Optics Communications 326 (2014) 64–69

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Spatial distribution of spontaneous parametric down-converted photons for higher order optical vortices

Shashi Prabhakar [a,*], Salla Gangi Reddy [a], A. Aadhi [a], Ashok Kumar [b], P. Chithrabhanu [a], G.K. Samanta [a], R.P. Singh [a]

[a] Physical Research Laboratory, Navrangpura, Ahmedabad 380009, India
[b] Instituto de Fisica, Universidade de Sao Paulo, Sao Paulo 66318, Brazil

ARTICLE INFO

Article history:
Received 28 January 2014
Received in revised form
7 April 2014
Accepted 8 April 2014
Available online 24 April 2014

Keywords:
Parametric down-conversion
Optical vortex
Entanglement
Critical beam-width

ABSTRACT

We make a source of entangled photons using spontaneous parametric down-conversion (SPDC) in a non-linear crystal and study the spatial distribution of photon pairs obtained through the down-conversion of different modes of light including higher order vortices. We observe that for the Gaussian pump, the thickness of the SPDC ring varies linearly with the radius of pump beam. However, in case of vortex carrying beams, two concentric SPDC rings are formed for beams above a critical radius. The full width at half maximum (FWHM) of SPDC rings increase with increase in the order of optical vortex beams. The presence of a critical beam width for the vortices as well as the observed FWHM of the SPDC rings are supported with our numerical results.

1. Introduction

The process of spontaneous parametric down-conversion (SPDC) has been used extensively for the generation of entangled photon pairs in many recent experiments. The purpose of these experiments range from Bell's inequality violation [1] to the implementation of quantum information protocols [2]. In the process of SPDC, a laser pump beam photon interacts with second-order nonlinear $\chi^{(2)}$ crystal, gets annihilated and gives rise to the emission of two photons. These two photons are generated simultaneously and follow the laws of energy and momentum conservation. The phenomena of SPDC was first observed by Burnham and Weinberg [3] and theoretically studied by Hong and Mandel [4].

The photon pairs generated through SPDC are entangled in the spatial degrees of freedom i.e. position-momentum entanglement [5] as well as entanglement in orbital angular momentum (OAM) [6]. This OAM entanglement can be described by a multi-dimensional Hilbert space [7–9], compared to the case of polarization entanglement which is limited to two dimensions only [10]. These photon pairs have been found to be entangled in time-bin also [11].

Optical vortices (OV) carry a dark core in a bright background [12]. If there is a phase change of $2\pi l$ around the point of darkness, it is called a vortex of topological charge l, where l is an integer. The sense of rotation determines the sign of topological charge of the vortex. A beam with such a phase structure has a helical wavefront and, therefore, carries an OAM of $l\hbar$ per photon [13] for a vortex of topological charge l. These beams have found a variety of applications, such as optical trapping of atoms [14], optical tweezing and spanning [15], optical communication [16], imaging [17], and quantum information and computation [8].

For any application of entangled photons generated through the SPDC, it is important to know the spatial distribution of photons arising from the SPDC process. For the Gaussian pump beam, the spatial distribution of SPDC photons has already been reported [18–21]. However, for photons generated by pumping with higher order vortices, it has not been reported so far. Although, the phase-matching by optical vortex pump beam has been studied theoretically by Pittman et al. [22].

With the availability of low noise and high quantum-efficiency electron-multiplying CCDs (EMCCD), the experiments with low photon level imaging have become possible [23]. To observe the shape of the SPDC ring formed by the Gaussian as well as optical vortex beams, we have carried out experimental studies using EMCCD. The observed experimental results are supported with our numerical results. The theory regarding the SPDC has been

---

* Corresponding author.
E-mail address: shaship@prl.res.in (S. Prabhakar).

http://dx.doi.org/10.1016/j.optcom.2014.04.017

discussed in Section 2, experiments performed in Section 3 and results in Section 4. Finally we conclude in Section 5.

2. Theory

The intensity distribution of an optical vortex of order l can be written as

$$I_l(x,y)=I_0(x^2+y^2)^{|l|}\exp\left(-\frac{x^2+y^2}{\sigma^2}\right), \tag{1}$$

where σ is the beam radius of host beam, I_0 is the maximum intensity in the bright ring. Clearly, Eq. (1) shows that the Gaussian beam is a special case of optical vortex with $l=0$.

The nonlinear effects in crystals have been exploited in a number of applications such as frequency doubling, optical parametric oscillation and the SPDC [24]. When a nonlinear crystal, for example Beta-Barium Borate (BBO), with non-zero second order electric susceptibility ($\chi^{(2)}$) is pumped by an intense laser, a *pump* photon (frequency ω_p and wave-vector $\mathbf{K}_p$) splits into a photon pair called *signal* and *idler*. The energy and momentum conservation provides us with

$$\hbar\omega_p=\hbar\omega_s+\hbar\omega_i, \tag{2}$$

$$\mathbf{K_p}=\mathbf{K_s}+\mathbf{K_i}, \tag{3}$$

where suffices s and i denote signal and idler photons respectively. The phase matching is determined by the frequency of pump laser beam and the orientation of crystal optic axis with respect to the pump. Eq. (2) can be simplified as

$$\frac{1}{\lambda_p}=\frac{1}{\lambda_s}+\frac{1}{\lambda_i}, \tag{4}$$

where λ_p, λ_s and λ_i denote wavelengths of pump, signal and idler photons respectively. We have considered e→o + o type (e: extraordinary, o: ordinary) interaction. Hence, Eq. (3) can be written as

$$\frac{2\pi n_e(\lambda_p,\Theta)}{\lambda_p}=\frac{2\pi n_o(\lambda_s)}{\lambda_s}\cos(\phi_s)+\frac{2\pi n_o(\lambda_i)}{\lambda_i}\cos(\phi_i) \tag{5}$$

$$\frac{2\pi n_o(\lambda_s)}{\lambda_s}\sin(\phi_s)=\frac{2\pi n_o(\lambda_i)}{\lambda_i}\sin(\phi_i) \tag{6}$$

where ϕ_s is the angle between $\mathbf{K_p}$ and $\mathbf{K_s}$, ϕ_i is the angle between $\mathbf{K_p}$ and $\mathbf{K_i}$ and Θ is the direction of optic axis with respect to $\mathbf{K_p}$. $n_e(\lambda_p,\Theta)$ and $n_o(\lambda_{s,i})$ are the extraordinary and ordinary refractive indices for respective wavelengths. They are obtained from the Sellmeier equations [24] and for the BBO crystal used in the experiment can be written as

$$n_o(\lambda)=\sqrt{2.7359+\frac{0.01878}{\lambda^2-0.01822}-0.01354\lambda^2} \tag{7}$$

$$n_e(\lambda)=\sqrt{2.3753+\frac{0.01224}{\lambda^2-0.01667}-0.01516\lambda^2} \tag{8}$$

$$n_e(\lambda,\Theta)=n_o(\lambda)\sqrt{\frac{1+\tan(\Theta)^2}{1+\left[\frac{n_o(\lambda)}{n_e(\lambda)}\times\tan(\Theta)\right]^2}} \tag{9}$$

where λ is in μm.

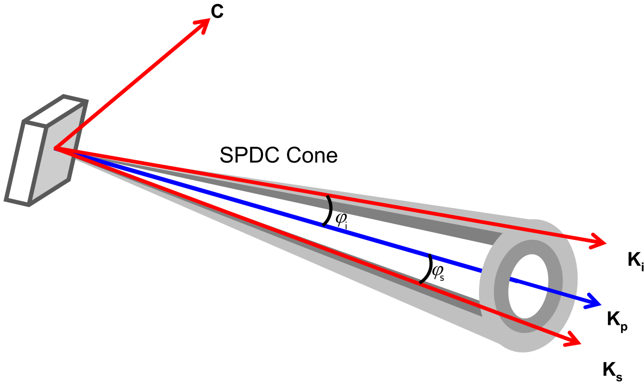

Fig. 1. Sketch diagram for the SPDC ring emission after passing the pump beam through the BBO crystal. Light and dark gray levels represent generation of idler and signal photon SPDC rings respectively.

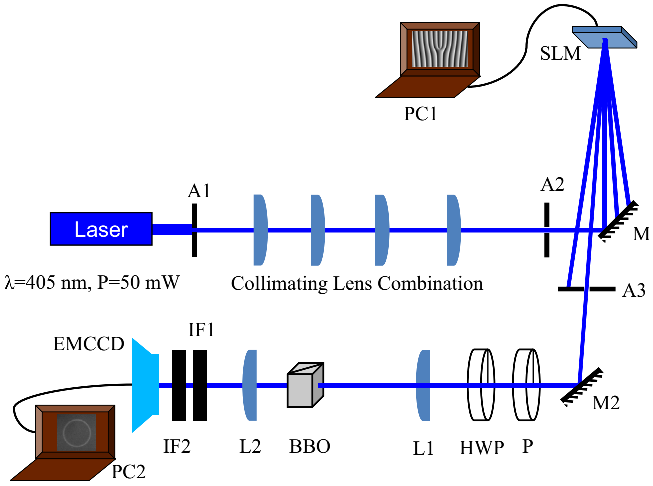

Fig. 2. Experimental setup for the study of SPDC photon pair distribution with an optical vortex as pump beam.

In Fig. 1, we have given a sketch of the SPDC photon pair generation in non-collinear type-I SPDC process. **C** denotes the crystal optic axis. The angular separation between $\mathbf{K_p}$ and $\mathbf{K_s}$ is due to energy and phase-matching conditions (Eqs. (5) and (6)) required for the SPDC process. We have also shown generation of a pair of signal and idler photons and formation of the ring centered around $\mathbf{K_p}$. In the present case, we have assumed that the pump beam has same horizontal and vertical widths.

We have used a negative-uniaxial BBO crystal with non-linear coefficient $d_{\text{eff}}=2.00$ pm/V, thickness 5 mm and optic axis $\Theta=29.7°$. The pump beam with wavelength λ_p=405 nm is incident normal to the crystal. We plan to study the degenerate or near-degenerate case in which the signal and idler photons have almost same wavelength $\lambda_{s,i}=810\pm5$ nm. The wavelength for down-converted photons is chosen from the interference filters (IF) used in the experiment. With these experimental parameters, Eqs. (5) and (6) have been solved to determine ϕ_s and ϕ_i by Runge–Kutta (RK) method for a particular value of λ_s and λ_i satisfied by Eq. (4).

Numerical simulations have been performed by first considering a particular value of λ_s and λ_i. Angles ϕ_s and ϕ_i are evaluated using RK method for chosen λ_s, λ_i and experimental parameters. The signal and idler photons are generated in cones having half-opening angle ϕ_s and ϕ_i as represented in Fig. 1 and appear as two rings on the detector plane. The center of these rings is concentric with the pump beam. Now, consider a single point on the intensity distribution of pump falling on the crystal. The stream of single photons passing through the chosen point generates SPDC rings whose radius depends on the distance between crystal and EMCCD. The intensity of the rings is proportional to the intensity at the selected point. The rings corresponding to signal and idler photons are then added to obtain the SPDC ring for the pump photons. In a similar way, rings for all other points of pump intensity distribution are obtained and added. The obtained spatial