Optics Communications 313 (2014) 290–293

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Airy beam with a hyperbolic trajectory

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ARTICLE INFO

Article history:
Received 26 June 2013
Accepted 13 October 2013
Available online 26 October 2013

Keywords:
Airy beam
Beam acceleration
Hyperbolic beam trajectory

ABSTRACT

The well-known Airy beams are generated when a transparency with a cubic phase put at the focal distance from a spherical lens is illuminated by a Gaussian beam. In this case, a diffraction-free Airy beam that propagates with acceleration on a parabolic path is generated behind the spherical lens's focus. We have shown that directly behind the cubic-phase transparency there is a path section on which the Airy beam is propagating with acceleration on a hyperbolic path. We refer to the Airy beam on the hyperbolic path section as a Hyperbolic Airy (HA) beam. The HA beams notably show the linear divergence, the nonuniform acceleration, rapidly decaying with distance, and the "center of gravity" shifting linearly with distance in the absence of the linear phase in the initial field, with the acceleration being by an order of magnitude higher than that of the Airy beam on the parabolic path section.
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1. Introduction

The solution to the paraxial equation of propagation in the form of Airy functions was first offered in Refs. [1–3]. The Airy beams discussed in Refs. [1–3] possess an infinite energy. In [4–6] finite-energy Airy beams in optics were analyzed. The solution of Eq.

$$2i\frac{\partial E}{\partial \xi}+\frac{\partial^2 E}{\partial s^2}=0 \quad (1)$$

has been found in the form of a function [4]

$$E\ (s,\xi)=\mathrm{Ai}(s-\xi^2/4+ia\xi)\ \exp\ (is\xi/2+ia^2\xi/2-i\xi^3/12-a\xi^2/2+as) \quad (2)$$

which takes the form of the exponentially apodized Airy function at the input, $\xi=0$:

$$E\ (s,\xi=0)=\mathrm{Ai}(s)\exp\ (as), a>0 \quad (3)$$

where $s=x/x_0$, $\xi=z/(kx_0{}^2)$ are the dimensionless transverse and longitudinal Cartesian coordinates, $k=2\pi/\lambda$ is the wavenumber, x_0 is an arbitrary transverse scale, $\mathrm{Ai}(x)$ is the Airy function, and a is constant. The Airy beam can be produced by passing a Gaussian beam through a phase transparency with cubic dispersion on the transverse coordinate, followed by implementing the Fourier transform with a spherical lens. This can be inferred from the Fourier image of the input field (3):

$$\tilde{E}(t)=\exp\ (-at^2)\exp(it^3/3-ia^2t+a^3/3) \quad (4)$$

The key peculiarity of the Airy beam is that its major lobe propagates along a curved trajectory, which has the form of a parabola. However, it has been shown [7] that the "center of gravity" of a finite-energy Airy beam in Eq. (3) is not displaced upon propagation, with the accelerating effect occurring only at small values of the parameter $a \ll 1$. Analytic expressions for different types of the accelerating beams, including the Airy beams in the ABCD optical system have been obtained [8–10]. Propagating beams with their rays forming a desired caustic curve have been examined [11–14]. Accelerating beams that propagate on a circular and elliptic trajectory have also been studied [15–17]. To our knowledge there have been no publications handling the Airy beams accelerating along a hyperbolic trajectory. In this work, we show that in an optical setup conventionally employed to generate the Airy laser beams of Eq. (2) there is a path section found immediately behind the transparency of Eq. (4) on which the Airy beam is propagating with acceleration on a hyperbola. We have termed such a beam as the Hyperbolic Airy (HA) beam. While lacking the property of being diffraction-free upon propagation on the hyperbolic path, the HA beam shows a number of other notable properties, such as having high acceleration (though rapidly decaying with distance due to highly curved path) and preserving its shape up to a scale, i.e. showing the linear divergence. We have shown that the hyperbolic path is observed for both a paraxial beam (Beam Propagation Method simulation, BPM) and a vector beam (Finite-Difference in Time Domain simulation, FDTD). On the hyperbolic path, the HA beams can also find use for micromanipulation [18] and generation of curved plasma channels [19].

2. Diffraction of the Gaussian beam by cubic phase element

To analyze a beam which is generated in the Fresnel diffraction zone of the phase transparency (4), let us write the complex amplitude of the Gaussian beam directly behind the phase

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http://dx.doi.org/10.1016/j.optcom.2013.10.044

transparency:

$$E(x,0)=\exp\left[-(x/w)^2+i\alpha(x/x_0)^3+i\beta(x/x_0)\right], \tag{5}$$

where w is the Gaussian beam's waist radius and α and β are dimensionless parameters of the phase transparency. Then, the paraxial approximation of the light field amplitude at distance z is given by

$$E(x,z)=\sqrt{\frac{-i2\pi k}{z}}\frac{x_0}{\sqrt[3]{3\alpha}}\exp\left[\frac{1}{3\alpha}\left(\frac{x_0}{w}\right)^2\left(\beta-\frac{kx_0x}{z}\right)+\frac{2}{27\alpha^2}\left(\frac{x_0}{w}\right)^6\left(1-3\frac{z_0^2}{z^2}\right)\right]\times\exp\left[\frac{ikx^2}{2z}-\frac{iz_0}{z}\frac{1}{3\alpha}\left(\frac{x_0}{w}\right)^2\left(\beta-\frac{kx_0x}{z}\right)\right]\exp\left[-\frac{2i}{27\alpha^2}\left(\frac{x_0}{w}\right)^6\left(3\frac{z_0}{z}-\frac{z_0^3}{z^3}\right)+ikz\right]$$
$$\times\mathrm{Ai}\left\{\frac{1}{\sqrt[3]{3\alpha}}\left[\beta-\frac{kx_0x}{z}+\frac{1}{3\alpha}\left(\frac{x_0}{w}\right)^4\left(1-\frac{iz_0}{z}\right)^2\right]\right\}, \tag{6}$$

where $z_0=kw^2/2$ is the Rayleigh range. The relationship for the complex amplitude (6) was derived by substituting the initial field (5) into the Fresnel transform, separating out the perfect cube in the integrand exponential function, and the subsequent representation of the Airy function in the integral form [20, identity 10.4.32].

The expression in Eq. (6) describes an HA beam. It is noteworthy that while the propagation of the Airy beam in an ABCD optical system was discussed in Ref. [8], a relationship similar to Eq. (6) was not deduced and the possibility of the beam acquiring the acceleration upon propagation on a hyperbolic path was not discussed. The relationship in Eq. (6) suggests that unlike the linear phase of the Airy beam (2), the HA beam has a quadratic phase, thus experiencing the divergence upon propagation. Besides, similarly to Eq. (2), the argument of the Airy function in Eq. (6) is complex, although the z-dependence is different: the value of the Airy function argument is in direct proportion to z^2 in Eq. (2) and in inverse proportion to z in Eq. (6). It is possible to obtain the infinite energy Airy beams if the cubic phase transparency is illuminated by a plane wave ($w\to\infty$) rather than a Gaussian beam. Then, we obtain, instead of Eq. (6):

$$E(x,z)=\sqrt{\frac{-i2\pi k}{z}}\frac{x_0}{\sqrt[3]{3\alpha}}\mathrm{Ai}\left[\frac{1}{\sqrt[3]{3\alpha}}\left(\beta-\frac{kx_0x}{z}-\frac{k^2x_0^4}{12\alpha z^2}\right)\right]\exp\left[\frac{ik}{2z}\left(x^2+\frac{kx_0^3x}{3\alpha z}-\frac{\beta x_0^2}{3\alpha}+\frac{k^2x_0^6}{54\alpha^2z^2}\right)+ikz\right] \tag{7}$$

Because the phase relation remains quadratic, the beam in Eq. (7) will diverge upon propagation. As opposed to the beam in Eq. (6), the Airy function argument has become real, making it possible to deduce an equation for the beam path. Putting the Airy function's argument equal to values y_m at which the function has local maxima, we find: $[\beta-kx_0x/z-k^2\times{}_0{}^4/(12\alpha z^2)]/(3\alpha)^{1/3}=y_m$. It leads to explicit expression of the trajectory of the HA beam maximum:

$$x=\frac{\left(\beta-y_m\sqrt[3]{3\alpha}\right)z}{kx_0}-\frac{kx_0^3}{12\alpha z} \tag{8}$$

3. Condition of beam acceleration

Unlike a parabolic trajectory of the beam (2), the HA beam propagates along a hyperbolic path (8). The beam exhibits acceleration on the path sections at which the first and second derivatives have the same sign, i.e. $(dx/dz)(d^{\,2}x/dz^2)>0$, or

$$\left[\frac{\beta-y_m\sqrt[3]{3\alpha}}{kx_0}+\frac{kx_0^3}{12\alpha z^2}\right]\left[\frac{-kx_0^3}{12\alpha z^3}\right]>0$$

Since both k and z are positive, considering two cases when x_0^3/α is positive or negative, we derive the following inequality:

$$\frac{1}{z^2}<\frac{12\alpha}{k^2x_0^4}\left(y_m\sqrt[3]{3\alpha}-\beta\right), \tag{9}$$

which can only be fulfilled if the right-hand side is positive, i.e. $\mathrm{sign}(\alpha)\beta<y_m(3|\alpha|)^{1/3}$. In this case, the acceleration will occur at distances

$$z>z_1=\frac{kx_0^2}{2\sqrt{3\alpha(y_m\sqrt[3]{3\alpha}-\beta)}} \tag{10}$$

Unlike the beams (2), the acceleration is not uniform and decreases as z^{-3}. Let us analyze the following parameters: $x_0=\lambda=532$ nm, $\alpha=-1$, $\beta=10$, $m=0$, $y_0=-1.01879$. In this case, the condition (9) is satisfied, so that the trajectory has the acceleration at $z>z_1\approx330$ nm. The intensity pattern of Eq. (7) is depicted in Fig. 1. The size of the computation domain in Fig. 1 is $-10\lambda\le x\le+10\lambda$, $0\le z\le4\lambda$. Fig. 2 depicts the intensity profiles in the planes (a) $z=\lambda$, (b) 2λ, and (c) 4λ.

4. Acceleration of the conventional Airy beam

For comparison, let us examine the Airy beam of Eq. (2) at $a=0$. Putting the Airy function's argument in Eq. (2) equal to values of y_m at which it has local maxima, the trajectory of the Airy beam maximum can be explicitly expressed as $x=x_0y_m+z^2/(4k^2\times{}_0{}^3)$. From the equation, the beam is seen to have a uniform acceleration equal to $1/(2k^2\times{}_0{}^3)$, while the Airy beam of Eq. (2) is seen to be diffraction-free, because $x_1-x_2=x_0(y_m-y_n)$ is independent of z. Meanwhile for the HA beam, from Eq. (7) it follows that $x_1-x_2=(3\alpha)^{1/3}z(y_m-y_n)/(kx_0)$ and the beam shows a linear divergence with increasing z (see Fig. 2). Fig. 3 shows the intensity pattern from the field (2) for the following parameters: $\lambda=532$ nm, $x_0=\lambda/2$. The acceleration of the beam (2) at $a=0$ equals $1/(\pi^2\lambda)$, whereas for the HA beam of Eq. (8) shown in Fig. 1, the acceleration at $z=z_1\approx330$ nm is about $19.87/(\pi^2\lambda)$. This is the reason why the beam's trajectory in Fig. 1 is more curved. Note that both beams in Fig. 1 and Fig. 3 have been computed under the same conditions. The beams

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