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Study photonic crystals defect model property with quantum theory

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ABSTRACT

In this paper, we have presented a quantum theory approach to study one-dimensional photonic crystals with and without defect layer. We give quantum dispersion relation, quantum transmissivity, reflectivity and absorptivity, and compare them with the classical dispersion relation, transmissivity, reflectivity and absorptivity. By the calculation, we find that the classical and quantum dispersion relation, transmissivity reflectivity and absorptivity are identical. With the quantum theory new approach, we can study two-dimensional and three-dimensional photonic crystals in the future.

1. Introduction

Photonic crystals have generated a surge of interest in the last decades because they offer the possibility to control the propagation of light to an unprecedented level [1–4]. In its simplest form, a photonic crystal is an engineered inhomogeneous periodic structure made up of two or more materials with very different dielectric constants. When an electromagnetic wave (EM) propagates in such a structure whose period is comparable to the wavelength of the wave, unexpected behaviors occur. Among the most interesting ones are the possibility of forming a complete photonic band gap (CPBG) [5,6], which forbids the radiation propagation in a specific range of frequencies. The existence of PBGs will lead to many interesting phenomena. In the past ten years has been developed an intensive effort to study and micro-fabricate PBG materials in one, two or three dimensions, e.g., modification of spontaneous emission [7,8] and photon localization [9–12].

The existence of PBGs will lead to many interesting phenomena, e.g., modification of spontaneous emission [13,14] and photon localization [15–17]. Thus numerous applications of photonic crystals have been proposed in improving the performance of optoelectronic and microwave devices such as high-efficiency semiconductor lasers, right emitting diodes, wave guides, optical filters, high-Q resonators, antennas, frequency-selective surface, optical limiters and amplifiers [18,19]. Other applications of PCs have been proposed and designed in the SLED to realize high power [20–22]. These applications would be significantly enhanced if the band structure of the photonic crystal could be tuned.

The theory calculations of PCs have many numerical methods, such as the plane-wave expansion method (PWE) [23], the finite-difference time-domain method (FDTD) [24], the transfer matrix method (TMM) [25], the finite element method (FE) [26], the scattering matrix method [27], the Green's function method [28], etc. These methods are classical electromagnetism theory. Obviously, the full quantum theory of PCs is necessary. In Refs. [29,30], the authors give the quantum wave equation of single photon. In Ref. [31], we give the quantum wave equations of free and non-free photon. In this paper, we have studied the 1D PCs by the quantum wave equations of photon [31], and given quantum dispersion relation, quantum transmissivity, reflectivity and absorptivity, and compare them with the classical dispersion relation, transmissivity, reflectivity and absorptivity. By the calculation, we find that the classical and quantum dispersion relation, transmissivity reflectivity and absorptivity are identical. With the new approach, we can study two-dimensional and three-dimensional photonic crystals.

2. The quantum wave equation and probability current density of photon

The quantum wave equations of free and non-free photon have been obtained in Ref. [31], they are

$$i\hbar\frac{\partial}{\partial t}\vec{\psi}(\vec{r},t)=c\hbar\nabla\times\vec{\psi}(\vec{r},t), \tag{1}$$

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and

$$i\hbar\frac{\partial}{\partial t}\vec{\psi}(\vec{r},t)=c\hbar\nabla\times\vec{\psi}(\vec{r},t)+V\vec{\psi}(\vec{r},t), \tag{2}$$

where $\vec{\psi}(\vec{r},t)$ is the vector wave function of photon, and V is the potential energy of photon in medium. In the medium of refractive index n, the photon's potential energy V is [31]

$$V=\hbar\omega(1-n). \tag{3}$$

The conjugate of Eq. (2) is

$$-i\hbar\frac{\partial}{\partial t}\vec{\psi}^*(\vec{r},t)=c\hbar\nabla\times\vec{\psi}^*(\vec{r},t)+V\vec{\psi}^*(\vec{r},t). \tag{4}$$

Multiplying Eq. (2) by $\vec{\psi}^*$, Eq. (4) by $\vec{\psi}$, and taking the difference, we get

$$i\hbar\frac{\partial}{\partial t}(\vec{\psi}^*\cdot\vec{\psi})=c\hbar(\vec{\psi}^*\cdot\nabla\times\vec{\psi}-\vec{\psi}\cdot\nabla\times\vec{\psi}^*)=c\hbar\nabla\cdot(\vec{\psi}\times\vec{\psi}^*), \tag{5}$$

i.e.

$$\frac{\partial\rho}{\partial t}+\nabla\cdot J=0, \tag{6}$$

where

$$\rho=\vec{\psi}^*\cdot\vec{\psi}, \tag{7}$$

and

$$J=ic\vec{\psi}\times\vec{\psi}^*, \tag{8}$$

are the probability density and probability current density, respectively.

By the method of separation variable

$$\vec{\psi}(\vec{r},t)=\vec{\psi}(\vec{r})f(t), \tag{9}$$

the time-dependent equation (2) becomes the time-independent equation

$$c\hbar\nabla\times\vec{\psi}(\vec{r})+V\vec{\psi}(\vec{r})=E\vec{\psi}(\vec{r}), \tag{10}$$

where E is the energy of photon in medium.

By taking curl in (10), when $\partial V/\partial x_i=0, (i=1,2,3)$, Eq. (10) becomes

$$(\hbar c)^2(\nabla(\nabla\cdot\vec{\psi}(\vec{r}))-\nabla^2\vec{\psi}(\vec{r}))=(E-V)^2\vec{\psi}(\vec{r}). \tag{11}$$

Choosing transverse gange

$$\nabla\cdot\vec{\psi}(\vec{r})=0, \tag{12}$$

Eq. (11) becomes

$$\nabla^2\vec{\psi}(\vec{r})+\left(\frac{E-V}{\hbar c}\right)^2\vec{\psi}(\vec{r})=0. \tag{13}$$

With Eqs. (12) and (13), we should study one-dimensional PCs by the quantum theory approach.

3. The quantum theory of one-dimensional photonic crystals

For one-dimensional photonic crystals, we should define and calculate its quantum dispersion relation and quantum transmissivity. The one-dimensional PCs structure is shown in Fig. 1.

In Fig. 1, $\vec{\psi}_I$, $\vec{\psi}_R$, $\vec{\psi}_T$ are the wave functions of incident, reflection and transmission photon, respectively, and they can be written as

$$\vec{\psi}(\vec{r},t)=\vec{\psi}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}=\psi_x\vec{i}+\psi_y\vec{j}+\psi_z\vec{k}, \tag{14}$$

By transverse gange $\nabla\cdot\vec{\psi}(\vec{r})=0$, we get

$$k_x\psi_x+k_y\psi_y+k_z\psi_z=0. \tag{15}$$

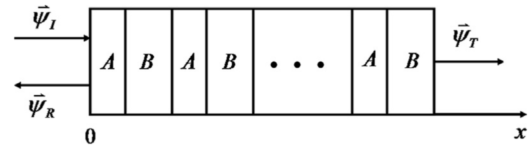

Fig. 1. The structure of one-dimensional photonic crystals.

In Fig. 1, the photon travels along with the x-axis, the wave vector $k_y=k_z=0$ and $k_x\neq 0$. By Eq. (15), we have

$$\psi_x=0, \tag{16}$$

so the total wave function of photon is

$$\vec{\psi}=\vec{\psi}_y\vec{j}+\vec{\psi}_z\vec{k}, \tag{17}$$

Eq. (13) becomes two component equations

$$\nabla^2\psi_y+\left(\frac{E-V}{\hbar c}\right)^2\psi_y=0, \tag{18}$$

and

$$\nabla^2\psi_z+\left(\frac{E-V}{\hbar c}\right)^2\psi_z=0. \tag{19}$$

In Fig. 1, the wave functions of incident, reflection and transmission photon can be written as

$$\vec{\psi_I}=F_y e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{j}+F_z e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{k}, \tag{20}$$

$$\vec{\psi_R}=F'_y e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{j}+F'_z e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{k}, \tag{21}$$

$$\vec{\psi_T}=D_y e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{j}+D_z e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{k}, \tag{22}$$

where F_y, F_z, F'_y, F'_z, D_y, and D_z are their amplitudes.

The component form of Eq. (1) is

$$\begin{cases} i\hbar\frac{\partial}{\partial t}\psi_x=\hbar c\left(\frac{\partial\psi_z}{\partial y}-\frac{\partial\psi_y}{\partial z}\right) \\ i\hbar\frac{\partial}{\partial t}\psi_y=\hbar c\left(\frac{\partial\psi_x}{\partial z}-\frac{\partial\psi_z}{\partial x}\right), \\ i\hbar\frac{\partial}{\partial t}\psi_z=\hbar c\left(\frac{\partial\psi_y}{\partial x}-\frac{\partial\psi_x}{\partial y}\right) \end{cases} \tag{23}$$

substituting Eqs. (14) and (16) into (23), we have

$$\psi_z=i\psi_y, \tag{24}$$

the probability current density becomes

$$J=ic\vec{\psi}\times\vec{\psi}^*=2c|\psi_z|^2\vec{i}=2c|\psi_{0z}|^2\vec{i}, \tag{25}$$

where

$$\psi_z=\psi_{0z}e^{i(\vec{k}\cdot\vec{r}-\omega t)}, \tag{26}$$

the ψ_{0z} is ψ_z amplitude.

For the incident, reflection and transmission photon, their probability current density J_I, J_R, J_T are

$$J_I=2c|F_z|^2, \tag{27}$$

$$J_R=2c|F'_z|^2, \tag{28}$$

$$J_T=2c|D_z|^2, \tag{29}$$

We can define quantum transmissivity T and quantum reflectivity R as

$$T=\frac{J_T}{J_I}=\left|\frac{D_z}{F_z}\right|^2, \tag{30}$$