



Exact iterative solution of simultaneous second-harmonic and third-harmonic generation in nonlinear photonic crystals

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ABSTRACT

A computational study on simultaneous second-harmonic and third-harmonic generation and enhancement through a $\chi^{(2)}$ three-wave mixing process in one-dimensional nonlinear photonic crystals is presented. The mathematical model partly overcomes the shortcoming of some existing models based on the undepleted pump approximation, which is derived from a nonlinear system of Maxwell equations. We introduce not only a variational approach that combined the finite element method and the fixed-point iteration to study the nonlinear frequency conversion but also a continuation technique depends on the weak formulation of the incremental fields to ensure the convergence of the iterative procedure when the nonlinearity is very strong. Two designed photonic crystals matched the phase by utilizing and balancing the interplay of material dispersion and the geometrical dispersion are reported, as well as a tested photonic crystal to show the validity and efficiency of our approaches. Numerical experiments indicate that the conversion efficiencies of SH and TH generation can be significantly enhanced when the frequencies of fundamental and harmonic waves are tuned at the photonic band edges or are located to the defect states.

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1. Introduction

Photonic crystals (PhCs) are artificially fabricated structures in which the index of the refraction varies alternatively between high-index regions and low-index regions on the wavelength scale [1]. The existence of photonic band gaps (PBG) in the periodic structure offers the possibility of confining and controlling the propagation of photons in these materials, which leads to many potential applications of PhCs [1–3]. The nonlinear frequency conversion has been researched for many years [4–7]. In the traditional nonlinear optical processes, the phase-matching (PM) condition was concerned for obtaining high conversion efficiency, which has a great restriction to the choice of birefringence material [4]. During the recent decades, the PhCs have attracted considerable interest in nonlinear frequency conversion due to the quasi-phase matching (QPM) technique [8]. The existing experiments demonstrated that QPM could be used to yield good conversion efficiencies for simultaneous multi-wavelength generation, and reopen the challenge for applications in many optical devices [9,10].

Utilizing the QPM technique, simultaneous second-harmonic (SH) and third-harmonic (TH) generation via a $\chi^{(2)}$ three-wave mixing process can occur with high conversion efficiencies in a one-dimensional (1D) PBG structure [11,12]. TH generation in such a single 1D nonlinear PhC can occur by directly using the third-order nonlinear susceptibility (direct THG, $\omega + \omega + \omega = 3\omega$) [13], as well as by a cascade manner of second-harmonic generation (SHG, $\omega + \omega = 2\omega$) and sum-frequency generation (SFG, $\omega + 2\omega = 3\omega$) as a result of quadratic susceptibility. Since the quadratic nonlinear coefficient is much higher than the third-order nonlinear coefficient such that one can simultaneously generate SH and TH in higher conversion efficiencies, we discuss the effect of quadratic nonlinear coefficient progress on SH and TH generation in this paper.

For the SH and TH generation problem, most existing models in periodic structures have relied on the undepleted-pump approximation (UPA). Under the UPA, the effect of the SH and TH waves on the fundamental frequency (FF) field and the effect of the TH wave on the SH field are ignored. The problem can be solved by well-established linear systems [14]. However, the UPA is invalid while the output energy becomes significant compared to the intense incident beam [4,15]. Here, we extend the techniques described in [16] to get a general nonlinear Helmholtz equations based on Maxwell equations, together with boundary conditions obtained from jumping conditions. Our full nonlinear problem has a unique solution, provided that the magnitude of the nonlinear susceptibility tensors is not too large.

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Numerical methods are essential to analyze nonlinear frequency conversion in photonic devices. In previous work, many numerical methods have been reported to solve the harmonic generation problems, such as the finite element method [15], the Green function method [17], the matrix transfer method [18], the multiple scale expansion approach [19], the finite difference time domain method [20], and the Dirichlet-to-Neumann map methods [21]. But few numerical methods on the problem of the simultaneous SH and TH generation and enhancement are investigated. Previously, we have studied an effective and simple continuation method for the SH generation in nonlinear PhCs [15]. In this paper, we want to extend the technique in our previous work to present a simple and efficient numerical method for analyzing simultaneous SH and TH generation through a $\chi^{(2)}$ three-wave mixing process in 1D PhCs based on full nonlinear system of equations.

Firstly, we develop a variational approach that combined the finite element methods and the fixed-point iteration to study simultaneous SH and TH generation in 1D nonlinear PhCs. This finite-element fixed-point iteration algorithm (FEFPIA) can accurately predict the field intensity distribution of both reflected and transmitted waves when an intense pump beam is incident on the surface of a PhC. Further, the transmission and reflection properties and conversion efficiencies of SH and TH generation in a wide variety of PBG structures can be exhibited. The FEFPIA can be extended to high dimensional structure, but may fail to produce convergent solution since the iteration scheme may break down when the nonlinearity is very strong [15,16].

To overcome the shortcoming of FEFPIA, we develop a continuation approach to ensure the convergence of the iterative procedure. Our continuation finite-element fixed-point iteration algorithm (CFEFPIA) of solving the underlying nonlinear Helmholtz equations is based on the weak formulation of the full nonlinear system. Since the method combining finite element methods with general fixed-point iteration is efficient when the nonlinearity is not so strong, we study the incremental fields due to small increase of the intensity of the incident wave, assuming all three fields continuously depend on the input. A new variational formulation for the increment fields is derived based on the perturbation technique. The incremental fields can be computed iteratively by the combination of finite element methods and the general fixed-point iteration algorithm. By increasing the input intensity successively, we can obtain the FF, SH and TH fields for strong incident pump. The continuation method can accurately predict the field intensity distribution of both reflected and transmitted waves. The transmission, reflection and conversion efficiencies of SH and TH generation in the case of strong incident pump for variety of PhCs are studied.

In our numerical simulations, the designed PhCs with much higher conversion efficiencies are partly based on the work of M. Centini and G. D'Aguanno et al. [11]. Generally, there are two methods to enhance simultaneous SH and TH generation in a PBG structure, one is to simultaneously adjust all frequencies of fundamental wave, SH and TH waves, located at the resonance peaks of the PBEs. The other is to introduce some defect states into a perfect PhC to give rise to strong localized fields in the defect states, where both the SH and TH generation can be greatly enhanced due to the defect states [22–24]. Simultaneous SH and TH generation in both perfect and defective PhCs is presented in our simulations, which show the convergence of our method is fast and giant enhancement of the simultaneous SH and TH generation can be obtained by designing a nonlinear PhC with QPM technique. To show our model with pump depletion is more applicable than the UPA, a comparison between two methods is also presented.

2. Nonlinear system and finite element model

Consider a representative 1D PhC sample $(AB)_m$ comprised of two non-ferromagnetic materials A and B alternatively in the domain $0 \leq z \leq L$, where m denotes the number of periods. The lattice constant is $a = l_A + l_B$, where l_A and l_B are the thickness of materials A and B , respectively.

Suppose that there is no external current or charge, and the electric and magnetic fields are time harmonic. It is further supposed that the incident plane wave is normally launched upon the surface of the structure along the z -axis direction. In the transverse electric (TE) polarization, the governing equations expressed by electric field in the domain $[0, L]$ can be deduced from the Maxwell equations as follows:

$$\begin{cases} \frac{d^2 E_1}{dz^2} + (k_0 n_1)^2 E_1 = -2k_0^2 \chi_1^{(2)} (\bar{E}_1 E_2 + \bar{E}_2 E_3), \\ \frac{d^2 E_2}{dz^2} + (2k_0 n_2)^2 E_2 = -(2k_0)^2 \chi_2^{(2)} (E_1^2 + 2\bar{E}_1 E_3), \\ \frac{d^2 E_3}{dz^2} + (3k_0 n_3)^2 E_3 = -2(3k_0)^2 \chi_3^{(2)} E_1 E_2, \end{cases} \quad (1)$$

where $k_0 = \omega/c$ is the wave number of free space, and c is the speed of light in a vacuum. E_1 , E_2 and E_3 are the electric field intensities, at the frequencies of ω , 2ω and 3ω , respectively. n_1 , n_2 and n_3 are the linear refractive index functions, corresponding to ω , 2ω and 3ω , respectively. $\chi_1^{(2)}$, $\chi_2^{(2)}$ and $\chi_3^{(2)}$ are three second-order nonlinear susceptibility tensors at ω , 2ω and 3ω , respectively. Note that the symbol \bar{E} denotes the complex conjugate of E .

The PhC structure is located in the middle of two linear media, that is, the refractive index of the background structure in $z < 0$ and $z > L$ are constants, denoted by $n_l (z < 0)$ and $n_r (z > L)$, respectively. Moreover, when the incident wave perpendicularly strikes the surface of the structure along the z -axis direction, the tangent components of electric field E and magnetic field H must be continuous at the interface of two homogeneous materials. Using the continue jumping boundary conditions inside the structure and general boundary conditions on the interfaces of the PhC [15], the boundary conditions can be derived as follows:

$$\begin{cases} \frac{dE_1}{dz}(0) = -2i\alpha_b E_0 + i\alpha_b E_1(0), \\ \frac{dE_2}{dz}(0) = i\beta_b E_2(0), \\ \frac{dE_3}{dz}(0) = i\gamma_b E_3(0), \\ \frac{dE_1}{dz}(L) = -i\alpha_0 E_1(L), \\ \frac{dE_2}{dz}(L) = -i\beta_0 E_2(L), \\ \frac{dE_3}{dz}(L) = -i\gamma_0 E_3(L), \end{cases} \quad (2)$$

where the incident wave is given by $(0, E_0 e^{-i\alpha_0 z}, 0)$, $\alpha_b = k_0 n_l$, $\beta_b = 2k_0 n_l$, $\gamma_b = 3k_0 n_l$, $\alpha_0 = k_0 n_r$, $\beta_0 = 2k_0 n_r$, and $\gamma_0 = 3k_0 n_r$.

Consequently, the problem of the nonlinear phenomena in 1D nonlinear PhCs is to solve the coupled inhomogeneous Helmholtz equations (Eq. (1)) in the domain $[0, L]$, integrated with the boundary conditions (Eq. (2)). This nonlinear system cannot be solved directly due to the inhomogeneous terms on the right-hand side of the equations. In this paper, the techniques described in [15] are extended to solve the full nonlinear problem by a combination of the finite element methods and the fixed-point iteration.

By using the classical variational technique, the corresponding variational problem to our nonlinear problem can be derived as

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