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Entanglement dynamics and maintenance for two atoms in coupled cavities

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ABSTRACT

We study the entanglement dynamics for two atoms held in separate cavities which are coupled. We calculate the negativity for the atoms by initially taking the atoms to be maximally entangled as well as taking the cavity fields to be in (i) maximally entangled state $\frac{1}{\sqrt{2}}(|01\rangle+|10\rangle)$ or (ii) completely unentangled state $|10\rangle$. Limited to the particular two examples, we find that in both of the cases a large coupling strength between the cavities maintains the maximal entanglement for the two atoms well; when the coupling strength is small, compared with case (ii), the initially maximally entangled fields only enhance the entanglement for the two atoms at certain times instead of having a total improvement on the entanglement. We also investigate the entanglement dynamics by taking into account the dissipation.

1. Introduction

Quantum entanglement is not only the significant feature of quantum mechanics [1], but also the essential element of quantum information processing (QIP) [2]. Since it plays such a vital role, many efforts have been devoted to the studies on it [3–11]. However, due to the restriction of experimental conditions and the inevitable interference from the environment, the entangled qubits always experience the decoherence which is the main obstacle hindering implementation of quantum technologies. Pellizzari et al. [12] showed how decoherence affects the computation within a realistic model of a quantum computer based on cavity QED system. Moreover, Yu and Eberly have theoretically showed entanglement sudden death (ESD), a phenomenon in which two entangled qubits became completely disentangled in a finite time [13,14]. Subsequently, Almeida et al. [15] experimentally demonstrated that the entanglement may suddenly disappear by using an all-optical experimental setup.

As decoherence is adverse to QIP, it is important to find ways to suppress it and protect the entanglement. The suppression strategies are proposed early such as quantum error-correction codes [16–18] and error-avoiding codes [19–21]. Recently, contrary to ESD, Ficek and Tanaś have found that two initially unentangled qubits can get entangled after a finite time which is called entanglement sudden birth (ESB) [22]. Afterwards, Xu et al. [23] experimentally showed that entanglement can be recovered after suffering from sudden death. In addition, Maniscalco et al. [24] used the quantum Zeno effect to resist the degradation of the entanglement. Furthermore, Paternostro et al. [25] showed us a protocol for generating and protecting steady-state entanglement by modulating the detunings in the interaction between a two-qubit system and a bosonic mode.

Recently, a coupled-cavity system which is promising to overcome the difficulty of individual addressing has become a hot topic [26–37]. And it offers the possibility for implementing distributed quantum computation. However, the fields which the entangled qubits interact with were mostly set to be unentangled initially in the previous studies. We here make an alternative and devote ourselves to the entanglement dynamics of a coupled-cavity system and discuss whether the initial maximal entanglement of the fields affects the entanglement of the qubits or not. So far, there have been several entanglement measures, such as the relative entropy of entanglement [38,39], the Wootters concurrence [40], the negativity [41], and so on. In this paper, we investigate the entanglement dynamics of two initially maximally entangled atoms in two coupled cavities. By calculating and numerically simulating the negativity for this atom–cavity system, we find that the atomic entanglement can be protected well under the case of large coupling strength of the cavities. By contrast with the situation that the fields are initially unentangled, the entanglement for the atoms can be enhanced at certain times rather than at the whole evolution. If the system dissipation is considered, we find that the entanglement for the atoms is more sensitive to the cavity loss than to the atomic decay when the fields are initially maximally entangled. But the consequence is the

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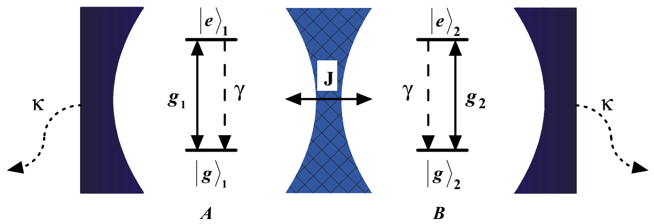

Fig. 1. The theoretical model. Two identical two-level atoms are trapped in two coupled cavities.

opposite when the fields are initially unentangled. It should be noted that our analysis is limited to the two particular examples with the fields initially in maximally entangled state $\frac{1}{\sqrt{2}}(|01\rangle+|10\rangle)$ or initially in completely unentangled state $|10\rangle$.

2. The model

Our theoretical model consists of two directly coupled cavities as shown in Fig. 1. Two identical two-level atoms 1 and 2 are trapped in two cavities and the atomic transition $|e\rangle_i \longleftrightarrow |g\rangle_i$ $(i=1,2)$ is resonantly coupled to the corresponding cavity mode. Photons can hop between the cavities. The Hamiltonian is given by $(\hbar=1)$

$$H=\sum_{i=1}^{2}[g_i(S_i^{\dagger}a_i+S_i^{-}a_i^{\dagger})]+J(a_1^{\dagger}a_2+a_1a_2^{\dagger}), \tag{1}$$

where $S_i^{\dagger}=|e_i\rangle\langle g_i|$ and $S_i^{-}=|g_i\rangle\langle e_i|$ $(i=1,2)$ with $|e_i\rangle$ and $|g_i\rangle$ being the excited and ground states for the ith atom, respectively. $a_i^{\dagger}$ and a_i are the creation and annihilation operators for the ith cavity field, respectively. g_i is the atom–cavity coupling strength. J describes the photon hopping strength between the two cavities.

The atoms are initially in the maximally entangled state $|\varphi(0)\rangle_a=\frac{1}{\sqrt{2}}(|eg\rangle+|ge\rangle)$. We denote the excitation number of the system with $n=\sum_{i=1}^{2}(|e\rangle_i\langle e|+a_i^{\dagger}a_i)$ which is conserved in the evolution process as $[n,H]=0$.

In our paper, we adopt the negativity as our entanglement measure, which is defined by

$$N(\rho)=\frac{\|\rho^T\|_1-1}{2}, \tag{2}$$

with $\|\rho^T\|_1$ being the trace norm of the partial transpose of the density matrix ρ [41]. Furthermore, for a two-qubit system, the negativity can be written as

$$N(\rho)=2\sum_j \max(0,-\mu_j), \tag{3}$$

where μ_j is the eigenvalue of the partial transpose ρ^T of the density matrix ρ of the system.

3. The evolution of entanglement for the atoms with initially entangled fields

Assume that the cavity fields are initially in the entangled state $|\varphi(0)\rangle_c=\frac{1}{\sqrt{2}}(|01\rangle+|10\rangle)$. Then the excitation number of the system is given by $n=2$ and the state evolution of the system can be spanned in the state subspace $\{|ee00\rangle, |eg01\rangle, |eg10\rangle, |ge01\rangle, |ge10\rangle, |gg11\rangle, |gg02\rangle, |gg20\rangle\}$. The state of the total system at time t can be expressed as

$$|\varphi(t)\rangle_s=y_1|ee00\rangle+y_2|eg01\rangle+y_3|eg10\rangle+y_4|ge01\rangle+y_5|ge10\rangle+y_6|gg11\rangle+y_7|gg02\rangle+y_8|gg20\rangle, \tag{4}$$

where the coefficients are given by

$$\begin{aligned} y_1 = {} & \cos(\lambda_4 t)\frac{g(J^2A+B-2g^2A)(A-3J^2+2g^2)}{24AJ(J^4+2g^4)} \\ & -i\sin(\lambda_4 t)\frac{g(A-3J^2+2g^2)}{A\sqrt{2(5J^2+A+6g^2)}} \\ & +\cos(\lambda_3 t)\frac{g(-J^2A+B+2g^2A)(A+3J^2-2g^2)}{24AJ(J^4+2g^4)} \\ & -i\sin(\lambda_3 t)\frac{g(A+3J^2-2g^2)}{A\sqrt{2(5J^2-A+6g^2)}}+\frac{Jg(J^2-g^2)}{J^4+2g^4}, \end{aligned} \tag{5}$$

$$\begin{aligned} y_2=y_5= {} & -i\sin(\lambda_4 t)\frac{\sqrt{2}(J^2A+B-2g^2A)(A-3J^2+2g^2)}{24AJ\sqrt{(J^4+2g^4)(5J^2+6g^2-A)}} \\ & +\cos(\lambda_4 t)\frac{(A-3J^2+2g^2)}{4A} \\ & +i\sin(\lambda_3 t)\frac{-\sqrt{2}(-J^2A+B+2g^2A)(A+3J^2-2g^2)}{24AJ\sqrt{(J^4+2g^4)(5J^2+6g^2+A)}} \\ & +\cos(\lambda_3 t)\frac{(A+3J^2-2g^2)}{4A}, \end{aligned} \tag{6}$$

$$\begin{aligned} y_3=y_4= {} & \cos(\lambda_4 t)\frac{(J^2A+B-2g^2A)(A-3J^2+14g^2)}{48Ag(J^4+2g^4)} \\ & -i\sin(\lambda_4 t)\frac{J(A-3J^2+14g^2)}{2A\sqrt{2(5J^2+A+6g^2)}} \\ & +\cos(\lambda_3 t)\frac{(-J^2A+B+2g^2A)(A+3J^2-14g^2)}{48Ag(J^4+2g^4)} \\ & -i\sin(\lambda_3 t)\frac{J(A+3J^2-14g^2)}{2A\sqrt{2(5J^2-A+6g^2)}}-\frac{J^2g^2}{J^4+2g^4}, \end{aligned} \tag{7}$$

$$\begin{aligned} y_6= {} & \cos(\lambda_4 t)\frac{g(J^2A+B-2g^2A)(A+9J^2+2g^2)}{24AJ(J^4+2g^4)} \\ & -i\sin(\lambda_4 t)\frac{g(A+9J^2+2g^2)}{A\sqrt{2(5J^2+A+6g^2)}} \\ & -\cos(\lambda_3 t)\frac{g(-J^2A+B+2g^2A)(-A+9J^2+2g^2)}{24AJ(J^4+2g^4)} \\ & +i\sin(\lambda_3 t)\frac{g(-A+9J^2+2g^2)}{A\sqrt{2(5J^2-A+6g^2)}}+\frac{Jg^3}{J^4+2g^4}, \end{aligned} \tag{8}$$

$$\begin{aligned} y_7=y_8= {} & -i\sin(\lambda_4 t)\frac{g(J^2A+B-2g^2A)}{2A\sqrt{(J^4+2g^4)(5J^2+6g^2-A)}}+\cos(\lambda_4 t)\frac{3Jg}{\sqrt{2}A} \\ & +i\sin(\lambda_3 t)\frac{g(-J^2A+B+2g^2A)}{2A\sqrt{(J^4+2g^4)(5J^2+6g^2+A)}}-\cos(\lambda_3 t)\frac{3Jg}{\sqrt{2}A}. \end{aligned} \tag{9}$$

Note here that A, B, λ_3 and λ_4 are given by

$$A=\sqrt{9J^4+60J^2g^2+4g^4}, \tag{10}$$

$$B=-9J^4+4g^4+4J^2g^2, \tag{11}$$

$$\lambda_3=\left(\frac{5}{2}J^2-\frac{A}{2}+3g^2\right)^{1/2}, \tag{12}$$

$$\lambda_4=\left(\frac{5}{2}J^2+\frac{A}{2}+3g^2\right)^{1/2}. \tag{13}$$