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Dynamics of entanglement density in photonic crystals

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ABSTRACT

We study the dynamics of entanglement between the atom and the photonic band-gap (PBG) reservoir modes, and also between the modes themselves by means of an entanglement density. We find that the hybrid entanglement between the atom and the bath modes is restricted by the atomic and bath mode populations as well as the detuning conditions. When the atomic transition frequency is outside the band gap, the bipartite states can be created between the reservoir modes for short times. We also compare the dynamics of entanglement between isotropic and anisotropic models and find out their differences and analogies. The theoretical results could be applied to the implementation of quantum information processing in nanostructured materials.

1. Introduction

Composite qubit–oscillator system has been extensively studied theoretically, due to its fundamental significance in modelling accurately a wide variety of physical phenomena. As a notable example, this system is a very successful tool for understanding the interaction between matter and quantized radiation [1]. Recently, the hybrid structures using both a qubit and a quantum harmonic oscillator have been widely studied on quantum communication and quantum information processing (QIP) [2–7]. This type of hybrid architecture may be used to make up for the weaknesses in both type of qubit structures, and has considerable applications in QIP. These include the realization of hybrid communication via qubit–oscillator states [8,9], the preparation of entangled states that are robust against thermal noise [10,11], the realization of hybrid quantum repeater [12], the measurement of geometric phases [13], the proposal for full quantum computation [14] and many others. In view of these and other proposals, current research focuses on engineering hybrid quantum systems with strong-coupling characteristics, and offering precise experimental control over their mutual interactions.

In this spirit, many experimental platforms can be used to realize qubit–oscillator models in the lab, such as the cavity QED [15–19], ion traps [20], nanomechanical resonators [21], photonic crystal materials [22], and so on. One of the greatest advantages of all these platforms is having capability of reproducing fundamental light-matter interactions on a larger scale, thus provides experimental conditions to explore these interactions in parameter regimes. In this paper, we focus on photonic crystal materials [23,24], which is one of the most promising platforms for hybrid quantum information processing. The main reason is that it can realize easily not only the strong coupling between the atom and its radiation field in photonic crystals, but also dynamical control of the qubit–oscillator interaction. It is well known that photonic crystals are artificial materials with periodic refractive index. Its PBG structures together with its unique dispersion relationship make the atom-light manipulation much more efficient [25–27]. Furthermore, electric field [28], temperature [29] and magnetic fields [30,31] can be used to tune the energy levels of atoms located in photonic crystals. So far, PBG materials have been extensively used as a method for implementing QIP ideas. For example, entanglement trapping [32,33], hybrid quantum information protocols [34], on-chip single-photon sources [35] and memory devices [36] can be realized by exploiting PBG materials. In particular, some special quantum phenomena have been found [37–39]. The ever increasing capability for exquisite control of interactions between the atom and the PBG reservoir on the experimental side, and the increasing applications of PBG materials on QIP, require ever increasing accuracy in understanding the coupling between atom and the PBG reservoir.

It is in this spirit that we study the hybrid entanglement between the atom and the PBG reservoir modes by means of entanglement density [40], with the stated purpose of better understanding this kind of interaction and investigating the new features of entanglement induced by strong coupling. In our model, a qubit is embedded in PBG materials, and the effects of decoherence on hybrid entanglement in both isotropic and anisotropic photonic crystals are considered. We find that, the detuning of the atomic transition frequency from the PBG and the dispersion

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properties plays a crucial role in controlling this qubit–oscillator hybrid entanglement. Furthermore, when the atomic transition frequency is outside the band-gap edge, the entanglement between reservoir modes can be induced. We also make a comparison to the entanglement characteristics between isotropic and anisotropic photonic crystals.

This paper is organized as follows: the theoretical model is given in Section 2. Section 3 discusses the method of entanglement density we adapted to characterize the hybrid entanglement. Our results on the dynamics of entanglement under decoherence are present in Section 4, including the total entanglement, the entanglement between the atom and reservoir modes, and also between the modes. We summarize our results in Section 5.

2. Dynamical model

we consider a two-level atom embedded in a photonic crystal coupled to the radiation field. The transition frequency ω_0 between the excited and the ground atomic states $|1\rangle$ and $|0\rangle$, respectively, is assumed to be near the band edge of the PBG. Performing the rotating wave approximation, the Hamiltonian for the system is [41] ($\hbar=1$)

$$H=\sum_k \omega_k a_k^\dagger a_k+\omega_0|1\rangle\langle 1|+\sum_k g_k(a_k^\dagger|0\rangle\langle 1|+a_k|1\rangle\langle 0|), \tag{1}$$

where $a_k^\dagger$ and a_k are the radiation field creation and annihilation operators for the kth radiation mode with frequency ω_k. The atom–field coupling constant takes the form

$$g_k=\omega_0 d\left(\frac{1}{2\varepsilon_0\omega_k V}\right)^{1/2}\vec{e}_k\cdot\vec{u}_d. \tag{2}$$

Here d and $\vec{u}_d$ are the absolute value and the unit vector of the atomic dipole moment, respectively, V is the sample volume, $\vec{e}_k$ are the two polarization unit vectors and ε_0 is the Coulomb constant.

We assume that the qubit is initially prepared in its excited state $|1\rangle$, and the radiation field is in the vacuum state $|\{0\}\rangle$. The state vector of the system at an arbitrary time t then has the form

$$|\psi(t)\rangle=f_a(t)e^{-i\omega_0 t}|1,\{0\}\rangle+\sum_k f_k(t)e^{-i\omega_k t}|0,\{1_k\}\rangle, \tag{3}$$

where $|1,\{0\}\rangle$ describes the atom in excited states $|1\rangle$ with no photons in field modes. The state $|0,\{1_k\}\rangle$ accounts for the mode k of the reservoir being excited and the atom in its ground state $|0\rangle$. Especially, the state vector $|\{1_k\}\rangle$ of the radiation field is generated in the process of spontaneous emission and may be regarded as a normalized single-photon state. Since there is only a single excitation in each reservoir mode, we can treat the reservoir states as a set of qubits [42]. Substituting (3) into the Schrödinger equation with Hamiltonian (1), we can obtain the following set of coupled equations:

$$i\frac{\partial}{\partial t}f_a(t)=\sum_k g_k f_k(t)e^{-i(\omega_k-\omega_0)t}, \tag{4}$$

$$i\frac{\partial}{\partial t}f_k(t)=g_k f_a(t)e^{i(\omega_k-\omega_0)t}. \tag{5}$$

By formal time integration of Eq. (5), and using the Laplace transform, we can obtain the Laplace transforms $\tilde{f}_a(s)$ for the amplitudes $f_a(t)$:

$$\tilde{f}_a(s)=\frac{1}{s+\Gamma}, \tag{6}$$

where $\Gamma=\sum_k g_k^2/[s-i(\omega_0-\omega_k)]$ is the Laplace transform of the delay Green's function[43] $G(t-t')=\sum_k g_k^2 e^{-i(\omega_k-\omega_0)t-t'}$. In the following subsections, we present the calculation of $f_a(t)$ for the isotropic and anisotropic photonic crystals.

2.1. The isotropic photonic crystal

In an isotropic photonic crystal, the photon dispersion relation can be expressed approximately by [41]

$$\omega_k\approx\omega_c+A(k-k_0)^2,\quad A\approx\omega_c/k_0^2\approx c^2/\omega_c, \tag{7}$$

where ω_c is the upper band-edge frequency and k_0 is a constant characteristic of the dielectric material. Such dispersion relation is valid for frequencies close to the upper photonic band edge and associates the band-edge wave vector with a sphere in k space, $|\mathbf{k}|=k_0$ [44]. Using this dispersion relation, and the method in Ref. [45], we have

$$\Gamma(s)=\frac{\beta^{3/2}}{i\sqrt{-is-\delta}}, \tag{8}$$

where $\beta^{3/2}=[(\omega_0 d)^2/6\pi\varepsilon_0](k_0^3/\omega_c^{3/2})$, and $\delta=\omega_0-\omega_c$. Note that the phase angle of $\sqrt{-is-\delta}$ has been defined as $-\pi/2<\arg(\sqrt{-is-\delta})<\pi/2$. The amplitudes $f_a(t)$ can be calculated by means of the inverse Laplace transforms

$$f_a(t)=\frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}\tilde{f}_a(s)e^{st}\,ds, \tag{9}$$

where σ is a real constant that exceeds the real part of all the singularities of $\tilde{f}_a(s)$. With the help of the residue theorem, the amplitudes $f_a(t)$ can be rewritten as [46]

$$f_a(t)=\sum_j\frac{1}{A'(x_j^{(1)})}e^{x_j^{(1)}t}+\sum_j\frac{1}{B'(x_j^{(2)})}e^{x_j^{(2)}t}+\frac{e^{i\delta t}\beta^{3/2}}{\pi}\int_0^\infty dx\frac{\sqrt{-ix}e^{-xt}}{i\beta^3-x(-x+i\delta)^2}, \tag{10}$$

where functions $A(x)$ and $B(x)$ are defined as

$$A(x)=x+\frac{\beta^{3/2}}{i\sqrt{-ix}-\delta}, \tag{11}$$

$$B(x)=x+\frac{\beta^{3/2}}{\sqrt{ix}+\delta}. \tag{12}$$

Here $x_j^{(1)}$ are the purely imaginary roots of the equation $A(x)=0$ in the region $[Re(x_j^{(1)})<0$ and $Im(x_j^{(1)})<\delta]$, $x_j^{(2)}$ are the complex roots of the equation $B(x)=0$ in the region $[Re(x_j^{(2)})<0$ and $Im(x_j^{(2)})<\delta]$, and the functions $A'(x)$ and $B'(x)$ are defined as $A'(x)=(d/dx)A(x)$ and $B'(x)=(d/dx)B(x)$.

2.2. The anisotropic photonic crystal

As one know, there is no physical PBG material with an isotropic gap, and it is instructive to consider a more realistic anisotropic model. In this model, the dispersion relation is modified strongly by the periodic dielectric structure, and the band edge is associated with a point $\mathbf{k}=\mathbf{k}_0^{(m)}$. The photon–dispersion relation can be expressed by $\omega_k=\omega_c+A(\mathbf{k}-\mathbf{k}_0^{(m)})^2$, characteristic of a three-dimensional phase space [47]. Using this dispersion relation, the parameter Γ can be written as

$$\Gamma(s)=\frac{i\gamma^{3/2}}{\sqrt{\omega_c}+\sqrt{-is-\delta}},$$

with

$$\gamma=\left[\frac{\omega_0^2 d^2\varepsilon_0\hbar A^{3/2}}{8\pi}\left(\sum_m \sin^2\theta_m\right)\right]^{2/3},$$

and θ_m is the angle between the atomic dipole moment $\vec{u}_d$ and the mth wave vector $\mathbf{k}_0^{(m)}$ [45]. In the same manner as isotropic calculation, the amplitudes $f_a(t)$ of the anisotropic photonic crystal