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Exact optical self-similar solutions in a tapered graded-index nonlinear-fiber amplifier with an external source

Jun-Rong He, Lin Yi *

School of Physics, Huazhong University of Science and Technology, Wuhan 430074, China

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ABSTRACT

We study the propagations of optical self-similar solutions in a tapered graded-index nonlinear-fiber amplifier with an external source through asymmetric twin-core fiber amplifiers. Various types of exact self-similar solutions, including the W-shaped and U-shaped solutions, trigonometric function solutions, and periodic wave solutions are found. The results show that these different types of self-similar optical structures can be generated and effectively controlled by modulating the amplitude of the source. The influences of nonlinear tunneling effects on the propagation of optical pulses are investigated as well. The obtained results may have potential applications in a tapered graded-index nonlinear-fiber amplifier with an external source.

1. Introduction

In the past few years, the self-similar waves or similaritons in nonautonomous systems and the asymptotic parabolic pulses in the gain amplifier media have been studied extensively due to their potential applications in nonlinearity and dispersion management systems [1–3]. The best known equation used to describe these problems is the nonlinear Schrödinger equation (NLSE), which has been analyzed from different points of view [4,5]. It is noted that the first soliton dispersion management experiment in a fiber with hyperbolically decreasing group velocity dispersion was realized in 1991 by Dianov's group at the General Physics Institute [6]. Therefore, the study for NLSE with distributed coefficients is significant. In recent years, studies of the NLSE with distributed coefficients have been widespread in [7–12]. Many exact optical self-similar waves, including the bright and dark soliton solutions [13], quasi-soliton solutions [14], and periodic wave solutions [15] to the NLSE model are found. These self-similar waves may be useful in real applications, since they can maintain their overall shapes but with their amplitudes and widths changing with the modulation of system parameters such as dispersion, nonlinearity, gain, and inhomogeneity.

It is noted that the above works for pulse propagation in nonlinear media are restricted to single core fibers. In fact, the twin-core fibers (TCFs), which originate from the linear coupling between the two fibers, can easily be fabricated [16,17]. Recently, the dynamics of self-similar solutions in TCFs has been investigated in Refs. [18–20], where the relevant equation is the NLSE driven by an external source. In addition, nonautonomous matter waves in Bose–Einstein condensates in the presence of an inhomogeneous source has been reported in Ref. [21]. More recently, the self-similar optical solutions in a graded-index nonlinear-fiber amplifier with an external source have been studied in Refs. [22,23]. Compression and propagation of dispersive and rectangular similaritons in asymmetric TCFs have been explored in Ref. [24]. In this paper, we will study the evolutions of self-similar solutions in a graded-index nonlinear-fiber amplifier with an external source through asymmetric TCF amplifiers. Based on the assumption in Refs. [17,19], the origin of the source in the model can be attributed to the built-in asymmetry of the TCF. By using the self-similar transformation, we present various types of exact self-similar solutions, including the W-shaped and U-shaped solutions, trigonometric function solutions, and periodic wave solutions. These self-similar solutions are obtained in the linearly chirped and unchirped cases by choosing different types of the tapering function $F(z)$. The results show that these different types of self-similar optical structures can be generated and effectively controlled by modulating the amplitude of the source. The main difference between the present work and that in Ref. [23] is that we introduce the inhomogeneity Kerr-nonlinearity function $R(z)$, which imposes the nonlinear tunneling effects on the propagation of the optical self-similar solutions.

2. The model and reduction

We consider the propagation of a continuous-wave optical beam inside two adjoining, closely spaced, nonidentical, single-mode fibers; the active one is a tapered, graded-index fiber inhomogeneity

* Corresponding author.
E-mail address: yilinhust@163.com (L. Yi).

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along the medium, whereas the passive one is a step-index fiber. The equation for the beam propagation in tapered, inhomogeneity-graded-index nonlinear fiber amplifiers with refractive index

$$n(z,x) = n_0 + n_1 F(z) x^2 + n_2 R(z) I(z,x), \tag{1}$$

is

$$i\frac{\partial \psi_1}{\partial z} + \frac{1}{2k_0}\frac{\partial^2 \psi_1}{\partial x^2} + \frac{1}{2}k_0 n_1 F(z) x^2 \psi_1 - \frac{i[g(z)-\alpha(z)]}{2}\psi_1 + k_0 n_2 R(z)|\psi_1|^2\psi_1 + \Gamma \alpha_{12}\psi_2 \exp[-i(kz-\omega x)] = 0, \tag{2}$$

where g and α denote the linear gain and loss, respectively. The wave number $k_0 = 2\pi n_0/\lambda$ with n_0 and λ being the linear index and wavelength of the optical source, respectively. n_1 is the linear defocusing parameter ($n_1 > 0$), and n_2 is the Kerr-type nonlinearity of the waveguide amplifier. The dimensionless profile function $F(z)$ can be negative or positive, which corresponds to the graded-index medium acting as a focusing or defocusing linear lens. The function $R(z) > 0$, which represents inhomogeneity of Kerr nonlinearity along medium. α_{12} and α_{21} [below in Eq. (3)] are the coupling parameter of the two nonidentical fibers and $\Gamma = \sqrt{\gamma_1/\gamma_2}$ is the ratio of the nonlinearity strengths in the two fibers. Here the parameter $\gamma_i = n_2\omega_i/(cA_i^{\rm eff})$, where $A_i^{\rm eff}$ is the effective core area, c is the speed of light, and ω_i is the carrier frequency in each fiber. The equation for the envelope of the pulse that propagates in step-index fibers is

$$i\left(\frac{\partial \psi_2}{\partial z} - \beta_1\frac{\partial \psi_2}{\partial x}\right) + \beta_2\frac{\partial^2 \psi_2}{\partial x^2} + |\psi_2|^2\psi_2 + \Gamma\alpha_{21}\psi_1 \exp[i(kz-\omega x)] = 0, \tag{3}$$

where β_1 is a measure of the difference in the group velocity in Eq. (3) from that in Eq. (2), and β_2 is the ratio of the dispersion coefficients of the two fibers. When considering the assumption in Refs. [17,19], Eqs. (2) and (3) can be written as the NLSE coupled to an external traveling wave field. In this case Eq. (2) modifies to

$$i\frac{\partial \psi}{\partial z} + \frac{1}{2k_0}\frac{\partial^2 \psi}{\partial x^2} + \frac{1}{2}k_0 n_1 F(z) x^2 \psi - \frac{i[g(z)-\alpha(z)]}{2}\psi + k_0 n_2 R(z)|\psi|^2\psi + \frac{\eta(z)}{|n_2|^{1/2}} \exp[i\varphi(x,z)] = 0, \tag{4}$$

where $\eta(z)$ is the source of the fiber which contains the amplitude part of ψ_2. By introducing the normalized variables $X = x/w_0$, $Z = z/L_D$, $G = [g(z)-\alpha(z)]L_D$, and $U = (k_0|n_2|L_D)^{1/2}\psi$, with $L_D = k_0 w_0^2$ being the diffraction length associated with the characteristic transverse scale $w_0 = (k_0^2 n_1)^{-1/4}$, Eq. (4) can be rewritten in a dimensionless form:

$$i\frac{\partial U}{\partial Z} + \frac{1}{2}\frac{\partial^2 U}{\partial X^2} + \frac{1}{2}F(Z)X^2 U + \sigma R(Z)|U|^2 U = \frac{iG(Z)}{2}U + s_0\eta(Z)\exp[i\Phi(X,Z) + i\Theta(X,Z)], \tag{5}$$

where $\sigma = n_2/|n_2| = \pm 1$, and $s_0 = -k_0^2 w_0^3$ denotes the source amplitude. In the scaled form, the original phase φ is divided into two parts, $\Phi(X,Z)$ and $\Theta(X,Z)$.

For the evolution of the optical pulse to be self-similar, the functional form of the pulse's intensity must remain unchanged at different propagation distances. When the pulse's width changes, the local expansion velocity of the self-similar pulse is $v = \ell_Z X/\ell$ (ℓ is a positive function of the propagation distance Z that characterizes the change in the pulse's width), which is related to the gradient of the pulse's phase as $v = \nabla\Phi$ [25]. Thus, the self-similar pulse possesses a quadratic phase $\Phi = \ell_Z X^2/(2\ell)$, which indicates that the pulse could be linearly chirped ($\ell_Z \neq 0$) or unchirped ($\ell_Z = 0$) for the self-similar evolution. Hence, the pulse's intensity can be written as $|U(Z,X)|^2 = \exp[\int_0^Z G(Z')\,dZ']|Q(\zeta,\xi)|^2/\ell$, with $\xi \equiv X/\ell$, where ζ is a function of Z, which refers to the transformed propagation distance.

Thus, by introducing the following self-similar transformation:

$$U(Z,X) = \sqrt{\frac{G'}{\ell}}Q(\zeta,\xi)\exp\left(i\frac{\ell_Z}{2\ell}X^2\right), \tag{6}$$

with $G' \equiv \exp[\int_0^Z G(Z')\,dZ']$, we can transform Eq. (4) into the form of

$$i\frac{d\zeta}{dZ}\frac{\ell}{G'R}Q_\zeta + \frac{1}{2\ell G'R}Q_{\xi\xi} + \sigma|Q|^2 Q + \frac{\ell^2}{2G'R}(F\ell - \ell_{ZZ})\xi^2 Q = \frac{s_0\eta}{R}\left(\frac{\ell}{G'}\right)^{3/2}\exp[i\Theta(X,Z)]. \tag{7}$$

From Eq. (7) it can be seen that by appropriately choosing the relations between the functions $F(Z), G'(Z), R(Z)$, and $\eta(Z)$, one can obtain various types of self-similar solutions for Eq. (5). For example, the exact self-similar solutions for Eq. (5) can be constructed by reducing Eq. (7) to the following driven NLSE with constant coefficients:

$$iQ_\zeta + \frac{1}{2}Q_{\xi\xi} + \sigma|Q|^2 Q - s_0\exp[i\Theta(X,Z)] = 0, \tag{8}$$

provided that the following relations,

$$\frac{d\zeta}{dZ} = \ell^{-2}, \quad R = \frac{1}{\ell G'}, \quad \eta = \ell^{-3}\left(\frac{1}{R}\right)^{1/2}, \ell_{ZZ} = F\ell. \tag{9}$$

The exact analytical solution of Eq. (8) can be searched for $Q = \rho(\theta)\exp[i\Theta(X,Z)]$ with $\theta = \xi - v\zeta$, which yields the stationary NLSE with a constant source:

$$\frac{1}{2}\rho'' + \mu\rho + \sigma\rho^3 - s_0 = 0, \tag{10}$$

where $\mu = \varpi + v^2/2, \Theta(X,Z) = v\theta - \varpi\zeta$, and the function $\rho(\theta)$ can be solved by using the Möbius transformations [26]. With this, various types of analytical solutions to Eq. (8) can be found, including the periodic wave solutions, trigonometric function solutions, and soliton solutions.

Therefore, the exact self-similar solution of Eq. (5) can be written as

$$U(Z,X) = \sqrt{\frac{G'}{\ell}}\rho(\theta)\exp\left[i\frac{\ell_Z}{2\ell}X^2 + i(v\theta - \varpi\zeta)\right], \tag{11}$$

which exhibits different features for the choice of σ. Interestingly, the bright soliton solution can be obtained for $\sigma = 1$, while the dark soliton solution can be obtained for $\sigma = -1$.

Here, as an example, we choose

$$R(Z) = 1 + h\,\mathrm{sech}^2[\delta(Z - Z_0)], \tag{12}$$

which can be used to investigate nonlinear tunneling of soliton through the nonlinear barrier (well) depending on the sign of the parameter h ($-1 < h < 0$ for well, and $h > 0$ for barrier) [8,27]. Here the parameters h, δ, and Z_0 describe the height of nonlinear barrier (well), the width, and the longitudinal location, respectively. It is noted that the last expression of Eq. (9) governs the modes of an inhomogeneous planar waveguide with the refractive index profile given by the function $F(Z)$. From the theory of sech^2-profile waveguides [28], the lowest-order mode of such a waveguide corresponds to

$$F(Z) = 1 - 2\,\mathrm{sech}^2(Z). \tag{13}$$

which yields $\ell = \mathrm{sech}(Z)$. In this situation, the gain function is of the form

$$G(Z) = \tanh(Z)\left\{1 + \frac{2h\delta\coth(Z)\tanh[\delta(Z-Z_0)]\,\mathrm{sech}^2[\delta(Z-Z_0)]}{1 + h\,\mathrm{sech}^2[\delta(Z-Z_0)]}\right\}. \tag{14}$$

Especially, when $h=0$, Eq. (12) presents a homogeneous nonlinear parameter, and the corresponding gain function $G(Z) = \tanh(Z)$. From the first expression in Eq. (9) we find that the transformed