Optics Communications 323 (2014) 1–5

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Discussion

# Creating three-mode entanglement of optical fields via effective coupling to separated atomic ensembles

Qing-Xia Mu [a,*], Jie Song [b]

[a] Mathematics and Physics Department, North China Electric Power University, Beijing 102206, China
[b] Department of Physics, Harbin Institute of Technology, Harbin 150001, China

ARTICLE INFO

*Article history:*
Received 29 October 2013
Received in revised form
2 January 2014
Accepted 23 February 2014
Available online 6 March 2014

*Keywords:*
Optical cavity
Three-mode entanglement
Atomic ensembles

ABSTRACT

We present a scheme to generate three-mode entanglement of optical fields based on concurrent interaction with separated atomic ensembles. The inseparability criterion for multimode entanglement [Phys. Rev. A 67 (2003) 052315] is well satisfied in a wide range of relevant parameters, which is sufficient to demonstrate genuine tripartite entanglement. Unlike most of the proposals that the environment will induce decoherence, here we show that the largest correlations of the three cavity modes are insensitive to the cavity decay rate. It is also found that the best correlations can be optimized by the effective coupling constants between the cavity fields and the collective atomic operators.

© 2014 Elsevier B.V. All rights reserved.

## 1. Introduction

The research of quantum entanglement is of great interest due to its wide applications to quantum information processing and quantum communication [1–5]. Of particular interest is the continuous variable (CV) entanglement owing to its unconditionality for the implementations of many information processes [6]. The recent researches show that the generation of entangled light has received a significant amount of attention. For example, by using the Kerr-nonlinearity in optical fibers with atomic ensembles one realized the two-mode squeezing [7,8]. A nondegenerate parametric oscillator below, near, and above threshold has shown to be one of the most efficient ways to produce CV entanglement [9,10]. However, the above researches are confined to bipartite systems.

Recently, along with the development on quantum information networks with many nodes, multimode entanglement is going to be the key ingredient for advanced multiparty quantum information processing due to its ability to address different nodes in a quantum network [11–13]. Therefore, much attention is paid to the generation of multipartite CV entanglement, especially for the three-mode case. Moreover, the criteria for the bipartite system have been generalized for genuine multipartite CV entangled state by van Loock and Furusawa [14]. Based on this sufficient condition, many linear optics [15] and nonlinear optics [16–18] schemes are proposed for creating multipartite CV entanglement. There are also experiments where entangled beams are produced by employing independent squeezed states and beam splitters [15,19]. However, it is difficult to obtain the entangled lights of different frequencies since beam splitter transformation is linear. On the other hand, the atom–field interaction is recognized as a fundamental mechanism for generating multipartite entanglement without initially prepared squeezing. There have been theoretical proposals to create three-mode entangled beams based on atomic coherent effects [20–25]. For example, Lü et al. [20] studied the generation of tripartite CV entanglement in a tripartite correlated emission laser. Liang et al. [24] showed that multimode squeeze operators can be engineered by using an optical cavity with an atomic ensemble.

Here we propose an alternative method to prepare tripartite entangled light with two separated atomic ensembles of four-level atoms. Each ensemble is driven by two classical fields and is coupled to two cavity modes. By utilizing the van Loock–Furusawa criterion [14], we show that the three-mode entanglement can be obtained at the output in a wide range of parameters. The most attractive feature of our scheme is that the maximum correlations of the output beams are insensitive to the cavity decay rate, but can be adjusted by the effective coupling constants between the cavity fields and the collective atomic operators.

## 2. The model and calculations

As shown in Fig. 1, we consider that two atomic ensembles are placed inside an optical cavity. Ensemble $j\ (j=1,2)$ consists of $N_j$ four-level atoms, with two upper levels $|2\rangle$ and $|3\rangle$ and two

* Corresponding author.
*E-mail address:* qingxiamu@ncepu.edu.cn (Q.-X. Mu).

http://dx.doi.org/10.1016/j.optcom.2014.02.050
0030-4018 © 2014 Elsevier B.V. All rights reserved.

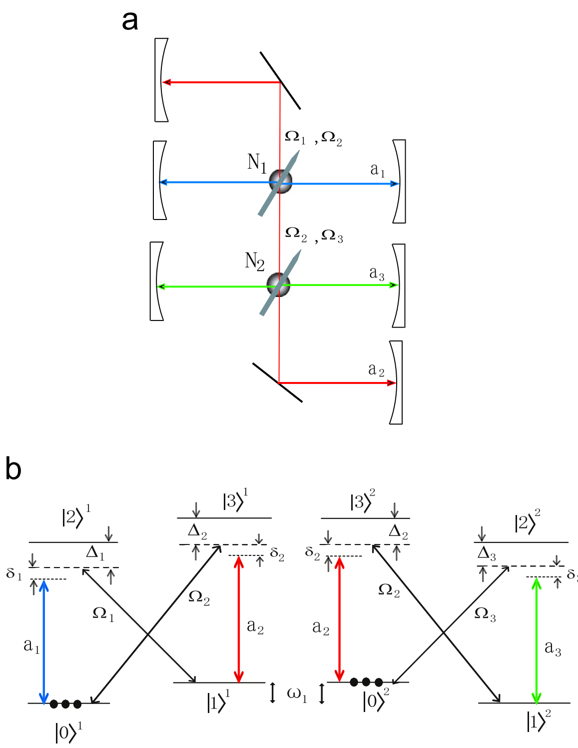


**Fig. 1.** (a) Possible cavity setup. Three cavity fields $a_1$, $a_2$ and $a_3$ are generated. Ensembles 1 and 2 contain $N_1$ and $N_2$ atoms respectively, with each atom coupled by two classical fields. (b) Atomic energy levels and transitions for ensemble 1 (left) and ensemble 2 (right). The energy of level $|0\rangle^1$ ($|1\rangle^2$) is taken to be zero as the reference point for atomic ensemble 1(2). The non-zero ground state has energy $\hbar\omega_1$, and the two excited states have energies $\hbar\omega_2$ and $\hbar\omega_3$, respectively.

lower levels $|0\rangle$ and $|1\rangle$. The levels $|0\rangle$ and $|1\rangle$ are stable with a long lifetime. Two classical fields, with Rabi frequencies $\Omega_1$ and $\Omega_3$, combined with two cavity modes $a_1$ and $a_3$ are coupled to two different atomic ensembles with coupling constants $g_1$ and $g_3$, while the classical field, with frequencies $\Omega_2$, combined with the cavity mode $a_2$ is applied to both ensembles with coupling constants $g_2^+$ and $g_2^-$, respectively. The two atomic ensembles are initially prepared in different ground states via separate optical pumping, but for convenience we relabel the ground states in ensemble 2 so that all atoms are initially in state $|0\rangle$ in our theoretical analysis.

In the rotating wave approximation, the Hamiltonian for this system in the interaction picture can be written as ($\hbar = 1$)

$$H = \sum_{i=1}^{N_1} [g_1 a_1 e^{i(\Delta_1+\delta_1)t} |2\rangle_i^1\langle 0| + g_2^+ a_2 e^{i(\Delta_2+\delta_2)t}|3\rangle_i^1\langle 1| + \Omega_1 e^{i\Delta_1 t}|2\rangle_i^1\langle 1| + \Omega_2 e^{i\Delta_2 t}|3\rangle_i^1\langle 0| + \mathrm{H.c.}] + \sum_{i=1}^{N_2} [g_2^- a_2 e^{i(\Delta_2+\delta_2)t}|3\rangle_i^2\langle 0| + g_3 a_3 e^{i(\Delta_3+\delta_3)t}|2\rangle_i^2\langle 1| + \Omega_2 e^{i\Delta_2 t}|3\rangle_i^2\langle 1| + \Omega_3 e^{i\Delta_3 t}|2\rangle_i^2\langle 0| + \mathrm{H.c.}], \tag{1}$$

where $\Delta_1 = \omega_2 - \omega_1 - \omega_{L1}$, $\Delta_2 = \omega_3 - \omega_{L2}$, and $\Delta_3 = \omega_2 - \omega_1 - \omega_{L3}$ are the frequency detunings between the atoms and the driven fields, with $\omega_{L1}(\omega_{L2}, \omega_{L3})$ being the frequency of classical field $\Omega_1(\Omega_2, \Omega_3)$. $\delta_1 = \omega_1 + \omega_{L1} - \omega_{a1}$, $\delta_2 = \omega_{L2} - \omega_1 - \omega_{a2}$, $\delta_3 = \omega_1 + \omega_{L3} - \omega_{a3}$, where $\omega_{a1}(\omega_{a2}, \omega_{a2})$ is the frequency associated with the cavity mode $a_1(a_2, a_3)$. In the large detuning limit, $|\Delta_k| \gg \{|g_1|, |g_2^+|, |g_2^-|, |g_3|, |\Omega_k|, |\delta_k|\} (k = 1, 2, 3)$, we can adiabatically eliminate the excited states $|2\rangle$ and $|3\rangle$ and get an effective Hamiltonian as follows:

$$H_{eff} = \left[-\delta_1 - \frac{|g_1|^2}{\Delta_1}\left(\frac{N_1}{2} - J_{z1}\right)\right] a_1^\dagger a_1 + \left[-\delta_3 - \frac{|g_3|^2}{\Delta_3}\left(\frac{N_2}{2} + J_{z2}\right)\right] a_3^\dagger a_3 + \left[-\delta_2 - \frac{|g_2^+|^2}{\Delta_2}\left(\frac{N_1}{2} + J_{z1}\right) - \frac{|g_2^-|^2}{\Delta_2}\left(\frac{N_2}{2} - J_{z2}\right)\right] a_2^\dagger a_2 - \frac{\Omega_1}{\Delta_1}\left(\frac{N_1}{2} + J_{z1}\right) - \frac{\Omega_2}{\Delta_2}\left(\frac{N_1}{2} - J_{z1}\right) - \frac{\Omega_2}{\Delta_2}\left(\frac{N_2}{2} + J_{z2}\right) - \frac{\Omega_3}{\Delta_3}\left(\frac{N_2}{2} - J_{z2}\right) - \left[J_1^-\left(\frac{g_1^* \Omega_1}{\Delta_1} a_1^\dagger + \frac{g_2^+ \Omega_2^*}{\Delta_2} a_2\right) + \mathrm{H.c.}\right] - \left[J_2^-\left(\frac{g_2^{-*} \Omega_2}{\Delta_2} a_2^\dagger + \frac{g_3 \Omega_3^*}{\Delta_3} a_3\right) + \mathrm{H.c.}\right], \tag{2}$$

where we have defined the collective atomic spin operators by

$$J_{jz} = \frac{1}{2}\sum_{i=1}^{N_j}(|1\rangle_i^j\langle 1| - |0\rangle_i^j\langle 0|), \quad J_j^+ = \sum_{i=1}^{N_j} |1\rangle_i^j\langle 0|. \tag{3}$$

In the Holstein–Primakoff transformation, we map the collective spin operators into harmonic oscillator annihilation and creation operators $c_j$ and $c_j^\dagger$ ($[c_j, c_j^\dagger] = 1$) via $J_j^+ \simeq \sqrt{N_j} c_j^\dagger$, $J_j^- \simeq \sqrt{N_j} c_j$, and $J_{jz} \simeq -N_j/2$ in the low-lying collective excitations. Then the effective Hamiltonian can be reduced to

$$H_{eff} = \left(\delta_1 + \frac{N_1|g_1|^2}{\Delta_1}\right) a_1^\dagger a_1 + \left(\delta_2 + \frac{N_2|g_2^-|^2}{\Delta_2}\right) a_2^\dagger a_2 + \delta_3 a_3^\dagger a_3 + \left[\sqrt{N_1} c_1 \left(\frac{g_1^* \Omega_1}{\Delta_1} a_1^\dagger + \frac{g_2^+ \Omega_2^*}{\Delta_2} a_2\right) + \mathrm{H.c.}\right] + \left[\sqrt{N_2} c_2 \left(\frac{g_2^{-*} \Omega_2}{\Delta_2} a_2^\dagger + \frac{g_3 \Omega_3^*}{\Delta_3} a_3\right) + \mathrm{H.c.}\right], \tag{4}$$

where the constant energy terms have been neglected. By choosing the appropriate detunings or laser intensities, we assume that the condition $\delta_1 + N_1|g_1|^2/\Delta_1 = \delta_2 + N_2|g_2^-|^2/\Delta_2 = \delta_3 = 0$ is satisfied. Then the effective Hamiltonian can be written as

$$H_{eff} = (i\lambda_1 a_1^\dagger + i\lambda_2 a_2) c_1 + (i\lambda_3 a_2^\dagger + i\lambda_4 a_3) c_2 + \mathrm{H.c.}, \tag{5}$$

where we have set $\lambda_1 = |\sqrt{N_1} g_1 \Omega_1/\Delta_1|$, $\lambda_2 = |\sqrt{N_1} g_2^+ \Omega_2/\Delta_2|$, $\lambda_3 = |\sqrt{N_2} g_2^- \Omega_2/\Delta_2|$ and $\lambda_4 = |\sqrt{N_2} g_3 \Omega_3/\Delta_3|$. Following the standard procedure which was obtained by Gardiner and Collett [26], the quantum Langevin equations for the three cavity modes can be expressed as

$$\begin{aligned} \dot{a_1} &= \lambda_1 c_1 - \frac{\kappa_1}{2} a_1 - \sqrt{\kappa_1} a_1^{in}, \\ \dot{a_2} &= -\lambda_2 c_1^\dagger + \lambda_3 c_2 - \frac{\kappa_2}{2} a_2 - \sqrt{\kappa_2} a_2^{in}, \\ \dot{a_3} &= -\lambda_4 c_2^\dagger - \frac{\kappa_3}{2} a_3 - \sqrt{\kappa_3} a_3^{in}, \\ \dot{c_1} &= -\lambda_1 a_1 - \lambda_2 a_2^\dagger, \\ \dot{c_2} &= -\lambda_3 a_2 - \lambda_4 a_3^\dagger, \end{aligned} \tag{6}$$

where $k_1$ ($k_2, k_3$) is the cavity decay rate. $a_1^{in}$ ($a_2^{in}, a_3^{in}$) is the operator associated with the vacuum input field satisfying the correlations $\langle a_i^{in}(t), a_{i'}^{in^\dagger}(t')\rangle = \delta_{ii'}\delta(t-t')$ for $i, i' = 1, 2, 3$.

From the Fourier transformation in rotating frame and combining with the boundary condition $a_i^{out} = a_i^{in} + \sqrt{\kappa_i} a_i$ ($i = 1, 2, 3$), the output fields can be determined by the function of the input fields

$$\begin{aligned} a_1^{out}(\omega) &= A(\omega) a_1^{in}(\omega) + B(\omega) a_2^{in\dagger}(-\omega) + C(\omega) a_3^{in}(\omega), \\ a_2^{out}(\omega) &= -B(\omega) a_1^{in\dagger}(-\omega) + D(\omega) a_2^{in}(\omega) + E(\omega) a_3^{in\dagger}(-\omega), \\ a_3^{out}(\omega) &= C(\omega) a_1^{in}(\omega) - E(\omega) a_2^{in\dagger}(-\omega) + F(\omega) a_3^{in}(\omega), \end{aligned} \tag{7}$$

where

$$A(\omega) = \left[i\omega\left(i\omega - \frac{\kappa_2}{2}\right)\left(i\omega - \frac{\kappa_3}{2}\right)\lambda_1^2 + i\omega\left(i\omega + \frac{\kappa_1}{2}\right)\left(i\omega - \frac{\kappa_3}{2}\right)(\lambda_3^2 - \lambda_2^2)\right.$$

Download English Version:

https://daneshyari.com/en/article/1534741

Download Persian Version:

https://daneshyari.com/article/1534741

Daneshyari.com