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# Diffraction of waves by a resistive half-plane

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ABSTRACT

The scattered waves by a resistive half-plane are investigated with defining reflection and transmission coefficients for the diffracted waves. The coefficients are determined according to suitable conditions that are derived from the boundary conditions and the limiting cases of the reflection and transmission coefficients of the geometrical optics fields. The resultant field expressions are examined and compared with the literature numerically.



## 1. Introduction

The resistive surfaces are introduced in order to model a thin dielectric layer. Some portion of the incident wave transmits through the surface and the other one reflects by being multiplied with a Fresnel type reflection coefficient. The resistivity of the surface $R$ is given by $j/[\omega d(\varepsilon-\varepsilon_0)]$ where $\omega$ is the angular frequency and $d$ width of the layer [1]. $\varepsilon$ represents the frequency dependent dielectric constant of the layer and $\varepsilon_0$ shows the permittivity of the free space. The resistive boundary condition that constitutes a relation between the magnetic and electric field intensities in terms of a jump relation over the surface is defined in terms of $R$ [2]. The solution of the scattering problem of waves by a resistive half-plane was first studied by Papadopoulos [3,4]. He considered a pulse as the incident wave and examined the problem in the time domain by using the Hilbert factorization method. The surface current, induced on a resistive half-screen by an E-polarized electromagnetic field, was evaluated by Senior [5]. The field behavior, in the neighborhood of a resistive half-plane, was investigated by Braver et al. in 1988 by the multipole expansion of the electromagnetic wave [6]. Volakis and Collins studied the diffraction problem of waves by a resistive half-plane between two dielectric media with the method of Wiener–Hopf factorization [7]. This problem is important since the dielectric interface affects structure of the transmitted wave from the resistive surface. Another scattering scenario, considered by Volakis, is the diffraction of waves by a resistive half-plane, located on a resistive sheet [8]. The diffracted field expressions were obtained by the same method. Finally a solution of the scattering problem by the half-plane with resistive boundary conditions was performed by Senior [9] with the method of plane wave spectrum integral [10]. He expressed the diffracted waves in terms of the Malyuzhinets function [11]. However, in Ref. [12] we showed that this solution is not exact, because the field expression of Senior is in the form

$$u=\nu(\phi)\frac{\exp(-jk\rho)}{\sqrt{k\rho}} \tag{1}$$

for $u$ is the $z$ component of the total diffracted electromagnetic field. $k$ is the wavenumber and $(\rho,\phi)$ are the length and angle elements of the polar coordinates, respectively. $\nu$ is a function of $\phi$. One of the resistive surface's boundary conditions can be written as

$$u^+|_S=\frac{1}{2jk\rho\eta}\left[\frac{\partial u^+}{\partial\phi}-\frac{\partial u^-}{\partial\phi}\right]_S \tag{2}$$

where $\eta$ is $Z_0/2R$. $Z_0$ and $R$ are the impedance of free space and resistivity of the scatterer, respectively. The superscripts $+$ and $-$ represent the upper and lower parts of the resistive surface $S$. We obtain the equation

$$v(0)=\frac{1}{2jk\rho\eta}\left(\left.\frac{\partial v}{\partial\phi}\right|_{\phi=0}-\left.\frac{\partial v}{\partial\phi}\right|_{\phi=2\pi}\right) \tag{3}$$

when Eq. (1) is used in Eq. (2) for a half-plane, located at $y=0$ and $x>0$. Note that $v$ can only satisfy Eq. (3) if it is also a function of $\rho$ besides $\phi$. Thus a diffracted field expression, as in Eq. (1), does not satisfy the resistive boundary condition. Furthermore the diffracted field representation of Senior does not compensate the discontinuities of the geometrical optics (GO) fields at the transition regions [12]. We extended the Malyuzhinets solution of the impedance half-screen problem for the resistive boundary conditions and showed numerically that the new diffracted field expression compensates

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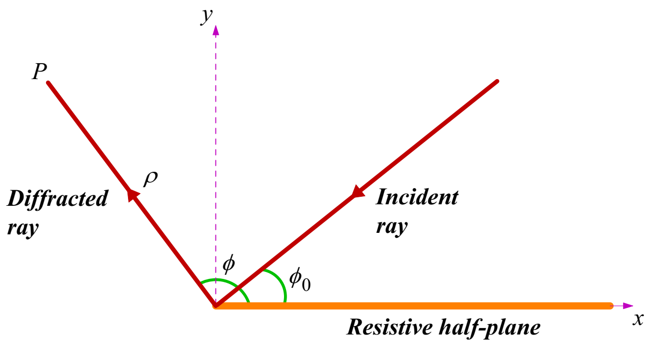


**Fig. 1.** Diffraction geometry of waves by a resistive half-plane.

the GO fields [13]. However this field also does not satisfy Eq. (2). For this reason it also can not be a rigorous solution of the problem.

Our aim in this paper is to obtain a solution of the diffraction problem of plane waves by a resistive half-plane that satisfies the boundary conditions on the surface of the scatterer. There are three GO waves in this problem, namely the incident, reflected and transmitted [14]. Thus there will also be three diffracted waves that compensate the discontinuities of the GO fields in the transition regions. We will use the geometrical theory of diffraction (GTD) fields for the diffracted waves and multiply them with unknown reflection and transmission coefficients that will be determined according to boundary conditions and limiting cases of the surface resistivity. The evaluated diffracted fields will be examined numerically with the GO and scattered waves.

The scattering problem of waves by a resistive half-screen is important, because the interaction process of waves by surfaces that transmits and reflects the incoming radiation has important areas of application in the literature. For example, Kats et al. discusses a perfect absorbing system by using a thin lossy dielectric layer on a perfect conductor [15]. They propose that the width of the layer is much smaller than the wavelength of the incident wave. A similar study put forth by Kats et al. by demonstrating samples with dielectric coatings [16]. The idea was also applied to thermal emittance of sapphire deposited with vanadium dioxide [17].

A time factor of $\exp(j\omega t)$ is considered and suppressed throughout the paper. $\omega$ is the angular frequency and $t$ represents time. $j$ is defined by $\sqrt{-1}$.

## 2. Definition of the problem and related conditions

We take into account a half-plane, located at $y=0$ and $x>0$. The edge of the half-screen coincides with the $z$ axis. A plane wave of

$$u_i = u_0 \exp\left[jk\rho\ \cos(\phi-\phi_0)\right] \tag{4}$$

is incident on the surface. $u_0$ is the complex amplitude and $k$ wavenumber. $\phi_0$ is the angle of incidence. The geometry of the problem is given in Fig. 1. $P$ is the observation point. The total field $u$ satisfies the resistive boundary conditions

$$u|_{\phi=0} = u|_{\phi=2\pi} \tag{5}$$

and

$$u|_{\phi=0} = \frac{1}{j2k\rho\eta}\left(\frac{\partial u}{\partial \phi}\bigg|_{\phi=0} - \frac{\partial u}{\partial \phi}\bigg|_{\phi=2\pi}\right) \tag{6}$$

on the surface of the half-plane [12]. The GO fields of the scattering problem by the resistive half-screen are known and can be written as

$$u_{GO} = u_{iGO} + u_{tGO} + u_{rGO} \tag{7}$$

for $u_{GO}$ is the total GO wave. $u_{iGO}$, $u_{tGO}$ and $u_{rGO}$ are the incident, transmitted and reflected GO fields and can be defined by the equations of

$$u_{iGO} = u_0 e_- U(-\xi_-), \tag{8}$$

$$u_{tGO} = \tau u_0 e_- U(\xi_-) \tag{9}$$

and

$$u_{rGO} = \gamma u_0 e_+ U(-\xi_+) \tag{10}$$

respectively [12,14]. $e_\pm$ represents $\exp[jk\rho\cos(\phi \pm \phi_0)]$. $\xi_\pm$ is $-\sqrt{2k\rho}\ \cos\left[(\phi \pm \phi_0)/2\right]$. $\tau$ and $\gamma$ are the transmission and reflection coefficients of the GO waves and can be defined by

$$\tau = \frac{\sin\ \phi_0}{\sin\ \phi_0 + \eta} \tag{11}$$

and

$$\gamma = -\frac{\eta}{\sin\ \phi_0 + \eta} \tag{12}$$

respectively. The total field, scattered by the resistive half-plane, is the scattered wave, which is the sum of the total GO and diffracted waves. The total scattered field can be written as

$$u = u_{GO} + u_d \tag{13}$$

where $u_d$ is the total diffracted wave that satisfies the boundary conditions, given by Eqs. (5) and (6), on the surface of the half-pane. Thus our problem is the determination of $u_d$. The total diffracted field can be decomposed into three diffracted wave components as

$$u_d = u_{id} + u_{td} + u_{rd} \tag{14}$$

for $u_{id}$, $u_{td}$ and $u_{rd}$ are the incident, transmitted and reflected diffracted fields that can be defined as

$$u_{id} = -\frac{\exp(-j\pi/4)}{2\sqrt{2\pi}} \frac{1}{\cos(\phi-\phi_0/2)} \frac{\exp(-jk\rho)}{\sqrt{k\rho}}, \tag{15}$$

$$u_{td} = \frac{\exp(-j\pi/4)}{2\sqrt{2\pi}} \frac{T(\phi)}{\cos(\phi-\phi_0/2)} \frac{\exp(-jk\rho)}{\sqrt{k\rho}} \tag{16}$$

and

$$u_{rd} = -\frac{\exp(-j\pi/4)}{2\sqrt{2\pi}} \frac{R(\phi)}{\cos(\phi+\phi_0/2)} \frac{\exp(-jk\rho)}{\sqrt{k\rho}} \tag{17}$$

respectively. We considered the relation between the GO and diffracted waves, outlined by the geometrical theory of diffraction [18], in the determination of the diffracted field components. The diffracted field compensates the discontinuity of the GO wave at the transition region. For this reason, the amplitude and phase of the diffracted wave varies with the GO field according to a determined ratio. For example the high frequency asymptotic form of the incident scattered field is given by

$$u_{is} = e_- U(-\xi_-) - \frac{\exp(-j\pi/4)}{2\sqrt{2\pi}} \frac{1}{\cos(\phi-\phi_0/2)} \frac{\exp(-jk\rho)}{\sqrt{k\rho}} \tag{18}$$

for a perfectly conducting half-plane [19]. If the incident GO wave is multiplied with a constant coefficient, the incident diffracted field must also be multiplied with a function that is equal to the constant coefficient at the shadow boundary, located at $\phi=\pi+\phi_0$. The transmitted and reflected diffracted waves are multiplied by the functions of $T(\phi)$ and $R(\phi)$ with this aim. These two functions are also dependent on $\rho$ because of the reason, mentioned in the context of Eq. (3). However this dependence will be not shown in the arguments of the functions. $T(\phi)$ and $R(\phi)$ must satisfy some