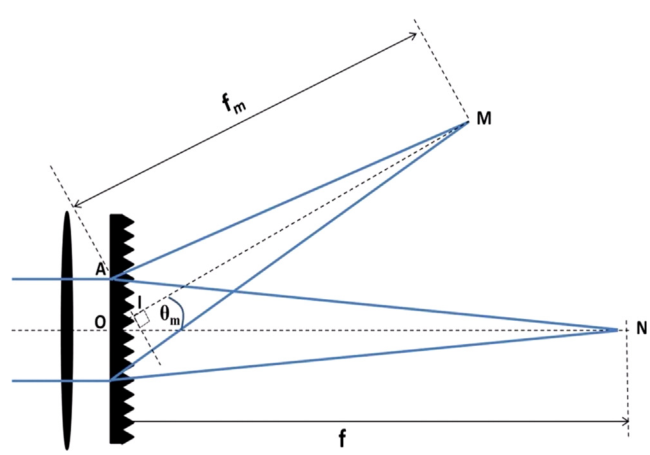

Fig. 2. Focusing of a beam diffracted to m-th order.