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Quantum metrology with Fock and even coherent states: Parity detection approaches to the Heisenberg limit

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ABSTRACT

Using an any number of photons Fock state $|m\rangle$ and an even coherent state as inputs of Mach–Zenhder interferometer (MZI), we investigate the quantum interference of output port on the basis of phase space theory. The statistical properties are discussed by explicitly deriving the Wigner function, photon-number distribution at individual output port. Moreover, the phase estimation is examined. It is shown that the parity signal is more super-resolved with the increase of m, n_c values (n_c is the square of amplitude of coherent state). The phase sensitivity has superior sensitivity in the vicinity of zero phase difference, and reaches below shot-noise limit when $m \geq 1$. It also approaches the Heisenberg limit with the increasing m.

1. Introduction

Accurate and efficient phase estimation of the optical fields is the basis of quantum metrology which has been widely studied with the rapid development of quantum information [1–3]. In order to enhance the phase estimation, people had proposed many theoretical schemes to surpass the standard quantum limit (SQL, $\delta\varphi \to 1/\sqrt{\overline{N}}$) and even to reach the Heisenberg limit (HL, $\delta\varphi \to 1/\overline{N}$) by using nonclassical quantum states, where $\overline{N}$ is the mean number of input photons [4–18]. For example, it is shown that the HL could be approached with the so-called NOON states as the input, together with an appropriate measurement scheme [1,5] (Here, these states are not the initial states of the MZI, but the states after the first beam-splitter). Twin-Fock states $|N\rangle_a|N\rangle_b$ inputs can also be considered as another possible way to reach the HL using an Mach–Zehnder interferometer (MZI) [19]. Generally, it is difficult to generate the NOON states and the twin Fock states with a large photon number. However, by feeding coherent and squeezed-vacuum states to the MZI, it is found that the phase sensitivity can reach the Heisenberg scaling, and is independent of the true value of the phase shift for arbitrary values of squeezing [6,20]. The interference between the two states can effectively produce path entangled NOON states with very high fidelities [7].

In some research works [21–24], an improved phase estimation scheme is presented employing entangled coherent states inputs and photon-number and parity measurements. Here, the entangled coherent states are obtained by superimposing coherent states and Schodinger cat states. It is shown that there is a noticeable improved sensitivity of phase estimation in a certain region (in comparison to NOON states), which demonstrates that superpositions could be very useful and robust for quantum metrology, although the coherent states are known as the most "classical-like" quantum states. Quantum optical metrology has Heisenberg-limited sensitivity of phase estimation as its goal. Nonclassical states of light can dramatically improve the sensitivity of interferometric phase estimations, which is crucial to the development of modern quantum optics [25,26]. In this paper, we consider an any number of photon Fock state and coherent state superpositions as inputs of the MZI, which can be seen as an example of mixed inputs with continuous- and discrete-variable quantum states [27,28]. It is found that the HL can be reached by using such two states. To the best of our knowledge, there is no relative report in literature before.

The paper is organized as follows. In Section 2, we briefly describe the the propagation of the MZI using the phase space function (Wigner function). In particular, we shall realize this purpose using the property that the order of Weyl ordered operators are invariance under similar transformations. This method is convenient for understanding the corresponding input–output process. In Section 3, we shall investigate statistical properties of the field at an output port, such as Wigner function, average photon numbers, fidelity between an output port and an

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input one (photon number distribution), and parity. Sections 4 and 5 are devoted to studying the phase sensitivity with parity measurement and intensity difference measurement schemes, respectively. Conclusions are given in Section 6.

2. Relation between input and output for MZI

Mach–Zehnder Interferometer (MZI) can be considered as a prototype and conventional interferometer of a linear lossless device with two input ports and two output ports (see Fig. 1), where two beam splitters are symmetric, i.e. the proportion of the transmissivity and the reflectivity of each beam splitter is 50:50. It is simplest to discuss the working of the interferometer in terms of the transformations of the field operators. As shown in Fig. 1, the Heisenberg operators a_o and b_o of the output fields are related to the Heisenberg operators a and b of the input fields by

$$\begin{pmatrix} a_o \\ b_o \end{pmatrix} = U \begin{pmatrix} a \\ b \end{pmatrix}, \tag{1}$$

where the transformation matrix U has to be unitary to ensure the validity of the communication relations, and

$$\begin{aligned} U &= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \\ &= \begin{pmatrix} -ie^{i(\varphi/2)}\sin\frac{\varphi}{2} & ie^{i(\varphi/2)}\cos\frac{\varphi}{2} \\ ie^{i(\varphi/2)}\cos\frac{\varphi}{2} & e^{i(\varphi/2)} i\sin\frac{\varphi}{2} \end{pmatrix} = \begin{pmatrix} -B & A \\ A & B \end{pmatrix}, \end{aligned} \tag{2}$$

which leads to $U^{-1} = \begin{pmatrix} -B^* & A^* \\ A^* & B^* \end{pmatrix}$, $(\det U = e^{i\varphi})$ and we have set $B = ie^{i(\varphi/2)}\sin(\varphi/2)$, $A = ie^{i(\varphi/2)}\cos(\varphi/2)$. Here the mirrors contribution phase shifts to both beams is ignored which only leads to an overall phase shift. From the transformation relation in Eq. (1), it is equaling to the following transformation:

$$\begin{pmatrix} a_o \\ b_o \end{pmatrix} = S \begin{pmatrix} a \\ b \end{pmatrix} S^\dagger, \tag{3}$$

where S is a unitary operator.

In order to build the relation between inputs and outputs, we consider the transformation of Wigner operator through the MZI shown in Fig. 1. For single-mode system ρ_a, the Wigner function (WF) is defined as $\mathrm{Tr}[\rho_a \Delta(\alpha)]$, where $\Delta_a(\alpha)$ is the single-mode Wigner operator [29,30] defined as

$$\begin{aligned} \Delta_a(\alpha) &= \frac{2e^{2|\alpha|^2}}{\pi}\int \frac{d^2 z}{\pi}|z\rangle\langle -z| e^{-2(z\alpha^* - z^*\alpha)} \\ &= \vdots\,\delta(\alpha - a)\delta(\alpha^* - a^\dagger)\,\vdots, \\ \Delta_b(\beta) &= \vdots\,\delta(\beta - b)\delta(\beta^* - b^\dagger)\,\vdots, \end{aligned} \tag{4}$$

where the symbol $\vdots\,\vdots$ denotes the Weyl ordering, $\alpha = (q_1 + \mathrm{i}p_1)/\sqrt{2}$ and $\beta = (q_2 + \mathrm{i}p_2)/\sqrt{2}$. Using the property that the order of Weyl ordered operators are invariance under similar transformations [29,31,32], which means

$$S\,\vdots(\circ\circ\circ)\vdots\,S^{-1} = \vdots\,S(\circ\circ\circ)S^{-1}\,\vdots, \tag{5}$$

as if the "fence" $\vdots\,\vdots$ did not exist, so S can pass through it, thus the Wigner operators under the transform (3) become

$$\begin{aligned} \Delta_{ao}(\alpha)\Delta_{bo}(\beta) &= S\,\vdots\,\delta(\alpha - a)\delta(\alpha^* - a^\dagger)\times\delta(\beta - b)\delta(\beta^* - b^\dagger)\,\vdots\,S^\dagger \\ &= \vdots\,\delta(\alpha_o - a)\delta(\alpha_o^* - a^\dagger)\times\delta(\beta_o - b)\delta(\beta_o^* - b^\dagger)\,\vdots \\ &= \Delta_a(\alpha_o)\Delta_b(\beta_o), \end{aligned} \tag{6}$$

which indicates that after the transformation of MZI, the Wigner operators of output ports still can be considered as the product of Wigner operators of input ports, related by the classical correspondence of quantum transformation as follows:

$$\begin{pmatrix} \alpha_o \\ \beta_o \end{pmatrix} = U^{-1}\begin{pmatrix} \alpha \\ \beta \end{pmatrix}. \tag{7}$$

According to the definition of WF $W_o(\alpha_o, \beta_o)$, the WF of the output port can be directly obtained from the input port WF $W_{in}(\alpha, \beta)$ by replacing α and β with $\alpha_o = \beta A^* - \alpha B^*$, and $\beta_o = \alpha A^* + \beta B^*$, respectively. Thus, for the light field propagating through the optical devices, the relation between the output WF and the input WF is given by

$$W_o(\alpha, \beta) = W_a(\beta A^* - \alpha B^*)W_b(\alpha A^* + \beta B^*). \tag{8}$$

When Fock state $|m\rangle$ is used as an input of MZI, whose WF is given by [33,34]

$$W_{|m\rangle}(\alpha) = \frac{2(-1)^m}{\pi}e^{-2|\alpha|^2}L_m(4|\alpha|^2), \tag{9}$$

which is related to a Laguerre–Gaussian function, and $L_m(x)$ is Laguerre polynomial whose generating function is

$$L_m(xy) = \frac{(-1)^m}{m!}\frac{\partial^{2m}}{\partial t^m \partial \tau^m}e^{-t\tau + tx + \tau y}|_{t=\tau=0}. \tag{10}$$

On the other hand, when the even coherent states are used as the other input, whose definition is given by $|\psi\rangle = N(|\beta_0\rangle + |-\beta_0\rangle)$, where $|\beta_0\rangle$ ($\beta_0 = \sqrt{n_c}e^{-i\varphi_c}$, $|\beta_0| = n_c$) is a coherent state and the normalization factor N is given by $N = [2(1 + e^{-2|\beta_0|^2})]^{-1/2}$. The corresponding WF is

$$W_{|\beta_0\rangle}(\beta) = \frac{2N^2}{\pi}\left\{e^{-2|\beta_0 - \beta|^2} + e^{-2|\beta_0 + \beta|^2} + (e^{-2|\beta_0|^2}e^{2(\beta_0^* - \beta^*)(\beta_0 + \beta)} + c.c)\right\}. \tag{11}$$

The corresponding WF of the input states is just the product of the respective WFs [32,33], i.e.

$$\begin{aligned} W_{in}(\alpha, \beta) = \frac{4N^2(-1)^m}{\pi^2}&e^{-2|\alpha|^2}L_m(4|\alpha|^2) \\ &\{e^{-2|\beta_0 - \beta|^2} + e^{-2|\beta_0 + \beta|^2} \\ &+ (e^{-2|\beta_0|^2}e^{2(\beta_0^* - \beta^*)(\beta_0 + \beta)} + c.c)\}. \end{aligned} \tag{12}$$

Thus using the transformation in Eq. (7), we can obtain the WF at the output fields by replacing α and β with $\alpha_o = \beta A^* - \alpha B^*$, and $\beta_o = \alpha A^* + \beta B^*$, i.e.

$$\begin{aligned} W_{out}(\alpha, \beta) = &\frac{4N^2(-1)^m}{\pi^2 e^{2|\beta_0|^2}}e^{-2|\alpha|^2 - 2|\beta|^2}L_m(4|\alpha_o|^2) \\ &\times\{W_+^{out} + W_-^{out} + e^{2|\beta_0|^2}W_{\inf}^{out}\}, \end{aligned} \tag{13}$$

where we have set (note $AB^* - BA^* = 0$, and $AA^* + BB^* = 1, AA^* - BB^* = \cos\varphi, 2AB^* = \sin\varphi$)

$$\begin{aligned} W_+^{out} &= \exp\left\{2\left(\alpha\beta_0^* A^* + \beta_0^* B^* \beta\right) + c.c\right\}, \\ W_-^{out} &= \exp\left\{2\left(-\alpha\beta_0^* A^* - \beta_0^* B^* \beta\right) + c.c\right\}, \\ W_{\inf}^{out} &= \exp\left\{2\left(\alpha\beta_0^* A^* + \beta\beta_0^* B^*\right) - c.c\right\} + c.c. \end{aligned} \tag{14}$$

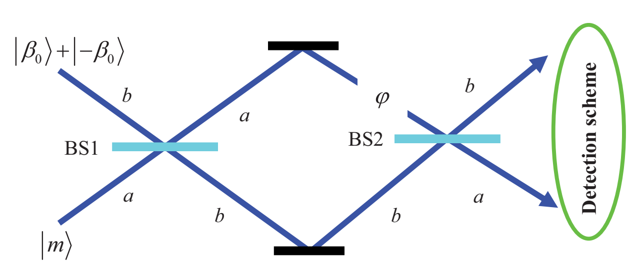

Fig. 1. The Mach–Zehnder interferometer with an input even coherent and Fock states.