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Limited-angle reverse helical cone-beam CT for pipeline with low rank decomposition

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ABSTRACT

In this paper, tomographic imaging of pipeline in service by cone-beam computed tomography (CBCT) is studied. With the developed scanning strategy and image model, the quality of reconstructed image is improved. First, a limited-angle reverse helical scanning strategy based on *C*-arm computed tomography (*C*-arm CT) is developed for the projection data acquisition of pipeline in service. Then, an image model which considering the resemblance among slices of pipeline is developed. Finally, split Bregman method based algorithm is implemented in solving the model aforementioned. Preliminary results of simulation experiments show that the projection data acquisition strategy and reconstruction method are efficient and feasible, and our method is superior to Feldkamp–Davis–Kress (FDK) algorithm and simultaneous algebraic reconstruction technique (SART).

1. Introduction

Cone-beam computed tomography (CBCT) plays an important part for the nondestructive testing of object. It is known to all that the internal transported chemicals and external chemicals from soil may result in corrosion of pipeline. Then it is necessary to implement regular maintenance with the help of nondestructive testing. Subject to practical conditions, projection data is available under certain range of angle. Then incomplete projection data is available. While in classical computed tomography (CT), sufficient projection data should be collected to estimate a numerically accurate tomographic image [1]. While insufficient projection data problems occur in some practical applications, such as limited-angle problem. Limited-angle problem arises in many CT applications and has caused attention since 1980s. For example, it is frequent in straight line CT, mammography, large scale object detection, dental CT, etc [2]. In fact, limited-angle problem is one kind of incomplete projection data problems. By and large, incomplete projection data problems in CT are classified into three types: limited-angle projections, limited-view projections and truncated projections [3]. Tomographic imaging of pipeline in service belongs to the first type because projection data is available subject to practical conditions. However, it is nearly impossible to acquire the projection data of pipeline in service under circular trajectory, let alone the helical trajectory. The problem lies in that the quality of reconstructed image may be degraded due to its insufficient projection data especially when the scope of scanning angle is less than 180°. Under this circumstance, traditional reconstruction methods including analytical and algebraic methods may not provide satisfactory result. And artifacts are inevitable. Then the primary mission is to suppress artifact for better image quality. Currently there are mainly two major methodologies for limited-angle problem [2].

The first methodology is to develop new reconstruction methods based on existing methods. A natural idea is to estimate the missing part of the projection data with the image reconstructed by the projection data in hand so that complete projection data is available [4]. Another method for estimate the missing projection data is extrapolation [5]. To reduce dose of radiation under dual energy CT, an algorithm based on discrete algebraic reconstruction technique (DART) is used to distinguish different components with limited number of scanning views within limited scanning angle interval [6]. Other methods include factorization methods like singular value decomposition (SVD) and wavelet decomposition, projection onto convex sets (POCS), etc [7–10].

The other methodology is to incorporate various kinds of prior information into the process of reconstruction. These methods tend to iterative ones [11–13]. The key of the kind of method is to put the prior information in a cost function properly. In CT reconstruction, prior information about object refers to those what

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are known in advance, such as density, boundary, etc. Image quality will be improved and reconstruction will be accelerated if prior information is utilized properly. Sometimes object is sparse itself or under proper transformation, so this characteristic is frequently utilized in CT reconstruction. And there arise a lot of methods inspired by Compressive Sensing (CS) theory. These methods use the l_1-norm of the image as the cost function, where l_1-norm is the sum of the absolute values of the image pixel values. Total variation minimization (TVM) utilizes l_1-norm of the gradient of image as the cost function, and it has been one of the most popular methods up to date. Some algorithms based on TVM include POCS combined with TVM, anisotropic TVM, etc [2,14,15].

Recently, a technique named low-rank matrix recovery is frequently mentioned and it has been applied in many areas such as machine learning and computer vision [16]. The method takes the rank of the image as the cost function, but it was a problem hard to be solved. Usually it is referred to as the NP hard problem [17,18]. Similar as Compressive sensing, nuclear norm can be safely introduced as the cost function to substitute the rank under certain conditions [19], and fast algorithms are developed to solve it [20,21].

In this paper, two major works have been done in the reconstruction of pipeline in service. First, limited-angle reverse helical scanning strategy is developed to obtain projection data based on the reverse helical scanning strategy proposed by Pan [22]. The trajectory of limited-angle reverse helical is a special trajectory that it doesn't change direction in every first half circle of a conventional helix and changes direction in every second half circle. This kind of scanning strategy is similar as the trajectory of *C*-arm CT in some image-guided radiation therapy (IGRT) applications. Then an image model is developed based on the robust PCA (principle component analysis)-based 4D CT model. In the image model, the 3D object is rearranged as a 2D image with each column is a single slice image along the axial direction. Then the rearranged image model is factorized into the sum of a low rank matrix and a sparse matrix. Finally, split Bregman based algorithm is implemented in solving the model aforementioned. By combing nuclear norm minimization with l_1-norm minimization, the reconstruction of pipeline in service is implemented.

The rest of the paper is organized as follows. In Section 2, models and algorithms about l_1-norm minimization and RPCA related models will be introduced, and then the developed scanning strategy and image model towards the reconstruction of pipeline will be described. Simulation experimental results will be shown in Section 3. Finally, Section 4 is the conclusion and discussion.

2. Models and algorithms

2.1. Models

There used to be two models about three dimensional (3D) objects for choice. One is in vector form and the other is in 3D form. These two models are as follows:

$$X=(x_{ijk})_{MNL}, \tag{1}$$

where M, N, L is the size of every dimension of 3D object, and

$$X=[x_1 \quad \cdots \quad x_i \quad \cdots \quad x_N], \tag{2}$$

where N is the number of total pixels in vector form of 3D object.

2.1.1. l_1-Norm minimization model

A general form of l_1-norm minimization [23] in CT reconstruction problem is as:

$$f=\arg\min_{f} ||f||_1, \text{ s.t.} ||Af-P|| \le \varepsilon, \tag{3}$$

where f is the image to be reconstructed, $||\cdot||_1$ is l_1-norm of the image, A is the system matrix which is determined by geometry of CT system, and P denotes the actual measured projection data.

2.1.2. RPCA and RPCA-4DCT model

RPCA is an efficient tool for data analysis in statistics. It is originally used to recover principle component from measurements and modeled as [24]:

$$(X_1,X_2)=\arg\min_{(X_1,X_2)} ||X_1||_* + r||X_2||_1, \text{ s.t.} X_1+X_2=X, \tag{4}$$

where X is the measurements matrix with n_1 and n_2 are the rows and columns separately, X_1 and X_2 corresponds to the low rank part and sparse counterpart, r is the parameter to balance the effect between nuclear norm $||\cdot||_*$ and l_1-norm $||\cdot||_1$, and it is selected as $r=1/\sqrt{\max(n_1,n_2)}$. As is shown in formula (5) and (6), the nuclear norm is defined as the sum of all singular values, and the l_1-norm is defined as the sum of the absolute value of all elements in a matrix:

$$||X_1||_* = \sum_m \sigma_m, \tag{5}$$

$$||X_2||_1 = \sum_j (\sum_i |X_{2,ij}|). \tag{6}$$

Inspired by the work in RPCA, a model RPCA-4DCT is applied in solving dynamic imaging problems by Gao [25]. In his work, all the images under different phases are modeled as a matrix:

$$X=[x_1 \quad \cdots \quad x_i \quad \cdots \quad x_p]=X_1+X_2, \tag{7}$$

where $x_i=[x_{i1} \quad \cdots \quad x_{ij} \quad \cdots \quad x_{iN}]^T$ is a single image corresponding to a certain phase, N is the number of total pixels in a single phase, p is the phase number. X_1 denotes the low rank matrix representing the ‘background’ which is stationary or similar over time and X_2 denotes the sparse matrix representing the variable component. Then 4DCT can be implemented by solving the optimization problem:

$$(X_1,X_2)=\arg\min_{(X_1,X_2)} ||A(X_1+X_2)-Y||^2 + \lambda_* ||X_1||_* + \lambda_1 ||WX_2||_1. \tag{8}$$

In RPCA-4DCT model, 4D object is modeled as a matrix with each column x_i is a single phase, and p is the phase number. Similar as RPCA, matrix decomposition is implemented so that a low rank matrix X_1 and a sparse matrix X_2 are available. But RPCA-4DCT model is different from RPCA model in two aspects:

(1) The measured data is acquired by a system matrix A.
(2) The matrix X_2 is sparse under a transformation W. Except for data fidelity term, corresponding regularization terms about X_1 and X_2 are also incorporated in the cost function.

2.1.3. Scanning strategy and developed image model

This paper developed a limited-angle reverse helical scanning strategy (Fig. 1.) based on *C*-arm CT and formed an image model for the reconstruction of pipeline in service.

In Pan's work, reverse helical scanning strategy is adopted for data acquisition [26]. That is, rotation direction changes between two consecutive turns comparing with the regular helix trajectory. While the limited-angle reverse helical trajectory changes direction in every half turn comparing with the conventional helix.

Similar to the RPCA-4DCT model, the 3D object is modeled from the perspective of matrix with each column is a single slice