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Effect of changing the initial excitation spatial profile on the radiation emitted from a slab of two-level atoms

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ABSTRACT

For the purpose of investigating the effect of the initial spatial distribution of the excited atoms in a slab on its emission characteristics, I compute, for different configurations of a 1D system consisting initially of equal number of atoms in the excited and ground states, the emitted field at both the front-end and back-end of the slab. I show that depending on the spatial configuration of the excited atoms/ground states atoms in the slab, the system may be at later times in a metastable state, emits equally from both end faces or emits preferentially from only one of these. These qualitatively distinct states of the system are due to the different spatial coherence present in each case, and/or the difference in the strong inter-modal coupling present and resulting from the nonlinearity in the Maxwell–Bloch equations.

1. Introduction

Dicke [1] in his seminal work on superradiance [see, for example, the review paper/books: [2–4] for more background] emphasized the non-applicability of the original theory simple results to the case of large samples; however it was always asserted, inter alia, that, for a 3-D sample with linear dimension ⋘ the wavelength of the radiation emitted in a transition from the excited state to the ground state of the two-level atom, the value of the cooperative decay rate (CDR) is independent of the particular geometry and distribution of the excited atoms in the sample considered.

Many extensions to Dicke's original work to generalize his results to extended samples were actively pursued by many groups. Rehler and Eberly [5] proposed a semiclassical treatment to obtain both the directional and temporal characteristics of the superradiant emission. Ernst and Stehle [6] proposed the idea that the emitted photons tend to form a ray. Bonifacio et al. [7–10] proposed the single mode treatment of the emission process. Bonifacio, Degiorgio, Glauber, Haake, and Narducci et al. [11–17] focused their attention on obtaining approximate solutions to the field-atoms equations. Parallel to these efforts, were the work of a number of authors that sought a better understanding of superradiance by obtaining the emission characteristics of a finite number of atoms [18–20]. Carmichael, Kim, Clemens et al. [21–24] developed the quantum trajectory theory of spontaneous emission.

Friedberg and the author pursued their investigation of the cooperative phenomena in an ensemble of two-level atoms by consistently using the formalism and techniques of quantum electrodynamics:

- We showed that the assumption that the decay time in the emission from a collection of N atoms, in small samples, to be simply N-times smaller than that of the isolated atom is not in general true [25,26]. We showed specifically that for configurations where the atomic density profile in a small sphere varied with the radial distance the value of the CDR depended on the details of the atomic spatial distribution.
- We obtained closed-form expressions for the CDR and its twin effect, the Cooperative Lamb Shift (CLS), at initial time, for different geometric configurations and sizes of the atomic sample. In [27], I summarized these expressions.
- We showed that an expansion in the eigenmodes of the Lienard–Weichert Green's function provided a powerful tool to obtain many interesting results in the linear regime of cooperative phenomena specifically to compute the time-dependence of effects resulting from cooperative phenomena in the linear regime, it proved useful to formulate the dynamics in these problems in term of an expansion of the physical quantities in a basis formed by the eigenfunctions of the Lienard–Wiechert Green's function. For example, using the 1D expression of the eigenfunction expansion, it was possible to compute for the slab geometry: (i) the Dynamical Lorentz Shift [28], (ii) the spectral distribution of the emission from a slab prepared by a delta pulse [29], and (iii) the Purcell–Dicke effect [30], which predicts manyfold enhancement in the CDR of a collection of N two-level

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identical atoms between two metals when the plasmon frequency in the metal and the polariton frequency of the two-level medium are in resonance. Furthermore, the expansion in eigenfunctions technique allowed us to obtain closed-form expression for the pumping rate and the lasing frequency for a stationary state of N two-level atoms near lasing threshold [31].

Investigating the cooperative effects in the nonlinear regime using the eigenmode analysis requires recasting the set of Maxwell–Bloch equations in the basis formed by the eigenmodes. In [32], I gave the details for obtaining the equations of motion for the expansion coefficients of the different physical quantities, and I solved these equations to analyze the details of the superradiance emission from a completely inverted system. The method proved extremely helpful in following physically the different phases of superradiance, encouraging one to explore deeper other effects of the nonlinearity, obtained through the standard numerical techniques.

In this paper, I analyze the problem of the emission from the slab for different configurations of the initial spatial distribution of the excited and ground state atoms for a slab with thickness of the order of the atomic transition wavelengtth. In this problem different initial spatial distributions of the atomic excitation leads to qualitatively different states of the system at later times. The cases that will be considered in my analysis have all in common the property that at the initial time the total number of excited atoms and ground state atoms in the system is the same. These configurations are of particular interest because in each of these cases, the linear theory (Beer's Law) would predict that a beam entering the slab normally at $T=0$ from one of its faces would exit from the other face neither amplified nor attenuated since $\int n(z,t=0)dz=0$.

The main results obtained in this this paper are that, depending on the initial spatial configuration of the excited atoms/ground states atoms, the two-level atom system may remain at later times in a metastable state (i.e. essentially no emission on the superradiance time-scale or equivalently having the Rabi frequency associated with the electric field at the slab exit plane to be of the same order as its initial value due to the quantum fluctuations), emits equally from both end faces of the slab or emits preferentially from only one of these faces.

The paper is organized as follows: In Section 2, I summarize the results obtained in [32] which transformed the nonlinear system of partial differential Maxwell–Bloch equations into an infinite set of first order coupled ordinary differential equations for the expansion coefficients of the dynamical variables. In Section 3, I give the results of integrating these equations for a number of initial spatial configurations. I conclude in Section 4.

2. New form of the 1D Maxwell–Bloch equations

As was previously shown in [32], if one decomposes the system's dynamical variables, i.e. the atomic polarization, the difference in population between the two atomic states and the Rabi frequency associated with the electric field, in the basis formed by the eigenfunctions of the integral equation:

$$\Lambda_s\varphi_s(Z)=\frac{u_0}{2}\int_{-1}^{1}dZ'\exp(iu_0|Z-Z'|)\varphi_s(Z'), \tag{1}$$

as follows

$$\psi(Z,T)=\sum_s e_s^o(T)\tilde{\varphi}_s^o(T)+\sum_s e_s^e(T)\tilde{\varphi}_s^e(T), \tag{2}$$

$$n(Z,T)=\sum_s \eta_s^o(T)\tilde{\varphi}_s^o(T)+\sum_s \eta_s^e(T)\tilde{\varphi}_s^e(T), \tag{3}$$

$$\chi(Z,T)=\sum_s p_s^o(T)\tilde{\varphi}_s^o(T)+\sum_s p_s^e(T)\tilde{\varphi}_s^e(T), \tag{4}$$

the set of Maxwell–Bloch equations reduce to an infinite set of coupled ordinary first order differential equations in the expansion coefficients. (The tilde over the eigenfunction indicates that the normalized eigenfunctions are used in the expansions format.)

In what follows, I shall use the system of units where all quantities are normalized to the parameter of interatomic cooperativity $C=4\pi N\wp^2/\hbar V$, where N is the number of particle, V is the slab volume, and $\wp$ is the reduced dipole moment of the atomic transition (its normalization is uniquely determined when given as function of the isolated atom decay rate, see below). In these units, the transverse decay rate Γ_2, due to the instantaneous dipole–dipole interaction between atoms, is equal to 2.33/4, and the normalized Lorentz shift is equal to 1/3. The isolated atom decay rate $\gamma_1=(4/3)\wp^2k_0^3/\hbar$ specifies the longitudinal decay rate of the system. The normalized coordinates are respectively: $Z=z/z_0 T=Ct \quad \Gamma_1=\gamma_1/C \quad \Gamma_2=\gamma_2/C \quad u_0=k_0\,z_0 \quad \Omega_L=\omega_L/C$. The slab thickness is $2z_0$, and $\Gamma_T=(\Gamma_1/2)+\Gamma_2$.

It is to be noted that the above eigenfunctions belong to one of two families, each with a definite parity (odd, even) in space, given respectively by

$$\varphi_s^o(Z)=\sin(v_s^oZ), \tag{5}$$

$$\varphi_s^e(Z)=\cos(v_s^eZ), \tag{6}$$

where the complex wavevectors (v_s^o, v_s^e) are solutions of the transcendental equations

$$\cot(v_s^0)=i\frac{u_0}{v_s^o}, \tag{7}$$

$$\tan(v_s^e)=-i\frac{u_0}{v_s^e}, \tag{8}$$

where s, a positive integer, is the index of the solution.

The eigenvalues associated with these eigenfunctions are given by:

$$\Lambda_s^{o,e}=i\frac{u_0^2}{u_0^2-(v_s^{o,e})^2}\cdot \tag{9}$$

I plot in Fig. 1, the real part and the imaginary part of the characteristic wave-vectors as function of the index, and the locus in the complex plane of the eigenvalues for $u_0=7\pi/4$. The important thing to note in these figures is that: (i) the real part of the characteristic vector for the odd and even solutions differ

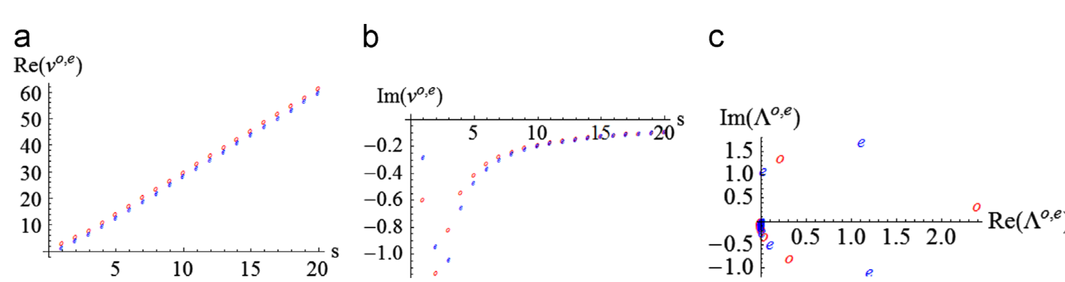

Fig. 1. The values of the odd ('o') and even ('e') wave-vectors and corresponding eigenvalues for a slab having thickness $2z_0$, and where $u_0=k_0z_0=7\pi/4$.