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Analytical vectorial structure of Gaussian beams carrying mixed screw–edge dislocations in the far field

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ABSTRACT

Based on the vectorial angular spectrum representation and stationary phase method, the explicit expressions for the TE and TM terms and energy flux distributions of Gaussian beams carrying mixed screw–edge dislocations in the far field beyond the paraxial approximation are derived and used to study their far-field properties. It is shown that there are phase singularities of electric or magnetic component and dark spots of energy flux distributions. By varying the controlling parameters such as the dislocation slope, off-axis distance, waist width, the motion and pair-annihilation of phase singularities and variations of energy flux distributions may appear. Under certain conditions, the phase singularities and energy flux distributions are symmetric about the origin. A comparison with the previous work is also made.

1. Introduction

The optical beams carrying phase singularities have been investigated extensively due to their theoretical importance and attractive applications in optical manipulation, optical communication, high resolution interference microscopy and high resolution metrology, etc. [1–4]. There are three types of phase singularities: the screw dislocation (vortex) with a spiral phase ramp around the point of phase singularity, the edge dislocation with π-phase shift located along a line in the transverse plane, and the hybrid dislocation [5,6]. Rozas et al. examined the interaction between optical vortices and the background field in linear and nonlinear media [7]. He et al. explored the interaction of two edge dislocations nested in a Gaussian beam in free-space propagation, which showed that the edge dislocations may vanish, and two noncanonical vortices with opposite topological charge may take place [8]. Petrov studied the vortex–edge dislocation interaction experimentally and theoretically and found that in the paraxial regime this interaction induced the nucleation of additional vortices of both topological charges in a linear media [9–11]. Yan analyzed vortex–edge dislocation interaction in the presence of an astigmatic lens and the dependence of vortex–edge dislocation on the astigmatic coefficient and off-axis distance [12].

The theoretical and applicative study of laser beams has increased from the paraxial regime to the non-paraxial regime and from the scalar field to the vector field [13–15]. The vectorial structure analysis of beams is important to the unique nature and potential applications such as divergence high-power laser diodes, microcavity lasers, etc. [16,17]. It is known that arbitrary polarized electromagnetic wave can be expressed as a superposition of TE and TM terms, which are orthogonal to each other and separable in the far field [18]. The analytical methods for studying the vectorial characteristics include perturbation expansion procedure, angular spectrum, partial differential operators, non-paraxial diffraction integral, etc. [19–24]. By means of the vectorial angular spectrum representation and method of stationary phase, the far-field properties of different beams such as Laguerre–Gaussian beams, non-paraxial four-petal Gaussian beams, Hermite–cosine–Gaussian beams, helical hollow Gaussian beams in free-space propagation were studied in detail [25–30]. The purpose of this paper is to explore the far-field properties of Gaussian beams carrying mixed screw–edge dislocations. In Section 2, analytical expressions for the TE and TM terms of Gaussian beams carrying mixed screw–edge dislocations in the far field beyond the paraxial approximation are derived by using the methods of vector angular spectrum and stationary phase. Sections 3 and 4 illustrate the phase singularities of electric component and energy flux

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distributions in the far-field with numerical examples. Finally, Section 5 summarizes the main results obtained in this paper.

2. Analytical vectorial structure

Considering a linearly polarized Gaussian beam carrying mixed screw–edge dislocations at the source plane $z=0$, the electric field is expressed as [6,10]

$$E_x(x,y,0)=(ax-y+d)(x-b+iy)\exp\left(-\frac{x^2+y^2}{w_0^2}\right), \tag{1a}$$

$$E_y(x,y,0)=0, \tag{1b}$$

where a is the slope of edge dislocation, w_0 is the waist width of Gaussian beam, and d and b denote off-axis distances of edge dislocation and vortex, respectively.

According to the Fourier transform, the vectorial angular spectrum of the field is given by the following equation [25–30]:

$$A_x(p,q)=\frac{1}{\lambda^2}\iint_\infty E_x(x,y,0)\exp[-ik(px+qy)]dx\,dy, \tag{2a}$$

$$A_y(p,q)=\frac{1}{\lambda^2}\iint_\infty E_y(x,y,0)\exp[-ik(px+qy)]dx\,dy, \tag{2b}$$

where k is the wave number related to the wavelength λ by $k=2\pi/\lambda$. The substitution from Eqs. (1a) and (1b) into Eqs. (2a) and (2b) yields

$$A_x(p,q)=\frac{\pi w_0^2}{4\lambda^2}\{2(a-i)w_0^2-[2d+i(q-ap)kw_0^2][2b+(ip-q)kw_0^2]\}\times\exp\left[-\frac{1}{4}(p^2+q^2)k^2w_0^2\right], \tag{3a}$$

$$A_y(p,q)=0. \tag{3b}$$

In accordance with the vectorial structure of electromagnetic beams, an arbitrary polarized electromagnetic field can be decomposed into the TE and TM terms, i.e. [18],

$$\mathbf{E}(\mathbf{r})=\mathbf{E}_{TE}(\mathbf{r})+\mathbf{E}_{TM}(\mathbf{r}), \tag{4a}$$

$$\mathbf{H}(\mathbf{r})=\mathbf{H}_{TE}(\mathbf{r})+\mathbf{H}_{TM}(\mathbf{r}), \tag{4b}$$

where [25]

$$\mathbf{E}_{TE}(\mathbf{r})=\iint_\infty\frac{1}{p^2+q^2}[qA_x(p,q)-pA_y(p,q)](q\mathbf{i}-p\mathbf{j})\times\exp[ik(px+qy+\gamma z)]dpdq, \tag{5a}$$

$$\mathbf{H}_{TE}(\mathbf{r})=\sqrt{\frac{\varepsilon}{\mu}}\iint_\infty\frac{1}{p^2+q^2}[qA_x(p,q)-pA_y(p,q)][p\gamma\mathbf{i}+q\gamma\mathbf{j}-(p^2+q^2)\mathbf{k}]\times\exp[ik(px+qy+\gamma z)]dp\,dq, \tag{5b}$$

$$\mathbf{E}_{TM}(\mathbf{r})=\iint_\infty\frac{1}{\gamma(p^2+q^2)}[pA_x(p,q)+qA_y(p,q)][p\gamma\mathbf{i}+q\gamma\mathbf{j}-(p^2+q^2)\mathbf{k}]\times\exp[ik(px+qy+\gamma z)]dp\,dq, \tag{5c}$$

$$\mathbf{H}_{TM}(\mathbf{r})=-\sqrt{\frac{\varepsilon}{\mu}}\iint_\infty\frac{1}{\gamma(p^2+q^2)}[pA_x(p,q)+qA_y(p,q)](q\mathbf{i}-p\mathbf{j})\times\exp[ik(px+qy+\gamma z)]dp\,dq, \tag{5d}$$

$\mathbf{r}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ with $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ being unit vectors in the x, y, z directions, respectively; $\gamma=(1-p^2-q^2)^{1/2}$; $\mathbf{H}$ denotes the magnetic field vector; ε and μ are the electric permittivity and the magnetic permeability of the medium, respectively.

In the far-field, the condition $kr=k(x^2+y^2+z^2)^{1/2}\to\infty$ is fulfilled. Therefore, the stationary phase method is applicable [25–30]. It follows from Eqs. (3a) and (3b) and (5a)–(5d) that

$$\mathbf{E}_{TE}(\mathbf{r})=-\frac{\pi yzw_0^2}{4\lambda r^2(x^2+y^2)}\left[2(1+ia)w_0^2-\left(2b+\frac{ix-y}{r}kw_0^2\right)\left(2id+\frac{ax-y}{r}kw_0^2\right)\right]\times\exp\left(-\frac{x^2+y^2}{4r^2}k^2w_0^2+ikr\right)(y\mathbf{i}-x\mathbf{j}), \tag{6a}$$

$$\mathbf{H}_{TE}(\mathbf{r})=-\frac{\sqrt{\varepsilon}\pi yzw_0^2}{4\sqrt{\mu}\lambda r^3(x^2+y^2)}\left[2(1+ia)w_0^2-\left(2b+\frac{ix-y}{r}kw_0^2\right)\left(2id+\frac{ax-y}{r}kw_0^2\right)\right]\times\exp\left(-\frac{x^2+y^2}{4r^2}k^2w_0^2+ikr\right)[xz\mathbf{i}+yz\mathbf{j}-(x^2+y^2)\mathbf{k}], \tag{6b}$$

$$\mathbf{E}_{TM}(\mathbf{r})=-\frac{\pi xw_0^2}{4\lambda r^2(x^2+y^2)}\left[2(1+ia)w_0^2-\left(2b+\frac{ix-y}{r}kw_0^2\right)\left(2id+\frac{ax-y}{r}kw_0^2\right)\right]\times\exp\left(-\frac{x^2+y^2}{4r^2}k^2w_0^2+ikr\right)[xz\mathbf{i}+yz\mathbf{j}-(x^2+y^2)\mathbf{k}], \tag{6c}$$

$$\mathbf{H}_{TM}(\mathbf{r})=\frac{\sqrt{\varepsilon}\pi xw_0^2}{4\sqrt{\mu}\lambda r(x^2+y^2)}\left[2(1+ia)w_0^2-\left(2b+\frac{ix-y}{r}kw_0^2\right)\left(2id+\frac{ax-y}{r}kw_0^2\right)\right]\times\exp\left(-\frac{x^2+y^2}{4r^2}k^2w_0^2+ikr\right)(y\mathbf{i}-x\mathbf{j}), \tag{6d}$$

Eqs. (6a)–(6d) represent the analytical vectorial expressions for the TE and TM terms of Gaussian beams carrying mixed screw–edge dislocations in the far field. They indicate that the far field properties depend on the slope of edge dislocation a, waist width of Gaussian beam w_0, off-axis distances of edge dislocation and vortex d, b, and the TE and TM terms are orthogonal to each other.

3. Phase singularities

The TE and TM terms of Gaussian beams carrying mixed screw–edge dislocations in Eqs. (6a)–(6d) are separable in the far field. The phase distributions of the electric and magnetic field components can be obtained from Eqs. (6a)–(6d). Taking the x component of $\mathbf{E}_{TE}$ as an illustrative example we study the phase singularities of electric field components. The contour lines of phase are determined by

$$\varphi=\arctan\left\{\frac{\mathrm{Im}[E_{TEx}(x,y,z)]}{\mathrm{Re}[E_{TEx}(x,y,z)]}\right\}=\mathrm{const.} \tag{7}$$

where Re and Im denote the real and imaginary parts of $E_{TE\,x}$, respectively. In the following numerical calculations, $z=1000\lambda$ and $\varepsilon/\mu=1$ (in free space) are kept fixed.

Fig. 1(a)–(c) gives the phase singularities of the x component $E_{TE\,x}$ for different values of the slope of edge dislocation a, where the calculation parameters are $b=d=0$, $w_0=0.5\lambda$, $a=0$ in (a), $a=2\lambda$ in (b), $a=4\lambda$ in (c). It shows that for $a=0$ two phase singularities A (0, 504.125) (marked "■") and B (0, −504.125) (marked "●") appear on the y axis in the region ($-600\le x/\lambda\le 600$, $-600\le y/\lambda\le 600$) in Fig. 1(a). They are located symmetrically about the x axis. By analyzing the vorticity of phase contours, the topological charges of the phase singularities are $m_A=m_B=+1$. By varying the slope of edge dislocation from $a=2\lambda$ in Fig. 1(b) to $a=4\lambda$ in 1(c), the phase singularities A (−450.903, 225.452), B (450.903, −225.452) move to A (−489.073, 122.268), B (489.073, −122.268), respectively, where A and B are symmetrical about the origin (0, 0). The positions (x, y) of phase singularities of E_{TEx} versus the slope of edge dislocation a in the region ($0\le a/\lambda\le 8$) are plotted in Fig. 1(d), which indicates that with increasing slope of the edge dislocation, the optical vortices approach the x axis, and move away from the y axis.

The positions of optical vortices of the x component of $\mathbf{E}_{TE}$ are determined by

$$\mathrm{Re}[E_{TEx}(x,y,z)]=0, \tag{8a}$$