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The effect on on-axis degree of polarization of stochastic vortex light beams by degree of coherence

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ABSTRACT

The cross-spectral density function of stochastic electromagnetic vortex beams in non-Kolmogorov turbulent atmosphere is presented on the basis of the unified theory of coherence and polarization, and the analytic expressions of the intensity and the degree of polarization of on-axis point are derived. It is shown that the evolution of intensity and polarization of vortex beams in atmosphere is different from that of non-vortex beams, and is influenced by the correlation length and the topological charge of the incident vortex beams. Some important results are concluded and discussed.

1. Introduction

The polarization property of laser beams through turbulent atmosphere is important in many applications such as remote sensing and optical communication. It is assumed for a long time that the state of polarization of light beams keeps invariant on propagation in free space. In 1994, James indicated that the state of polarization of a partially coherent beam changes as the beam propagates in free space, the coherent property of the field in the source plane affects the degree of polarization of the radiated beam [1,2]. Since then, change of polarization of light beams on propagation has been the focus of many investigations [3–8]. The degree of polarization of an electromagnetic Gaussian Schell-model beam tends to its value at the source plane after a sufficiently long propagation distance in turbulent atmosphere [3,5]. The polarization evolution of beams in oceanic turbulence resembles that in atmospheric turbulence, but asymptotic saturation of polarization in the oceanic turbulence occurs at a shorter distance [9]. In recent years, modulation of polarization of laser beams has been proposed. For example, the degree of polarization can be modulated by introducing a slit aperture [10]. Polarization modulation in a one-dimensional compound photonic crystal was investigated as well [11].

Beams with a phase term of $\exp(im\theta)$ are referred to vortex beams. It is predicted that each photon of vortex beams with rotational symmetry carries orbital angular momentum (OAM) of $m\hbar$, where m is the topological charge [12]. The OAM of vortex beams can be used to encode the information, which may have potential applications in free-space information transmission. The application provides a possibility to increase the information density with inherent security enhancement [13,14]. Owing to these advantages, the propagation of vortex beams in a turbulent atmosphere has attracted much interest [15,16]. Gbur and Tyson indicated that the topological charge of vortex beams is a robust quantity that can be transmitted over a significant distance without loss [17]. In optical communication, misalignment such as shift and tilt of the beam may lead to error encoding [13]. A method to overcome this problem was proposed, which uses the mean square value of OAM as an indicator to realize misalignment correction [18]. In this study, based on the unified theory of coherence and polarization, the polarization property of stochastic electromagnetic vortex beams passing through the turbulent atmosphere will be studied.

2. Theoretical analysis

In order to investigate the polarization property of stochastic electromagnetic beams, the unified theory of coherence and polarization is employed [19]. According to the theory, the second-order coherence and polarization property of a stochastic, statistically stationary electromagnetic beam may be expressed by a 2×2 electric cross-spectral density matrix [19]:

$$\overleftrightarrow{W}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega)\equiv[W_{ij}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega)]=[\langle E_i^*(\boldsymbol{\rho}_1;\omega)E_j(\boldsymbol{\rho}_2;\omega)\rangle],\quad (i,j=x,y),\tag{1}$$

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where $E(\boldsymbol{\rho},\omega)$ represents the electric field at a point $\boldsymbol{\rho}$, at frequency ω and the angular brackets denote the average taken over the ensemble of realizations of the electric field in the sense of coherence theory in the space frequency domain.

Vortex beams are characterized by their spiral phase, which can be written as

$$E_i(\rho,\varphi)=A_i(\rho,\varphi)\exp(-im_i\varphi), \tag{2}$$

where (ρ,φ) denote the radial and azimuthal coordinates, m_i is the topological charge of vortex beams, and $A_i(\rho,\varphi)$ is the field amplitude. Assumed that the initial field amplitude is a Laguerre–Gaussian mode with radial mode characteristic number $p=0$ [20]:

$$A_i(\rho,\varphi)=E_{i0}\left(\frac{\rho}{\sigma}\right)^{m_i}\exp\left(-\frac{\rho^2}{\sigma^2}\right)\exp(i\beta), \tag{3}$$

where E_{i0} is the amplitude, σ is a constant representing the beam size, and β means an arbitrary phase.

For the sake of computational convenience and without loss of generality, we assume that the off-diagonal elements of the electric cross-spectral density matrix of the beam in the source plane have zero value.

On substituting from Eqs. (2) and (3) into Eq. (1), and with the assumption that the statistical distribution of the phase corresponds to a Gaussian-Schell correlator, the cross-spectral density function can be expressed as

$$W_{ii}^{(0)}(\rho_1,\rho_2,\varphi_1,\varphi_2;\omega)=I_{i0}\left(\frac{\rho_1}{\sigma}\right)^{m_i}\left(\frac{\rho_2}{\sigma}\right)^{m_i}\exp\left[-\frac{(\rho_1^2+\rho_2^2)}{\sigma^2}\right]$$
$$\times\exp[-im_i(\varphi_1-\varphi_2)]\exp\left[-\frac{(\rho_1^2+\rho_2^2-2\rho_1\rho_2\cos(\varphi_1-\varphi_2))}{\delta_{ii}^2}\right], \tag{4}$$

where $I_{i0}=E_{i0}^2$, δ_{ii} is a positive constant characterizing the correlation length.

The cross-spectral density function in the plane $z>0$, which contains atmosphere governed by non-Kolmogorov statistic, can be given by [21]

$$W_{ii}(\mathbf{r}_1,\mathbf{r}_2;\omega)=\left(\frac{k}{2\pi z}\right)^2\int\int\int\int W_{ii}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega)K(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\mathbf{r}_1,\mathbf{r}_2;\omega)d\boldsymbol{\rho}_1 d\boldsymbol{\rho}_2 \quad (i=x,y), \tag{5}$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ represent two points located in the plane $z>0$, $k=\omega/c=2\pi/\lambda$ is the wavenumber with λ being the wavelength of the incident beam, and $K(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\mathbf{r}_1,\mathbf{r}_2;\omega)$ is the propagator, depending on the Green's function of the random medium of the form [21,22]:

$$K(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\mathbf{r}_1,\mathbf{r}_2;\omega)=\exp\left\{-i\frac{k}{2z}[(\mathbf{r}_1-\boldsymbol{\rho}_1)^2-(\mathbf{r}_2-\boldsymbol{\rho}_2)^2]\right\}$$
$$\times\exp\left\{-\frac{4\pi^4 z}{3\lambda^2}[(\mathbf{r}_1-\mathbf{r}_2)^2+(\mathbf{r}_1-\mathbf{r}_2)(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)+(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)^2]\int_0^\infty \kappa^3\Phi_n(\kappa)d\kappa\right\}, \tag{6}$$

where $\Phi_n(\kappa)$ being the spatial power spectrum of the refractive index fluctuations, for non-Kolmogorov case, can be given by [23]

$$\Phi_n(\kappa)=A(\alpha)\tilde{C}_n^2\frac{\exp(-\kappa^2/\kappa_m^2)}{(\kappa^2+\kappa_0^2)^{\alpha/2}},\quad 0<\kappa<\infty,\quad 3<\alpha<4, \tag{7}$$

where the term $\tilde{C}_n^2$ is a generalized structure constant, and

$$\kappa_0=\frac{2\pi}{L_0},\quad \kappa_m=\frac{c(\alpha)}{l_0},\quad c(\alpha)=\left[\frac{2\pi}{3}\Gamma\left(\frac{5-\alpha}{2}\right)A(\alpha)\right]^{1/(\alpha-5)},$$
$$A(\alpha)=\frac{1}{4\pi^2}\Gamma(\alpha-1)\cos\left(\frac{\alpha\pi}{2}\right),$$

with L_0 and l_0 being the outer and the inner scales of turbulence, respectively, and $\Gamma(x)$ the Gamma function. For the power spectrum of Eq. (7), the integral in Eq. (6) can be simplified as [16,23]

$$T=\int_0^\infty\kappa^3\Phi_n(\kappa)d\kappa=\frac{A(\alpha)}{2(\alpha-2)}\tilde{C}_n^2$$
$$\times\left\{\kappa_m^{2-\alpha}(2\kappa_0^2-2\kappa_m^2+\alpha\kappa_m^2)\exp\left(\frac{\kappa_0^2}{\kappa_m^2}\right)\Gamma\left(2-\frac{\alpha}{2},\frac{\kappa_0^2}{\kappa_m^2}\right)-2\kappa_0^{4-\alpha}\right\}. \tag{8}$$

From Eqs. (4) to (8), and employing the following formulae: [24]

$$\exp[ia\cos(\theta-\varphi)]=\sum_{l=-\infty}^{\infty}i^l J_l(a)\exp[il(\theta-\varphi)], \tag{9}$$

$$\int_0^{2\pi}\exp[-im\varphi_1+b\cos(\varphi_1-\varphi_2)]d\varphi_1=2\pi\exp(-im\varphi_2)I_m(b), \tag{10}$$

$$\int_0^{2\pi}\exp(-in\varphi)d\varphi=\begin{cases}2\pi \text{ if } n=0\\ 0 \text{ if } n\neq 0\end{cases} \tag{11}$$

The elements of cross-spectral density function in the output plane are given by

$$W_{ii}(r_1,r_2,\theta_1,\theta_2,z)=I_{i0}\left(\frac{k}{z}\right)^2$$
$$\times\exp\left\{-i\frac{k}{2z}[(r_1^2-r_2^2)]\right\}\sum_{l=-\infty}^{\infty}\iint\left(\frac{\rho_1}{\sigma}\right)^{m_i}\left(\frac{\rho_2}{\sigma}\right)^{m_i}$$
$$\times\exp\left[-\left(\frac{1}{\sigma^2}+\frac{1}{\delta_{ii}^2}+\frac{4\pi^4 zT}{3\lambda^2}\right)(\rho_1^2+\rho_2^2)\right]\exp\left\{-i\frac{k}{2z}[(\rho_1^2-\rho_2^2)]\right\}$$
$$\times I_{m_i+l}\left[\left(\frac{1}{\delta_{ii}^2}+\frac{4\pi^4 zT}{3\lambda^2}\right)2\rho_1\rho_2\right]J_l\left(\frac{kr_1\rho_1}{z}\right)J_l\left(\frac{kr_2\rho_2}{z}\right)$$
$$\times\exp[-il(\theta_1-\theta_2)]\rho_1\rho_2\,d\rho_1\,d\rho_2, \tag{12}$$

The intensity can be obtained by letting $r_1=r_2=r$ and $\theta_1=\theta_2=\theta$ of the above equation, which can be written as

$$I_i(r,\theta,z)=W_{ii}(r,r,\theta,\theta,z), \tag{13}$$

For on-axis point, the above equation can be further simplified as

$$I_i(r=0,z)=I_{i0}\left(\frac{k}{z\sigma^{m_i}}\right)^2 2^{-2-m_i}C_i(z)^{m_i}\left(A_i(z)-\frac{C_i(z)^2}{4B_i(z)}\right)^{-1-m_i}$$
$$\times B_i(z)^{-1-m_i}\Gamma(1+m_i), \tag{14}$$

where

$$A_i(z)=\frac{1}{\sigma^2}+\frac{1}{\delta_{ii}^2}+\frac{4\pi^4 zT}{3\lambda^2}+\frac{ik}{2z}, \tag{15}$$

$$B_i(z)=\frac{1}{\sigma^2}+\frac{1}{\delta_{ii}^2}+\frac{4\pi^4 zT}{3\lambda^2}-\frac{ik}{2z}, \tag{16}$$

$$C_i(z)=2\left(\frac{1}{\delta_{ii}^2}+\frac{4\pi^4 zT}{3\lambda^2}\right). \tag{17}$$

According to the definition of the unified theory of coherence and polarization, the degree of polarization on axis in the turbulent atmosphere can be expressed as [19]

$$P(r=0,z)=\frac{|I_x(r=0,z)-I_y(r=0,z)|}{I_x(r=0,z)+I_y(r=0,z)}. \tag{18}$$

The degree of polarization of the on-axis point depends on the following conditions: generalized structure constant $\tilde{C}_n^2$ and spectral power-law exponent α of the atmosphere, the correlation length δ_{xx}, δ_{yy} and the topological charge m_i of the incident beam. To study the polarization property of stochastic electromagnetic vortex beams in atmosphere, we use the following parameters for the source and the atmosphere, unless other parameters are specified in the figure captions: $\alpha=3.5$, $\sigma=2$ cm, $\tilde{C}_n^2=1\times10^{-14}m^{3-\alpha}$, $m_i=1$, $\lambda=1$ μm, $\delta_{xx}=1$ cm, $\delta_{yy}=6$ mm, $L_0=1$ m, $l_0=1$ mm, $I_{x0}=1$, and $I_{y0}=1/3$.