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Propagation of Airy beam passing through the misaligned optical system with hard aperture

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ABSTRACT

By expanding the hard aperture function into a finite sum of complex Gaussian functions, an approximate analytical expression for the two-dimensional Airy beam passing through an apertured and misaligned paraxial ABCD optical system has been derived. Based on the derived formula, the propagation properties of Airy beam through the misaligned optical system with hard aperture are illustrated numerically. From the numerical results we find that the center of the output beam is deviated from the optical axis and the position of the output beam is affected by the misaligned parameters, and can be controlled by adjusting them. Our results provide an effective and fast way for studying the paraxial propagation of Airy beam through apertured and misaligned paraxial ABCD optical systems.

1. Introduction

The techniques for reducing diffractive spreading have long been pursued in optical communication and optical design [1]. In 2007, a specific type of nondiffracting beam, namely, self-accelerating Airy beam was introduced into optics [2,3]. Airy beams propagate along parabolic trajectories (self accelerating) [4] and are endowed with nondiffracting and self-healing properties [5,6]. In the past six years, Airy beams have been studied intensively both in theory and experiment. Various methods of generating Airy beams have been developed, such as phase plate [7,8], liquid crystal cell [9], cylindrical lenses [10], three-wave mixing processes [11] and microchip laser [12]. Generation of Airy light bullets [13,14], femtosecond self-healing Airy pulse [15], Airy Plasmon [16–18], and electron Airy beam [19] have also been reported. The applications of Airy beams have also been proposed and demonstrated, including clearing optically mediated particle [20], producing curved plasma channels [21], trapping and guiding microparticles [22,23], electron capture and acceleration driven [24,25] and so on. The propagation properties [26] of Airy beams have been extensively studied. The different optical systems such as free space [27], water [28], nonlinear medium [29,30], turbulent atmosphere [31–33], a four-level electromagnetic induced transparency atomic vapor [34], uniaxial crystals [35] and with a spiral phase [36] are studied in recent years. Moreover, new laser modes relate to Airy beams such as Airy complex variable function Gaussian beams [37], abruptly autofocusing Airy beam with optical vortices [38] and finite energy Airy–Hermite–Gaussian beam [39] are also been studied.

In practical cases, most optical system are slightly misaligned more or less. Therefore it is necessary to take the misalignment of optical system into consideration. In the past decade, the effects of misaligned optical system have been investigated widely such as misaligned optical resonators [40], misalignment on holographic image or optical images [41] and misalignment in Talbot interferometry and Fabry–Perot interferometry [42,43]. In 1988, Wang and Ronchi had derived the generalized diffraction integral formula for treating the propagation of a laser beam through a slightly misaligned optical system [44]. Up to now, the propagation properties of various laser beams through an optical system with aperture or misalignment have been studied widely such as partially coherent astigmatic laser beam [45,46], hollow Gaussian beam [47], high-order Bessel–Gaussian [48,49] and general-type beam [50], elliptical Gaussian beam [51,52], flat beam [53], Hermite–Gaussian beam [54,55], Laguerre–Gaussian beam [56] and so on. To the best of our knowledge, however, no literature has been reported on the propagation of Airy beam passing through the misaligned paraxial optical with hard aperture. In this paper, we derive the generalized diffraction integral formula for treating the propagation of an Airy beam through a paraxial misaligned optical system with hard aperture. Based on the derived formula, we derived an analytical propagation formula for an Airy beam passing through a slightly misaligned optical system with hard aperture by expanding the hard aperture into a finite sum of complex Gaussian functions. The numerical examples are given.

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2. Propagation formula of airy beam through the misaligned optical system with hard aperture

The optical field distribution of Airy beam at the plane of $z=0$ in the Cartesian coordinate system is given by

$$E(x_1,y_1,0)=Ai\left(\frac{x_1}{w_{x0}}\right)\exp\left(\frac{ax_1}{w_{x0}}\right)Ai\left(\frac{y_1}{w_{y0}}\right)\exp\left(\frac{ay_1}{w_{y0}}\right), \tag{1}$$

where $Ai(\cdot)$ is the Airy function. The parameters w_{x0} and w_{y0} are transverse scales, and a is the exponential truncation factor; they characterize the width and curvature of the beams.

Assuming a hard-edge circular aperture in located at the $z=0$ plane, the corresponding hard aperture function is

$$T(x_1,y_1)=\begin{cases}1 & |x_1^2+y_1^2|\le b^2\\ 0 & others\end{cases}, \tag{2}$$

where b denotes the aperture width. We can expand the hard aperture function into a finite sum of complex Gaussian functions

$$T(x_1,y_1)=\sum_{n=1}^{N}F_n\exp\left[-\frac{G_n(x_1^2+y_1^2)}{b^2}\right], \tag{3}$$

and here F_n and G_n denote the expansion and Gaussian coefficients, respectively, which could be obtained by optimization-computation directly and a table of F_n and G_n can be found in [57].

Consider an Airy beam passing through a misaligned system as shown in Fig. 1 where $RP_{1,2}$ are the alignment reference planes, $RP_{1m,2m}$ are the misalignment reference planes, ε denotes the transverse offset, and ε' is the tilted angle. Within the framework of the paraxial approximation, the following generalized Collins formula can be used to study the propagation of a laser beam through an apertured misaligned ABCD optical system in the rectangular coordinate system [44].

$$E_2(x_2,y_2,z)=\frac{-i}{\lambda B}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}T(x_1,y_1)E_1(x_1,y_1)$$
$$\exp\left\{\frac{ik}{2B}[A(x_1^2+y_1^2)-2(x_1x_2+y_1y_2)+D(x_2^2+y_2^2)+Ex_1+Fy_1+Gx_2+Hy_2]\right\}dx_1dy_1, \tag{4}$$

where $k=2\pi/\lambda$ is the wave number with λ being the wavelength. A, B, C and D are elements of the transfer matrix. The parameters E, F, G, and H are defined by

$$\begin{cases}E=2(\alpha_T\varepsilon_x+\beta_T\varepsilon'_x)\\ F=2(\alpha_T\varepsilon_y+\beta_T\varepsilon'_y)\\ G=2(B\gamma_T-D\alpha_T)\varepsilon_x+2(B\delta_T-D\beta_T)\varepsilon'_x\\ H=2(B\gamma_T-D\alpha_T)\varepsilon_y+2(B\delta_T-D\beta_T)\varepsilon'_y\end{cases}, \tag{5}$$

where $\varepsilon_x,\varepsilon_y,\varepsilon'_x,\varepsilon'_y$ denote the two-dimensional misalignment parameters, $\varepsilon_x,\varepsilon_y$ are the displacement in the x and y directions, respectively, $\varepsilon'_x,\varepsilon'_y$ are the tilting angles of the element in the x and y directions, respectively. $\alpha_T,\beta_T,\gamma_T$ and δ_T represent the misaligned matrix elements and are determined by

$$\alpha_T=1-A;\quad \beta_T=l-B;\quad \gamma_T=-C;\quad \delta_T=\pm 1-D, \tag{6}$$

where l is the axial distance between the input plane and the output plane. For forward- (or backward-) going optical elements, the sign "+" (or "−") in δ_T should be chosen.

Applying the following integral formula

$$Ai(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\exp\left(\frac{iu^3}{3}+ixu\right)du, \tag{7}$$

$$\int_{-\infty}^{\infty}\exp(px^2+qx)dx=\frac{\sqrt{\pi}\exp[-q^2/4p]}{\sqrt{-p}};\quad if\quad \mathrm{Re}[p]<0, \tag{8}$$

and after performing the integration, we get the final output field distribution

$$\begin{aligned}E_2(x_2,y_2,z)=&\frac{i\pi}{\lambda B}\exp\left\{\frac{ik}{2B}[D(x_2^2+y_2^2)+Gx_2+Hy_2]\right\}\sum_{n=1}^{N}\frac{F_n}{p}\exp\left(-\frac{l_x^2+l_y^2}{4p}\right)\\ &\times\exp\left(\frac{l_x}{8p^2w_{x0}^3}-\frac{1}{96p^3w_{x0}^6}\right)\exp\left(\frac{l_y}{8p^2w_{y0}^3}-\frac{1}{96p^3w_{y0}^6}\right),\\ &\times Ai\left(\frac{1}{16p^2w_{x0}^4}-\frac{l_x}{2pw_{x0}}\right)Ai\left(\frac{1}{16p^2w_{y0}^4}-\frac{l_y}{2pw_{y0}}\right)\end{aligned} \tag{9}$$

where

$$p=\frac{iAk}{2B}-\frac{G_n}{b^2};\quad l_x=\frac{a}{w_{x0}}-\frac{ikx_2}{B}+\frac{ikE}{2B};\quad l_y=\frac{a}{w_{y0}}-\frac{iky_2}{B}+\frac{ikF}{2B}.$$

The Eq. (9) can be reduced to following expression of the Airy beam passing through the ideal unapertured ABCD optical system with $b\to\infty$ and $E=F=G=H=0$.

$$\begin{aligned}&E_2(x_2,y_2,z)=\frac{1}{A}\exp\left\{\frac{ik}{2B}[D(x_2^2+y_2^2)]\right\}\\ &Ai\left(\frac{x_2}{Aw_{x0}}-\frac{B^2}{4A^2k^2w_{x0}^4}+\frac{iBa}{Akw_{x0}^2}\right)\\ &\exp\left(-\frac{ikx_2^2}{2AB}+\frac{ax_2}{w_{x0}}+\frac{iBx_2}{2A^2kw_{x0}^3}-\frac{aB^2}{2A^2k^2w_{x0}^4}-\frac{iB^3}{12A^3k^3w_{x0}^6}+\frac{ia^2B}{2Akw_{x0}^2}\right)\\ &Ai\left(\frac{y_2}{Aw_{y0}}-\frac{B^2}{4A^2k^2w_{y0}^4}+\frac{iBa}{Akw_{y0}^2}\right)\\ &\exp\left(-\frac{iky_2^2}{2AB}+\frac{ay_2}{w_{y0}}+\frac{iBy_2}{2A^2kw_{y0}^3}-\frac{aB^2}{2A^2k^2w_{y0}^4}-\frac{iB^3}{12A^3k^3w_{y0}^6}+\frac{ia^2B}{2Akw_{y0}^2}\right).\end{aligned} \tag{10}$$

The Eq. (10) agrees with the result of Eq. (11) of reference [27]. If the Airy beam passes through the free space, i.e. $A=1,\ B=z,\ C=0,\ D=1$, the expression is same with the result of Eq. (5) of reference [2].

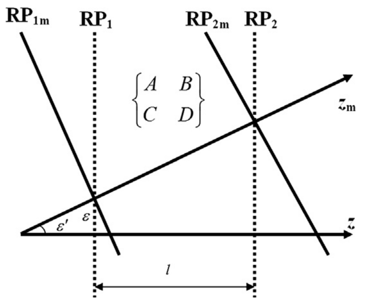

Fig. 1. Misaligned diagram for a two-dimensional forward-going system.

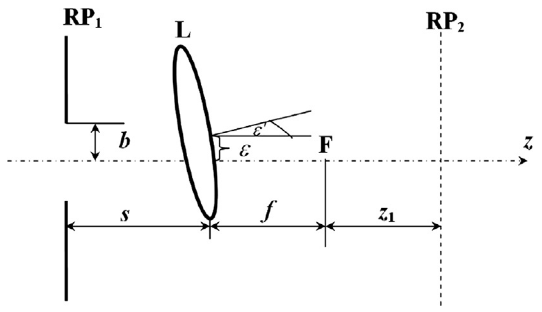

Fig. 2. A hard-aperture misaligned lens system.