Optics Communications 315 (2014) 374–380

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Modeling the spatial shape of nondiffracting beams: Experimental generation of *Frozen Waves* via holographic method

Tárcio A. Vieira [a], Michel Zamboni-Rached [b,*], Marcos R.R. Gesualdi [a]

[a] Universidade Federal do ABC, Santo André, SP, Brazil
[b] University of Campinas, Campinas, SP, Brazil

ARTICLE INFO

Article history:
Received 25 August 2013
Received in revised form
29 October 2013
Accepted 3 November 2013
Available online 16 November 2013

Keywords:
Nondiffracting waves
Diffraction
Waves
Optical beams
Propagation
Holography

ABSTRACT

In this paper we experimentally implement the spatial shape modeling of nondiffracting optical beams via computer generated holograms reconstructed optically by spatial light modulators. The results reported here are an experimental confirmation of the so-called *Frozen Wave* method, developed a few years ago. Optical beams of this type have potential applications in optical tweezers, medicine, atom guiding, remote sensing, etc.

1. Introduction

A few years ago, in a series of papers [1–4], an interesting theoretical method was developed, capable to furnish nondiffracting beams whose longitudinal intensity shape can be freely chosen a priori.

This approach is based on suitable superpositions of equal frequency and co-propagating Bessel beams, and the resulting wave fields are called Frozen Waves[1] (FWs), since they are moreover endowed with a pre-chosen *static* envelope, within which only the carrier wave propagates. Besides a strong control on the longitudinal intensity pattern, this method also allows a certain control on the transverse shape of the resulting beam.

Due to their unique characteristics, in particular their nondiffracting and spatial modeling properties, the FWs are quite interesting for many applications such as optical tweezers, remote sensing, atom guiding, medical purposes, etc. [5–7]

Very recently [8] the FW method was experimentally verified through the experimental generation by a holographic method of some FWs freely chosen in advance.

In this paper we present the experimental generation of several new and very interesting FWs through the implementation of amplitude computer generated holograms (CGHs) in spatial light modulators (SLMs). Our results confirm, once more, the theoretical predictions of the method developed in [1,2], and open exciting possibilities on the applicability of these very especial beams.

In the next section we make a synthesis of the theoretical FW method. After this, in Section 3, we show the experimental results concerning the generation of several nondiffracting beams whose spatial shapes are chosen in advance. The experimental generation is made by amplitude computer generated holograms implemented in two types of spatial light modulators, transmission and reflective.

2. Summarizing the theoretical Frozen Wave method

The theory of FWs was formulated in [1] and further improved in [2–4].

Here we summarize the method without entering into the mathematical details, which can be found in the references above.

To be brief, what we wish is to construct exact solutions to the wave equation representing nondiffracting beams whose longitudinal intensity pattern, $|F(z)|^2$, in the interval $0 \le z \le L$ can be freely chosen a priori.

This can be done by considering a superposition of equal frequency and co-propagating Bessel beams of order ν

$$\Psi(\rho,\phi,z,t) = e^{-i\omega t} \sum_{n=-N}^{N} A_n J_\nu(k_{\rho n}\rho) e^{ik_{zn}z} e^{i\nu\phi} \quad (1)$$

with

$$k_{\rho n}^2 = k^2 - k_{zn}^2 \quad (2)$$

* Corresponding author.
E-mail address: mzamboni@dmo.fee.unicamp.br (M. Zamboni-Rached).

[1] This approach is also called *Frozen Wave method*.

0030-4018/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.optcom.2013.11.001

where k, $k_{\rho n}$ and k_{zn} are the total, the transverse and the longitudinal wave numbers, respectively, of the nth Bessel beam in superposition (1).

In expression (1) the following choice is made:

$$k_{zn} = Q + 2\pi n/L \tag{3}$$

where Q is a constant such that

$$0 \le Q + (2\pi/L)n \le \omega/c \tag{4}$$

for $-N \le n \le N$.

The condition given by (4) ensures forward propagation only, with no evanescent waves. The constant parameter Q can be arbitrarily chosen, provided that (4) is obeyed, and it plays an important role in determining the spot-size of the resulting beam.

Still considering Eq. (1), we adopt the following choices for the coefficients A_n:

$$A_n = \frac{1}{L}\int_0^L F(z)e^{-i(2\pi/L)nz}\,dz \tag{5}$$

where, as we said, $|F(z)|^2$ is the desired longitudinal intensity pattern in the interval $0 \le z \le L$.

Now, it is important to notice that this longitudinal intensity pattern can be concentrated, as much as we wish (respecting the diffraction limit), over the propagation axis ($\rho = 0$), or over a cylindrical surface.

In the case we wish such intensity to be concentrated over the propagation axis, $\rho = 0$, zero order Bessel beams (i.e. $\nu = 0$) are to be used in the fundamental superposition (1). It is also possible to choose the spot radius, $\Delta\rho_0$, of the resulting beam by making $Q = (\omega^2/c^2 - 2.4^2/\Delta\rho_0^2)^{1/2}$.

Alternatively, if we wish this intensity configuration to be concentrated over a cylindrical surface, then higher order Bessel beams, with $\nu \ge 1$, are to be used in (1). In this case, the radius ρ_0 of the cylindrical surface can be approximately chosen, if we pick up the value of Q given by

$$\left[\frac{d}{d\rho}J_\nu(\rho\sqrt{\omega^2/c^2 - Q^2})\right]|_{\rho=\rho_0} = 0 \tag{6}$$

3. Experimental generation of Frozen Waves via holographic method

The use of spatial light modulator devices in holographic setups makes possible interesting applications in image phase correction, code signal encrypted and generation of optical beams [9–12,8,13,14].

The experiments conducted by us to generate interesting types of FWs are based on the holographic method. With the desired beams described by analytical exact solutions of the wave equation [15], we created an amplitude Computer Generated Hologram (CGHs), which is reconstructed by a nematic liquid crystal spatial light modulator (LC-SLM).

More specifically, once we have chosen the desired beam spatial shape (i.e., the beam's longitudinal intensity pattern, $|F(z)|^2$, and its spot radius or the radius of its cylindrical form), it can be approximately described by the analytical and exact FW solution in Eqs. (1) and (3)–(5). The amplitude CGH is constructed from the FW complex field $\Psi(\rho,\phi,z,t)$ (called FW-CGH) at the origin of the propagation axis, i.e. at ($z = 0$), and it is given by the transmittance hologram equation

$$H(x,y) = 1/2\{\beta(x,y) + \alpha(x,y)\cos[\phi(x,y) - 2\pi(\xi x + \eta y)]\} \tag{7}$$

where $\alpha(x,y)$ and $\phi(x,y)$ are amplitude and phase of the FW complex field $\Psi(\rho,\phi,z,t)$, respectively. For reducing the noise of the spectrum hologram signal, the conventional bias function $\beta(x,y) = [1 + \alpha^2(x,y)]/2$ is taken as a soft envelope of the amplitude $\alpha(x,y)$ [12]. To separate the different diffraction orders from the encoded complex field $\Psi(\rho,\phi,z,t)$, the off-axis reference plane wave $\exp[i2\pi(\xi x + \eta y)$ is used. In the Fourier plane, the center of the signal information is shifted to values (ξ,η) of the spatial frequencies and should be chosen according to diffraction efficiency and bandwidth of the SLM [11,8].

To guarantee the efficient generation of the FW in the chosen interval, we have used (to the FW-CGH) a circular aperture of minimum diameter D given by

$$D_{min} \ge 2L\left[\left(\frac{k}{k_{zn=-N}}\right)^2 - 1\right]^{1/2} \tag{8}$$

The parameters Q and L give us, via Eq. (4), the maximum number, $2N_{max}+1$, of Bessel beams in the superposition (1). If we consider $Q > k/2$ (as usually occurs), then

$$N_{max} = [L(k-Q)/2\pi] \tag{9}$$

where $[\cdot]$ is the floor function, i.e., N_{max} is the greatest integer smaller than or equal to $L(k-Q)/2\pi$.

3.1. Holographic experimental setups

We have experimentally generated six different and interesting FWs. In four of which it was used a *transmission* SLM, being the other two created with a *reflection* one. The most significant differences between them is the pixel resolution, and consequently the bandwidth and effective display areas. We will see later the implications of this difference in the holographic reconstruction processes of the FW complex field.

In the experimental holographic setups for FW generation [see Fig. 1(a) for the transmission SLM (Setup 1), and Fig. 1(b) for the reflection SLM (Setup 2)], we adopt a He–Ne laser (632.8 nm) that is expanded and collimated ("Exp") on a SLM device. Here we use the amplitude modulation with the polarizer *Pol* (angle 0°) and analyzer *Anl* (angle 90°), measured with respect to the input axis in the SLM. The 4-f spatial filtering system is used for the FW experimental generation.

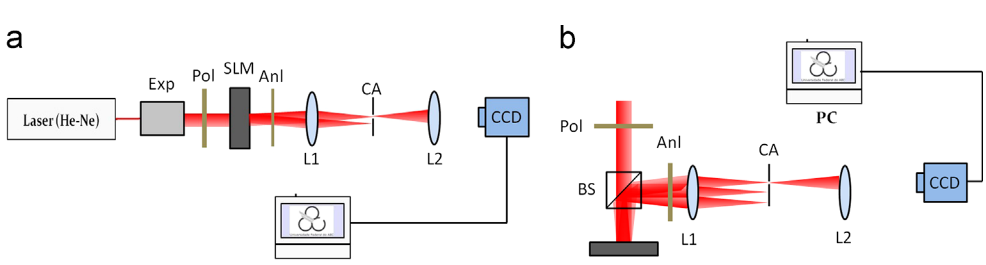

Fig. 1. (a) Experimental Setup 1; (b) Experimental Setup 2, for FW generation, where SF is a spatial filter, the L's are lenses, Pol is the polarizer, Anl is the analizer, CA is a circular aperture mask, and CCD is the camera. In (a) SLM is a LC-2002 transmission modulator and (b) SLM is a LC-R1080 reflective modulator.