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# Optical interband absorption and Stark shift in a quantum ring on a sphere

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ABSTRACT

An analytical solution of the quantum problem of an electron on a spherical segment with angular confinement potential in the form of rectangular impenetrable walls is presented. It is shown that the problem is reduced to finding solution of hypergeometric equation. On the basis of the obtained results the optical interband transitions in this system are discussed, and the threshold frequency and absorption coefficient both for single structure and non-interacting ensemble are calculated. The influence of electric field on system is discussed and it is shown that in such type of structure linear Stark shift takes place. Quantum transitions in the presence of electric field are considered and it is shown that the selection rule for an orbital quantum number analogue is removed.



## 1. Introduction

Investigation of quantum mechanical problems on curved surfaces initially had purely academic character and was perceived as interesting model problems. The exactly solvable problems of particle states on curved surfaces of different symmetries were discussed [1–8]. In particular, in Ref. [1] the path integral formulations for the Smorodinsky–Winternitz potentials in two- and three-dimensional Euclidean spaces are presented. In Ref. [2] the path integral formulations for Smorodinsky–Winternitz potentials, respectively, to systems with accidental degeneracies on the two- and three-dimensional sphere, and a complete classification of super-integrable systems on spaces of constant curvature are presented. All coordinate systems, which separate the Smorodinsky–Winternitz potentials on a sphere, and state the corresponding path integral formulations are mentioned. In Ref. [3] the basis functions for classical and quantum mechanical systems on the two-dimensional hyperboloid that admit separation of variables in at least two coordinate systems are examined. In Ref. [4] the free quantum motion on the three-dimensional sphere in ellipsocylindrical coordinates is studied, where a distinction between prolate elliptic and oblate elliptic coordinates is made. In Ref. [5] generalizations to spheres of Levi-Civita, Kustaanheimo–Stiefel and Hurwitz regularizing transformations in Euclidean spaces of dimensions 2, 3 and 5 are constructed. The corresponding classical and quantum mechanical analogues of the Kepler–Coulomb problem on these spheres are discussed. It is shown in Ref. [6] that oscillators on the sphere and the pseudosphere are related by the so-called Bohlin transformation, with the Coulomb systems on the pseudosphere.

On the other hand, in the last decade an interest to such problems has grown abruptly due to experimental realization of nanostructures of different geometries [9–11]. The motion of particles on such surfaces should be described via quantum mechanics on curved spaces [12–15]. Besides single-electron states two-electron states on spherical surfaces were also considered; in other words, the so-called spherical helium atoms were discussed [16–20].

The theoretical investigation of electron states in layered nanostructures is originated from the pioneering works of Chakraborty and Pietilainen [21–23]. Authors have considered one-electron and many-electron states in quantum rings in the presence of impurities, as well as under the influence of a magnetic field. At the same time, taking into account that in the radial direction the movement of electron is restricted both on internal and external radiuses, Chakraborty and Pietilainen have suggested a model of confining potential having the form of a two-dimensional shifted oscillator:

$$V_{conf}(r) = \alpha(r-r_0)^2,$$

where $\alpha$ characterizes the intensity of electron localization. Further optical, kinetic, spin, etc. properties of charge carriers localized in circinate and cylindrical layered nanostructures (see for example, [24–31]) were studied. Recently, especially linear and nonlinear optical properties of ring-shaped layered nanostructures [32–37] were intensively studied. In particular, in Ref. [34] the optical absorptions of an exciton in a quantum ring are studied. A similar problem is studied in the presence of a strong magnetic field [35].

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In Ref. [36] the linear and nonlinear intra-band optical absorption coefficients in GaAs/$Ga_{1-x}Al_xAs$ two-dimensional concentric double quantum rings are investigated. The analogical problem in the presence of electron–donor–impurity complex is discussed in Ref. [37].

Nanolayers of spherical symmetry were studied in Refs. [38–42]. The important peculiarity of spherical nanolayers is more flexible control of energy spectrum by changing both inner and outer radiuses (instead of only one outer radius as it is for the case of spherical quantum dot). Moreover, in particular cases, the results obtained for spherical nanolayer can be adapted for systems such as quantum well and spherical quantum dot. In Refs. [43–45] one and two electron states, as well as optical properties of spherical nanolayers, are investigated. In Refs. [41,42] it is assumed that small thicknesses of the spherical layer mean that particle is localized on a spherical surface with some effective radius $R_1 < R_{eff} < R_2$, where $R_1$ and $R_2$ are, respectively, the inner and outer radiuses of the layer.

In our recent work [42] we have examined electron states localized in a quantum ring on a spherical surface (see Fig. 1a, b). As a confinement potential we have chosen the singular analog of the so-called $\mathbb{C}P^N$-oscillator suggested by Bellucci and Nersessian in [46]

$$V(\theta) = 4\beta r_0^2 \tan^2\frac{\theta}{2} + \frac{\alpha}{4r_0^2\tan^2\frac{\theta}{2}},$$

which is a spherical generalization of the Smorodinsky–Winternitz potential [47]. On the other hand, the profile of semiconductor heterostructure confining potential depends on the growth conditions: temperature, pressure, etc. This is the reason why we have discussed the same angular problem of electron states in a quantum ring on a spherical surface for the case when the angular confinement potential is chosen in the form of rectangular impenetrable walls.

Besides the academic interest, the investigation of mentioned systems is connected with their direct application in semiconductor devices of new generation. In this context it is important to examine optical and spectral properties of layered structures of spherical symmetry. Particularly, in Refs. [48,49] optical interband transitions in spherical nanolayers are discussed. In the current work we have examined the spectral properties of considered structures. In particular, analytical expressions for the absorption coefficient and threshold frequency of quantum transitions are found. Besides that, we have considered such transitions for the non-interacting ensemble of discussed structures.

In addition, we have considered the influence of external electric field on the electronic and optical properties of system. It turned out that in this type of systems linear Stark shift takes place. Besides that, we have shown that the presence of electric field completely changes the behavior of the quantum transitions.

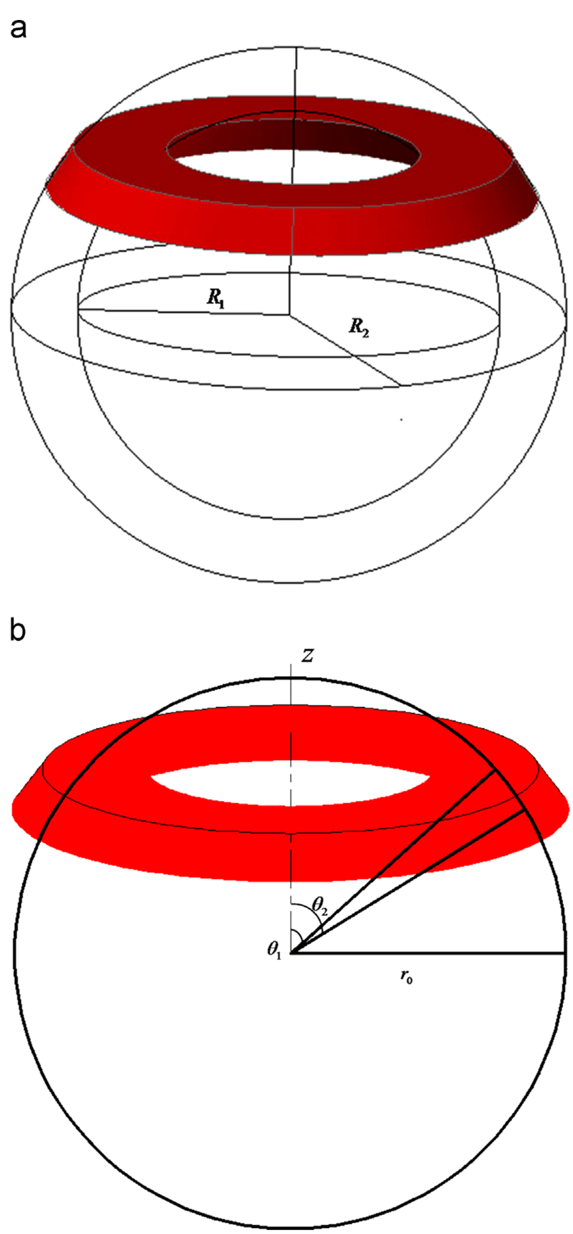


**Fig. 1.** (a) Ring-type layered heterostructure on sphere. (b) Spherical segment.

## 2. Theory

The Schrödinger equation of the system can be written as follows:

$$-\frac{\hbar^2}{2\mu}\left[\frac{\partial^2}{\partial r^2}+\frac{2}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\nabla^2_{\theta,\varphi}\right]\psi(r,\theta,\varphi)+\left(V^{rad}_{conf}(r)+V^{ang}_{conf}(\theta)\right)\psi(r,\theta,\varphi)=E\psi(r,\theta,\varphi), \tag{1}$$

where

$$V^{rad}_{conf}(r)=\begin{cases}0, & R_1<r<R_2,\\ \infty, & r<R_1, r>R_2,\end{cases} \tag{2}$$

$$V^{ang}_{conf}(\theta)=\begin{cases}0, & \theta_1<\theta<\theta_2,\\ \infty, & \theta<\theta_1, \theta>\theta_2,\end{cases} \tag{3}$$

and $\mu$ is the effective mass of electron. As it was mentioned above, the small thickness of the layer lets us assume that the electron will be in the ground state on the radial direction and move on the spherical surface of radius $r_0 = R_{eff} = (R_1+R_2)/2$ [41,42]. Then for the radial wave function of the ground state we can write [41]

$$R_0(r)=\sqrt{\frac{\pi\lambda}{2r}}\left(D_1J_{1/2}(\lambda r)+D_2J_{-1/2}(\lambda r)\right), \tag{4}$$

where $\lambda=\sqrt{2\mu E_0^{rad}/\hbar^2}$, $J_\nu(x)$ is the Bessel function, $D_1$ and $D_2$ are normalization constants. $R_0(r)$ and $E_0^{rad}$ are the ground state wave function and energy of the equation, respectively:

$$-\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}\right)R(r)+V^{rad}_{conf}(r)R(r)=E^{rad}R(r), \tag{5}$$

where for $E^{rad}$ we have [41]

$$E^{rad}=\frac{\pi^2\hbar^2N^2}{2\mu(R_2-R_1)^2}, \quad N=1,2,3,\dots. \tag{6}$$

Here $N$ is the quantum number of the radial quantization.