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Effect of aberrations on the parameters of an Airy beam

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ABSTRACT

The parameters of Airy beams generated using a cubic phase are analytically investigated using Fourier transforms. The effects of the incident beam, the cubic phase and the optical system on the parameters of an Airy beam are analyzed. Meanwhile, the effects of low order aberrations on the generation of an Airy beam are analytically studied. With the help of the relations between the aberrations and the parameters of an Airy beam, a corrected Airy beam can be generated by controlling these parameters.

1. Introduction

Airy beam is a member of the family of localized waves [1–4] and can exhibit many interesting properties, such as acceleration, traveling along a parabolic trajectory [5–7], the self-healing ability [8,9], and resilience against perturbations of media [10,11]. These properties of Airy beams make them of interest in modern optics. In past years, much attention has been paid on the generation and applications of Airy beam in many areas.

Airy beam can be generated by many methods [5,6,12–17], such as using a spatial light modulator (SLM) [5,6] and diffraction grating [12] that imposes a phase and amplitude distribution onto the diffracted light, or using three-wave mixing processes [14], which occur in asymmetric nonlinear photonic crystals.

It is well known that Airy beam can be generated by the Fourier transform of a cubic aberration which can be decomposed into elementary aberrations based on Zernike polynomials. S. Vo et al. have studied the intensity profiles with different expressions for a cubic aberration and the lateral shift of Airy beams during propagation in the context of the three-dimensional caustic representation [18]. It shows that the intensity profiles are very different with different combinations of the elementary aberrations.

However, in practice the parameters such as the truncation and transverse scale of Airy beam which satisfy the requirement in application are also important. For example, the relations between the parameters of Airy beams and the aberrations can help us in the generation of Airy beams. Besides the aberrations, these parameters depend on the experimental setup, i.e., the angle deviation, position deviation and aberration of optical elements in experiment will cause the distortion of the Airy beam. In the present paper, the effects of the experimental setup, the deviation and the aberration on the generation of an Airy beam are analytically studied. The relations of the parameters among the incident beam, optical system and the Airy beam are given. With the help of the relations, the parameters of Airy beam which satisfy the requirement in application can be obtained.

2. Generation of Airy beam by using cubic phase

2.1. Theoretical study

Because the Airy beams are solutions of the paraxial wave equation for a "source" Airy pattern, the analysis in the paper is performed within the context of the paraxial approximation. Therefore, based on the Huygens–Fresnel diffraction integral the fields of an Airy beam passing through an ABCD optical system can be expressed by [19]

$$E(x,y)=\left(\frac{k}{2\pi iB}\right)\iint E_0(x_0,y_0)\times\exp\left\{\frac{ik}{2B}[A(x_0^2+y_0^2)+D(x^2+y^2)-2(xx_0+yy_0)]\right\}dx_0dy_0 \quad (1)$$

where $E(x,y)$ and $E_0(x_0,y_0)$ are the optical field at the receiver and source planes, respectively; (x,y) and (x_0,y_0) denote the transverse coordinates of the receiver plane and the source plane; $k=2\pi/\lambda$ is the wave number and λ is wavelength. If the source plane is located in the first focal plane of a thin lens, and the receiver plane is

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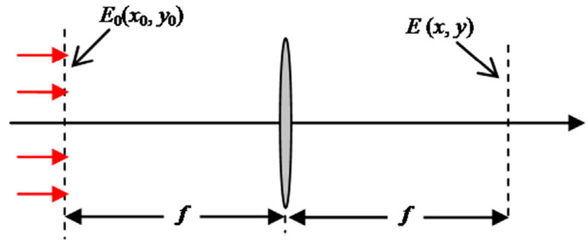

Fig. 1. Propagation geometry of the Fourier transform by a thin lens.

located in the second focal plane, the ABCD matrices can be expressed as [20]

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & f \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & f \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & f \\ -\frac{1}{f} & 0 \end{pmatrix} \tag{2}$$

where f is the focal length. Therefore, Eq. (1) can be rewritten as

$$E(x,y) = \frac{k}{if} F\{E_0(x_0, y_0)\}(\kappa_x, \kappa_y) \tag{3}$$

where $\kappa_x = kx/f$ and $\kappa_y = ky/f$ are the transverse coordinates of the spatial frequency domain, $F\{\ \}$ denotes the Fourier transform, please see Fig. 1.

Because the Airy beam with an exponential truncation is expressed as

$$E_0(x_0, y_0) = Ai\left(\frac{x_0}{\omega_0}\right)\exp\left(a\frac{x_0}{\omega_0}\right)Ai\left(\frac{y_0}{\omega_0}\right)\exp\left(a\frac{y_0}{\omega_0}\right), \tag{4}$$

by using the well-known integral

$$\int_{-\infty}^{+\infty} \exp\left[-i\left(\frac{t^3}{3} + rt^2 + st\right)\right] = 2\pi\exp\left[-ir\left(\frac{2r^2}{3} - s\right)\right]Ai(s - r^2) \tag{5}$$

we can obtain the Fourier transform of a Gaussian beam with a cubic phase as

$$\begin{aligned} E(x,y) &= \frac{k}{if} F\left\{\exp\left(-\frac{x_0^2 + y_0^2}{w_0^2}\right)\exp\left[-i\frac{c^3}{3}(x_0^3 + y_0^3)\right]\right\} \\ &= \frac{2\pi}{i\omega_0 c}\exp\left(-\frac{2a^3}{3}\right)Ai\left(\frac{x'}{\omega_0}\right)Ai\left(\frac{y'}{\omega_0}\right)\exp\left[\frac{a}{\omega_0}(x' + y')\right] \end{aligned} \tag{6}$$

where $\omega_0 = fc/k$, $a = 1/c^2 w_0^2$, $x' = x + a^2\omega_0$, $y' = y + a^2\omega_0$ and c is the coefficient of the cubic phase. It should be noted that the constant term before the Airy function in Eq. (6) is a factor which keeps the energy conserved. Eq.6 shows that the transverse scales of Airy beam are determined by the coefficient of the cubic phase and the ratio of the focal length to the wave number; the exponential truncation factor is small with large c and large waist width of the Gaussian beam; the optical field can be obtained from Eq. (4) by translation along the axis by the vector $(-a^2\omega_0, -a^2\omega_0)$. From Eq. (6) we also can see that ideal Airy beam is the Fourier transform of a plane wave with a cubic phase, and the Gaussian part can be regarded as a Gaussian aperture located at the first focal plane which causes the exponential truncation at the second focal plane.

2.2. Generation of Airy beam

To generate an Airy beam, an SLM is often used to form the cubic phase map. The experimental setup for the generation is shown in Fig. 2 [16]. As an example, a 512 × 512 BNS (Boulder Nonlinear Systems, Inc.) *XY* phase series SLM (array size 7.68 × 7.68 mm, pixel pitch 15 × 15 μm, please see Fig. 3 which is taken from a BNS booklet for *XY* series spatial light modulators) is used to form a cubic surface.

The phase response range of SLM is from 0 to 2π. But in practice, a maximum phase stroke larger than 2π is required which is anyway a necessity for the overdrive method to achieve

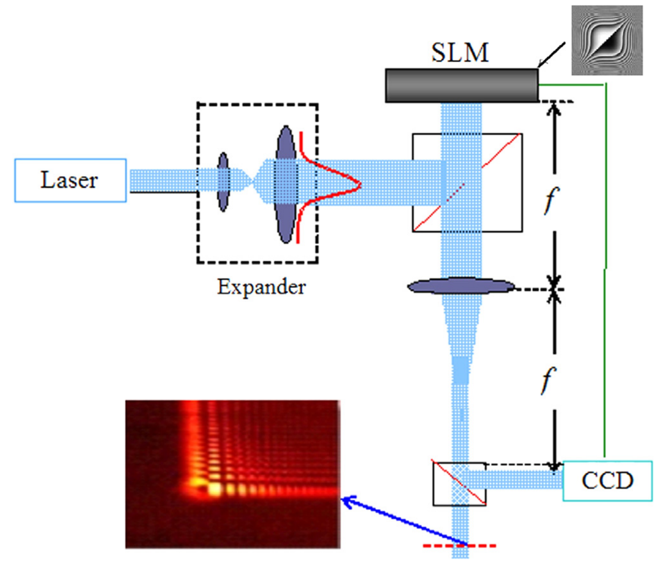

Fig. 2. Experimental setup for the generation of Airy beam by using SLM to form a cubic phase map.

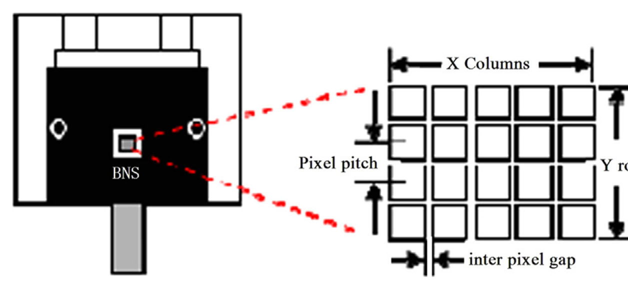

Fig. 3. BNS manufactures and square pixels arranged in an *XY* pattern.

optimum speed [21]. If we assume that the maximum phase is n times 2π, we can obtain the value of c from the expression of cubic phase in Eq. (6):

$$c = \sqrt[3]{6n\pi}/x_{0\ \max} = 693\sqrt[3]{n}\,\frac{c^3}{3} \tag{7}$$

where $x_{0\max} = 3.84$ mm is the half width of the SLM. If $n = 10$, as shown in Fig. 4a (please see Fig. 1.6b in Ref. [16]), we can get $c = 1493\ \mathrm{m}^{-1}$.

Therefore, the waist width w_0 of the Gaussian beam can be obtained by the requirement of exponential truncation factor. For example, if we want to obtain an Airy beam with $a = 0.2$, a Gaussian beam with $w_0 = 1.5$ mm is needed (see the circle in the phase map). If we want to get a nearly ideal Airy beam ($a \to 0$), a plane wave is needed. The variation of the truncation factor with the waist width of Gaussian beam is shown in Fig. 4b. The transverse scales which determine the size of the Airy beam can be obtained by $\omega_0 = fc/k$. If $f = 2$ m and $\lambda = 532$ nm, we can get $\omega_0 = 0.25$ mm.

3. Effect of low order aberrations

Optical systems and the elements always lead to aberrations, and it is important to know the effect of the aberrations on the generation of Airy beam. Aberration is often expressed through Zernike polynomials and it is shown that the effect of the low order aberration is larger than high order aberration. Because Airy beam has *X*–*Y* symmetry, only one-dimensional Airy beam