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Nonlinear scattering by magnetically biased semiconductor layer

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ABSTRACT

The nonlinear interaction of waves in the semiconductor slab illuminated by the plane waves of two tones or two Gaussian pulses with different central frequencies and lengths is examined in the self-consistent problem formulation, taking into account the nonlinear dynamics of carriers. The three-wave mixing technique is applied to study the nonlinear processes. It has been shown that the nonlinearity in passive weakly nonlinear semiconductor medium has the resistive nature associated with the dynamics of carriers. It has been shown that the nonlinear response of the magnetoactive semiconductor layer is strongly enhanced by the magnetic bias and the combinations of layer physical and geometrical parameters.

1. Introduction

The phenomena of nonlinear scattering have been the subject of extensive research in optics and condense matter physics for a long time. The efficient frequency generation, conversion and control are of paramount importance for design of high-performance devices and systems. However they still present enormous challenges not only at optical frequencies but also in the millimetre wave and terahertz (THz) ranges where the nonlinear wave interactions with solids are usually weak. Nonlinear artificial materials and metamaterials have recently attracted increasing interest [1–6] owing to their unique functional capabilities at millimetre, THz and optical frequencies.

The semiconductor structures represent an important class of electromagnetic media possessing natural nonlinearity, and the study of pulsed signals in such structures has not only enormous theoretical interest but also paves the way to design of innovative nonlinear and active devices for mm-wave, THz and optical applications. Most studies of semiconductor structures and their prospective applications to date have been concerned with the linear wave phenomena and time-harmonic signals [7–9]. The study of nonlinear phenomena in the semiconductor based active metamaterials is expected to offer a new means for maintaining integrity of the pulsed signals in the physical layer of hardware employed, e.g., for information processing (modulation [7,10], heterodyning [11,12] and harmonic generation [1,13]), spectroscopy [14] and material characterisation [15,16]. Dispersion of semiconductor material has profound impact on the nonlinear response of the medium despite being an essentially linear characteristic. The phase matching between pump wave and its harmonic is necessary requirement for efficient frequency conversion. The role of the dispersion in the phase matching is crucial for the efficient nonlinear wave interactions, frequency conversion, and harmonic generation [17,18].The recent studies have indicated that nonlinear artificial media have significant potential for controlling the dispersion relations of interacting waves which could meet the quasi-phase-matching conditions. The basic artificial structures with the necessary characteristics can be realized in the stratified medium with the periodic modulation of linear and nonlinear susceptibilities. The dispersion of the composite medium can be designed to compensate phase mismatch between the interacting waves and the frequency conversion efficiency may be increased by several orders of magnitude [19,20]. It was demonstrated in [19] that in the case of harmonic generation in semiconductor photonic crystal the intensity of the transmitted second harmonic (SH) reaches its maximum when the wavelengths of both the pump and SH waves are near the electromagnetic bandgap edges. The high density of modes at the band edge of photonic crystal (PhC) provides more favourable quasi-phase matching conditions for harmonic generation. However, it is necessary to note that the SH generation enhancement at the band edges is considerably impaired by the increased losses. The magnetoactive PhC controlled by external magnetic stimuli have recently demonstrated significant potential for efficient SH and third harmonic generation [21–25]. It was demonstrated that the SHG enhancement for two orders of magnitude has been achieved at the band edge of nonlinear magnetophotonic crystals. However the problems of distributed frequency mixing and the related processes of

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nonlinear scattering by layers and films illuminated by two or more plane waves of different frequencies incident at different angles still remain scarcely studied and poorly understood.

Although the properties of an artificial material are determined by the microscopic features of the constituent particles and their arrangements, the macroscopic response of the medium can be described by the effective constitutive parameters, when the feature size of the unit cell is much smaller than the wavelength. Since majority of the currently available metamaterials are fabricated as stacked layers, it is expedient to investigate their nonlinear characteristics in the planar layered structures. Such an approach often provides insight in the fundamental mechanisms and phenomenology of the distributed nonlinear wave interactions. The recent works in this area have been primarily concerned with the second and third harmonic generation in dielectric layers.

The effect of the medium properties on evolution of the pulse waveform and ability to control its shape are the critical attributes for any application. It is necessary to mention that the works in this area have been primarily concerned with the problems for which pulse length was much longer or much shorter than the thickness of the artificial structure (very long or very short pulses). In our work we consider the problem of pulse scattering from the commensurate nonlinear semiconductor layer.

In this paper, the problem of the weakly nonlinear magnetoactive semiconductor slab illuminated by the 2 different waves is considered, and the properties of reflected and refracted waves of the combinatorial frequencies are examined. The dynamics of charges in the semiconductor layers are taken into account. The problem statement and the solution of the respective boundary value problem for the case of frequency mixing of the two-tone plane waves or two Gaussian pulses with different central frequencies and lengths incident at oblique angles on the semiconductor layer are obtained by the three-wave interaction method [26] and outlined in Section 2. The results of the numerical analysis and the properties of plane waves and pulsed waveforms scattered by a nonlinear semiconductor slab are discussed in Section 3. The fundamental role of carrier collisions in the combinatorial frequency generation is discussed. The main features of the frequency mixing in the nonlinear passive semiconductor layer and effect of external magnetic field are summarised in Conclusion.

2. Three-wave nonlinear scattering by semiconductor layer

Let us consider the semiconductor layer of thickness L surrounded by a homogeneous linear medium with dielectric permittivity ε_a at $z \le 0$ and $z \ge L$. Let the layer be exposed to an external magnetic field $\mathbf{H}_0$ parallel to the y axis. The geometry of the problem is shown in Fig. 1.

The nonlinearity under the study is due to the nonlinearity of the free carrier current. The nonlinear set of equations consists of Maxwell's equations in which the nonlinear current is found from the current continuity equations, describing the nonlinear dynamics of charges, as follows

$$\begin{cases} \mathrm{rot}\mathbf{H} = \frac{\varepsilon_L}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j}, \\ \mathrm{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \\ e_0\frac{\partial n}{\partial t} + div\mathbf{j} = 0, \\ \frac{\partial \mathbf{v}}{\partial t} + \nu\mathbf{v} + (\mathbf{v}\nabla)\mathbf{v} = \frac{e_0}{m}\mathbf{E} + \frac{e_0}{mc}[\mathbf{v}\mathbf{H}_0] + \frac{e_0}{mc}[\mathbf{v}\mathbf{H}], \\ \mathbf{j} = e_0(n_0+n)\mathbf{v} \end{cases} \quad (1)$$

where c is the free space speed of light, n_0 is the carrier concentration at equilibrium; e_0 and m are the carrier charge and mass; ν is the collision frequency; $\mathbf{v}$, $\mathbf{j}$ and n are the velocity, current and concentration of carriers, respectively; ε_L is the permittivity associated with the semiconductor lattice. We assume

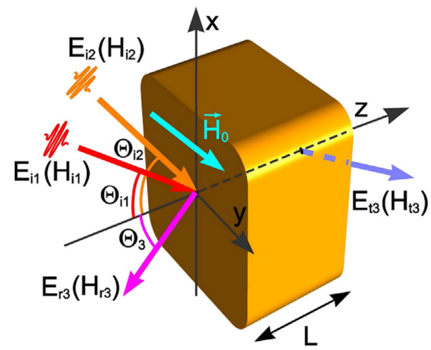

Fig. 1. Geometry of the problem: $E_{i1,i2}(H_{i1,i2})$ are electric (magnetic) fields of 2 pump waves incident at angles Θ_{i1} and Θ_{i2}; $E_{r3,t3}(H_{r3,t3})$ electric (magnetic) fields of the reflected and refracted waves outgoing from the layer into the surrounding media as the result of the nonlinear scattering at combinatorial frequency, Θ_3 is the angle of these waves' scattering; $\vec{H}_0$ is the external magnetic field.

that layer is isotropic in the x–y plane. Therefore the incident waves of the TE and TM polarisations with the fields independent of the y-coordinate ($\partial/\partial y = 0$) can be treated separately. Only the case of TM-polarization with the electric $E_{x,z}$ and magnetic H_y field components is discussed here. The TE waves with the components E_y, H_x, H_z are unaffected by the external magnetic field and all equations in the system (1) for TE waves are linear.

The tensor of equivalent permittivity for magnetoactive semiconductor layer can be written as [24]

$$\varepsilon_{xx}(\omega) = \varepsilon_{zz}(\omega) = \varepsilon_{\parallel}(\omega) = \varepsilon_L\left(1 - \frac{\omega_p^2(\omega+i\nu)}{\omega((\omega+i\nu)^2-\omega_H^2)}\right)$$

$$\varepsilon_{xz}(\omega) = -\varepsilon_{zx}(\omega) = \varepsilon_{\perp}(\omega) = \varepsilon_L\frac{i\omega_p^2\omega_H}{\omega((\omega+i\nu)^2-\omega_H^2)}, \quad (2)$$

where $\omega_p = (4\pi e_0^2 n_0/\varepsilon_L m)^{1/2}$ is the plasma frequency of semiconductor layer, $\omega_H = (e_0/mc)H_{0y}$ is the cyclotron frequency.

2.1. Basic equations for plane wave mixing

For beginning let us consider combinatorial frequency generation by the nonlinear semiconductor slab illuminated by the plane waves of two tones. Let us assume that semiconductor layer is illuminated by two plane waves of frequencies ω_1 and ω_2 incident at angles Θ_{i1} and Θ_{i2}, respectively. To study nonlinear processes, the method of three-wave interaction is used [26]. The main assumption of the method is the smallness of the nonlinear terms. This means that the energy of nonlinear interaction is lower than that of interacting waves. Thus at the combinatorial frequency $\omega_3 = \omega_1 + \omega_2$, the system of nonlinear Eq. (1) can be reduced to the following inhomogeneous Helmholtz equation for H_y

$$\left(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2}\right)H_y(\omega_3) + \varepsilon_f(\omega_3)k(\omega_3)^2H_y(\omega_3) = \frac{\varepsilon_{\perp}(\omega_3)}{\varepsilon_{\parallel}(\omega_3)}\frac{\partial}{\partial z}(\eta_2(\omega_1,\omega_2))$$
$$-\frac{\partial}{\partial z}(\eta_1(\omega_1,\omega_2)) + \frac{\varepsilon_{\perp}(\omega_3)}{\varepsilon_{\parallel}(\omega_3)}\frac{\partial}{\partial x}(\eta_1(\omega_1,\omega_2)) + \frac{\partial}{\partial x}(\eta_2(\omega_1,\omega_2))$$

$$\eta_1(\omega_1,\omega_2) = -i\frac{4\pi}{c}e_0n_0\left\{\frac{\omega_3+i\nu}{(\omega_3+i\nu)^2-\omega_H^2}\left[\omega_H\frac{i}{\omega_3+i\nu}\rho_1(\omega_1,\omega_2)\right.\right.$$
$$\left.\left.+\rho_2(\omega_1,\omega_2)\right] + \frac{1}{\omega_1}div\mathbf{v}(\omega_1)\mathrm{v}_x(\omega_2) + \frac{1}{\omega_2}div\mathbf{v}(\omega_2)\mathrm{v}_x(\omega_1)\right\},$$

$$\eta_2(\omega_1,\omega_2) = -i\frac{4\pi}{c}e_0n_0\left\{\frac{\omega_3+i\nu}{(\omega_3+i\nu)^2-\omega_H^2}\left[\omega_H\frac{i}{\omega_3+i\nu}\rho_2(\omega_1,\omega_2)\right.\right.$$
$$\left.\left.-\rho_1(\omega_1,\omega_2)\right] + \frac{1}{\omega_1}div\mathbf{v}(\omega_1)\mathrm{v}_z(\omega_2) + \frac{1}{\omega_2}div\mathbf{v}(\omega_2)\mathrm{v}_z(\omega_1)\right\},$$