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Efficient single-photon entanglement concentration for quantum communications

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ABSTRACT

We present two entanglement concentration protocols for single-photon entanglement. The first protocol is implemented with linear optics. With the help of the 50:50 beam splitter, variable beam splitter and an auxiliary photon, a less-entangled single-photon state can be concentrated into a maximally single-photon entangled state with some probability. The second protocol is implemented with the cross-Kerr nonlinearity. With the help of the cross-Kerr nonlinearity, the sophisticated single photon detector is not required. Moreover, the second protocol can be reused to get higher success probability. All these advantages may make the protocols useful in the long-distance quantum communication.

1. Introduction

Entanglement plays an important role in the current quantum information processing [1,2]. Most quantum information protocols such as quantum teleportation [3], quantum dense coding [4], quantum state sharing [5–7] and other protocols [8–13] need the entanglement to set up the quantum channel. Among all the entanglement forms, the single-particle entanglement with the form of $\frac{1}{\sqrt{2}}(|0,1\rangle_{AB}+|1,0\rangle_{AB})$ may be the simplest one. It corresponds to a superposition state in which the single particle is in two different locations A and B. Here $|0\rangle$ and $|1\rangle$ mean none particle and one particle, respectively [14–18]. In an optical system, as pointed by Lee and Kim, it can be generated by illuminating a beam splitter with a single photon. In 2000, they discussed the quantum teleportation and Bell's inequality using single-particle entanglement [14]. In 2004, Hessmo et al. reported their experiment about the single photon nonlocality [16]. The entanglement purification for single-photon entanglement was also proposed [17,18].

The most important application of the single-photon entanglement may be the quantum repeater protocol in long-distance quantum communication. For example, in the well known Duan–Lukin–Cirac–Zoller (DLCZ) repeater protocol [19,20], with one pair source and one quantum memory at each location, the quantum repeater can entangle two remote locations A and B. It can be written as $\frac{1}{\sqrt{2}}(|e\rangle_A|g\rangle_B+|g\rangle_A|e\rangle_B)$, where the $|e\rangle$ and $|g\rangle$ represent the excited state and the ground state of the atomic ensembles, respectively. Recently, Gottesman et al. proposed a protocol for building an interferometric telescope based on the single-photon entanglement [21]. The protocol has the potential to eliminate the baseline length limit, and allows in principle the interferometers with arbitrarily long baselines.

Unfortunately, similar to other types of entanglement, during the practical transmission process, the single-photon entanglement will also suffer from the noise. As pointed by Ref. [19], it will cause the photon loss. In this way, they have developed the quantum repeaters to solve such long-distance communication problem [20]. On the other hand, the single-photon entanglement is also sensitive to the channel length fluctuations, while will make the $\frac{1}{\sqrt{2}}(|0,1\rangle_{AB}+|1,0\rangle_{AB})$ suffer from the phase noise, and become a mixed state with the other ingredient of $\frac{1}{\sqrt{2}}(|0,1\rangle_{AB}-|1,0\rangle_{AB})$. Therefore, the group of Gisin discussed the entanglement purification for single-photon entanglement in both theory and experiment [17,18]. However, both the quantum repeaters and entanglement purification cannot completely solve the noisy problem. It still exists another error, which will make the maximally single-photon entanglement become a less-entangled state, with the form of $\alpha|0,1\rangle_{AB}+\beta|1,0\rangle_{AB}$, with $|\alpha|^2+|\beta|^2=1$. It may also come from the photon loss, or the different excitation

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probability in the atomic ensembles [19]. In the entanglement connection stage, if we use such less-entangled states $\alpha|0,1\rangle_{AB}+\beta|1,0\rangle_{AB}$ and $\alpha|0,1\rangle_{CD}+\beta|1,0\rangle_{CD}$ to connect the entanglement with the entanglement swapping, we will obtain the lesser quality quantum entanglement channel $\alpha^2|0,1\rangle_{AD}+\beta^2|1,0\rangle_{AD}$ [19,22]. Therefore, in the practical application, we need to recover the less-entangled state into the maximally entangled state.

Entanglement concentration is a powerful way to recover the pure less-entangled state into a maximally entangled state probabilistically [22–38]. Most entanglement concentration protocols (ECPs) are focused on the two-particle entanglement, such as the Schimidit projection method [23], the ECP based on the entanglement swapping [24], the ECP using unitary transformation [25], and the ECPs with linear optics and cross-Kerr nonlinearity [22,26–30,32,33]. In 2010, Sheng et al. have proposed an ECP for single-photon entanglement with the help of cross-Kerr nonlinearity [22]. In that protocol, after the two parties Alice and Bob picking up the successful case, they should send the photon in the spatial mode a_2b_2 to make a collective measurement. Moreover, during each concentration step, they require two pairs of nonlocal single-photon entanglement states and after the measurement at least one pair of entangled state can be remained. Though the protocol of Ref. [22] can reach a higher success probability than the protocol of Ref. [26], it is not an optimal one. The main reason is that the protocol is not based on the local operation and classical communication (LOCC). One of the single-photon entanglements should be sent back to perform a Bell-state measurement. If we consider a practical application, such process will also suffer from the noise, and make the whole protocol unsuccessful. Moreover, in order to obtain a higher success probability, it requires the phase shift of the coherent state to reach a large value of π in the single-photon level. However, natural cross-Kerr nonlinearities are extremely weak, which makes it difficult to realize the protocol under current experimental conditions [39,40].

Fortunately, in 2012, we developed a different way of entanglement concentration for two-photon polarized entangled state [29]. Two pairs of less-entangled state are not necessary. Only one pair of less-entangled state and a local single photon can also be used to complete the concentration task. In the same year, with the help of the local single photons, the entanglement concentration for arbitrary less-entangled W state was completed [34]. Interestingly, inspired by the previous works, the auxiliary single photon is also suitable for the concentration of single-photon entanglement, which can greatly improve the work of Ref. [22].

In this paper, we present two efficient ECPs for less-entangled single-photon state with local single photon. Both protocols only require one less-entangled single-photon state and a conventional single photon. In the first protocol, the linear optical elements are adopted to complete the task and it can reach the same success probability as Ref. [26]. In the second protocol, the weak cross-Kerr nonlinearity is adopted to improve the first protocol, which makes it can be reused to further concentrate the discarded items in the first protocol and get a higher success probability. In order to perform both the protocols successfully, the initial coefficients α and β should be known in advance to prepare the state of single photon. Actually, some previous ECPs in Refs. [25,29,30,34–37], also have this requirement. In a practical operation, one can measure an enough amount of samples to obtain the exact values of α and β [30].

This paper is organized as follows: In Section 2, we would present the first ECP with linear optics. In Section 3, we would present the second ECP with weak cross-Kerr nonlinearity. In Section 4, we briefly discuss the future experiment, calculate the total success probability and make a conclusion.

2. Single-photon entanglement concentration with linear optics

The basic principle of the first ECP is shown in Fig. 1. Suppose that the single photon source S_1 emits a photon and sends it to Alice and Bob in the spatial modes a_1 and b_1, which can create a less-entangled single-photon state $|\phi_1\rangle_{a_1b_1}$. $|\phi_1\rangle_{a_1b_1}$ can be written as

$$|\phi_1\rangle_{a_1b_1}=\alpha|1,0\rangle_{a_1b_1}+\beta|0,1\rangle_{a_1b_1}, \tag{1}$$

where α and β are the coefficients of the initial entangled state, $|\alpha|^2+|\beta|^2=1$.

Then another single photon source S_2 emits an auxiliary photon and sends it to Bob in the spatial mode b_2. Bob makes this auxiliary photon pass through a variable beam splitter (VBS) with the transmission of t, which can create another single-photon entangled state between the spatial mode c_1 and c_2 of the form

$$|\phi_2\rangle_{c_1c_2}=\sqrt{1-t}|1,0\rangle_{c_1c_2}+\sqrt{t}|0,1\rangle_{c_1c_2}. \tag{2}$$

In this way, the state of the whole two-photon system can be written as

$$\begin{aligned}|\phi\rangle_{a_1b_1c_1c_2}&=|\phi_1\rangle_{a_1b_1}\otimes|\phi_2\rangle_{c_1c_2}\\&=\alpha\sqrt{1-t}|1,0,1,0\rangle_{a_1b_1c_1c_2}+\beta\sqrt{t}|0,1,0,1\rangle_{a_1b_1c_1c_2}\\&\alpha\sqrt{t}|1,0,0,1\rangle_{a_1b_1c_1c_2}+\beta\sqrt{1-t}|0,1,1,0\rangle_{a_1b_1c_1c_2}.\end{aligned} \tag{3}$$

Then, Bob makes the photons in the spatial modes b_1 and c_1 enter the 50:50 beam splitter (BS), which can make

$$\begin{aligned}\hat{b}_1^{\dagger}|0\rangle&=\frac{1}{\sqrt{2}}(\hat{d}_1^{\dagger}|0\rangle-\hat{d}_2^{\dagger}|0\rangle)\\\hat{c}_1^{\dagger}|0\rangle&=\frac{1}{\sqrt{2}}(\hat{d}_1^{\dagger}|0\rangle+\hat{d}_2^{\dagger}|0\rangle).\end{aligned} \tag{4}$$

Here, the $\hat{b}_j^{\dagger}$, $\hat{c}_j^{\dagger}$ and $\hat{d}_j^{\dagger}$ with $j=1,2$ are the creation operators for the spatial mode b_j, c_j and d_j, respectively. After the BS, the whole two-photon system can evolve to

$$\begin{aligned}|\phi\rangle_{a_1d_1d_2c_2}=&\frac{\alpha\sqrt{1-t}}{\sqrt{2}}|1,1,0,0\rangle_{a_1d_1d_2c_2}+\frac{\beta\sqrt{t}}{\sqrt{2}}|0,1,0,1\rangle_{a_1d_1d_2c_2}\\&+\frac{\alpha\sqrt{1-t}}{\sqrt{2}}|1,0,1,0\rangle_{a_1d_1d_2c_2}-\frac{\beta\sqrt{t}}{\sqrt{2}}|0,0,1,1\rangle_{a_1d_1d_2c_2}\\&+\alpha\sqrt{t}|1,0,0,1\rangle_{a_1d_1d_2c_2}+\frac{\beta\sqrt{1-t}}{\sqrt{2}}|0,2,0,0\rangle_{a_1d_1d_2c_2}\end{aligned}$$

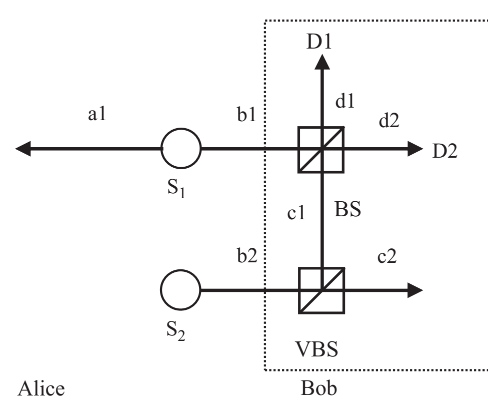

Fig. 1. A schematic drawing of the first ECP for the single-photon entanglement with linear optics. It is constructed by a 50:50 beam splitter (BS) and a variable beam splitter (VBS). Alice and Bob share a less-entangled single-photon state in the modes a_1 and b_1. Another single photon source emits an auxiliary photon in the mode b_2. The BS is located in the middle of Alice and Bob and it is used to couple the mode a_1 and c_2. The VBS is used to adjust the coefficients of the entangle state between the modes a_1 and c_2, and finally to attain the maximally entangled state.