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Spreading of spiral spectrum of Bessel–Gaussian beam in non-Kolmogorov turbulence

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ABSTRACT

The analytical formulas for the spiral spectrum of Bessel–Gaussian (BG) beam and the channel capacity of a communication system based on orbital angular momentum (OAM) in non-Kolmogorov turbulence have been derived. The influence of the azimuthal index, wavelength, exponent parameter α, inner scale and outer scale on spiral spectrum is investigated. Numerical results reveal that a spiral spectrum of BG beam in non-Kolmogorov turbulence is more affected by turbulence with larger azimuthal index, shorter wavelength, smaller inner scale and larger outer scale. It is demonstrated that the spiral spectrum also depends on the propagation distance and structure constant. The spiral spectrum of BG beam in non-Kolmogorov turbulence spreads significantly with the increasing of exponent parameter α and spreads slightly after reaching a maximum point. It is showed that the variation of channel capacity of the OAM-based communication system, with respect to parameter α, has the same trend as the spiral spectrum.

1. Introduction

In recent years, propagation of optical vortex beams with orbital angular momentum (OAM) through a turbulent atmosphere has attracted a lot of attention because of their applications in quantum information encoding and communication systems [1–7]. A key motivation for this idea is that the transverse spatial modal basis of vortex beams defines an infinite dimensional Hilbert space, allowing more information per photon to be used in quantum information applications [8]. The turbulence strongly affects the properties of the vortex beams propagating through it and consequently limits the performance of these systems based on OAM. Understanding the propagation properties of vortex beams through a turbulent atmosphere is critical in the optimization of these systems.

Previously, Laguerre–Gaussian (LG) beams were treated in most of the studies [9–11]. However, the modal basis used in quantum information experiments is seldom pure LG modes [12]. The optical manipulation and detection of LG modes require control of the radial index, which governs the radial shape of each LG mode's intensity profile. We shall refer to the modes that are detected without this radial control as azimuthal modes, such as Bessel–Gaussian (BG) modes [13]. It is well known that BG beams also possess an amount of OAM equal to $\ell\hbar$ per photon, where ℓ is the azimuthal index of the mode.

To the best of our knowledge, the propagation properties of BG beams in Kolmogorov turbulence have been well investigated [14–19]. However, the studies cited above have been mainly restricted to the effects of turbulence on intensity fluctuations, the degree of coherence, or the M^2-factor of BG beams. Not much attention has been paid to the effect on OAM states of these types of beams as they propagate through non-Kolmogorov turbulence. It is experimentally demonstrated that the Kolmogorov turbulence will induce the spreading of the input mode power over neighboring OAM modes, or spiral spectrum, resulting in crosstalk between the channels [20], and will affect the channel capacity of an OAM-based communication system drastically [21]. Further study on the spreading of spiral spectrum of BG beams in non-Kolmogorov turbulence, which is our goal in this paper, is required. Comparing with the results in [10,11], both inner scale l_0 and outer scale L_0 effects are included in our paper. It will be demonstrated later that the spreading of spiral spectrum is different from the result when the inner and outer scales are neglected. Furthermore, we investigate not only the influence of the exponent parameter α on the detection probability of OAM states, but also the influence of α on the channel capacity of the OAM-based communication system. The dependence of the spiral spectrum on the transverse wave number k_r of BG beams is also investigated.

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In Section 2 the expressions of the weight of detected OAM state and the capacity in the non-Kolmogorov turbulent channel are derived by use of the Huygens–Fresnel principle. In Section 3 some numerical simulations are performed to illustrate the influence on the spreading of spiral spectrum by both the beam parameters and the non-Kolmogorov spectrum and conclusions are given in Section 4.

2. Analytical formulas

In the case of the radial coordinates (s, ϕ_s) and the wave propagation in the z-axis, the electric field of a BG mode on the source plane can be given by [13,22]

$$E_\ell^{BG}(s,\phi_s,0) = \sqrt{\frac{2}{\pi}} J_\ell(k_r s)\exp(i\ell\phi_s)\exp\left(-\frac{s^2}{w_0^2}\right). \tag{1}$$

where $J_\ell(\cdot)$ is the Bessel function of the first kind, the signed integer ℓ is the azimuthal mode index and is also the OAM quantum number of photon in the mode, k_r is the transverse component of wave number $k = 2\pi/\lambda$ (λ is the laser wavelength). The initial radius at the beam waist is w_0.

With the Huygens–Fresnel integral, on a receiver plane, having the radial coordinates of ρ and ϕ, located at an axial distance of z away from the source plane in free space, the complex amplitude of BG beams is $u_{\ell 0}(\rho,\phi,z) = -2ik\int\int_{-\infty}^{\infty} G(\mathbf{s},\boldsymbol{\rho};z)E_\ell^{BG}(s,\phi_s,0)\,d^2s$, where $G(\mathbf{s},\boldsymbol{\rho};z)$ is the free space Green's function, with $\boldsymbol{\rho}$ and $\mathbf{s}$ denoting the transverse component of two points $\mathbf{R} = (\boldsymbol{\rho}, z)$ and $\mathbf{S} = (\mathbf{s}, 0)$ in space. In general, the free space Green's function is a spherical wave which, under the paraxial approximation, can be expressed as [23]

$$G(\mathbf{s},\boldsymbol{\rho};z) = \frac{\exp(ik|\mathbf{R}-\mathbf{S}|)}{4\pi|\mathbf{R}-\mathbf{S}|} \cong \frac{1}{4\pi z}\exp\left(ikz + \frac{ik}{2z}|\mathbf{s}-\boldsymbol{\rho}|^2\right). \tag{2}$$

After a straightforward calculation one obtains

$$\begin{aligned} u_{\ell 0}(\rho,\phi,z) = &-\sqrt{\frac{2}{\pi}} i^{\ell+1}\left(\frac{w_0}{w}\right)^2\left(i+\frac{z}{z_R}\right)\exp(ikz) \\ &\times\exp\left(-\frac{1k_r^2 z w_0^2}{4z - iz_R}\right)\exp\left(i\frac{k\rho^2}{2R(z)}\right) \\ &\times\exp\left(-\frac{\rho^2}{w^2}\right) I_\ell\left(\frac{z_R k_r \rho}{z - iz_R}\right)\exp(i\ell\phi), \end{aligned} \tag{3}$$

where the beam radius is $w = w_0[1+(z/z_R)^2]^{1/2}$ and the curvature radius of the wave front is $R(z) = z[1+(z_R/z)^2]$ with the Rayleigh range $z_R = kw_0^2/2$.

Using the Rytov approximation [23], the BG beam which propagates in weak turbulence at distance z can be represented as

$$u_\ell(\rho,\phi,z) = u_{\ell 0}(\rho,\phi,z)\exp[\psi(\rho,\phi,z)], \tag{4}$$

where $\psi(\rho,\phi,z)$ is the complex phase perturbation of the field due to random inhomogeneous along the propagation channel.

To elucidate the OAM content, or spiral spectrum, any field distribution can be decomposed into a superposition of spiral harmonics $\exp(im\phi)$ in cylindrical coordinates as [8]

$$u_\ell(\rho,\phi,z) = \frac{1}{\sqrt{2\pi}}\sum_{m=-\infty}^{\infty} a_{\ell m}(\rho,z)\exp(im\phi), \tag{5}$$

with the expansion coefficient $a_{\ell m}(\rho,z) = (1/\sqrt{2\pi})\int_0^{2\pi} u_\ell(\rho,\phi,z)\exp(-im\phi)\,d\phi$. The energy carried by the corresponding light beam can be written as $U_\ell = 2\varepsilon_0\sum_{-\infty}^{\infty} C_{\ell m}$, where $C_{\ell m} = \int_0^\infty |a_{\ell m}(\rho,z)|^2\rho\,d\rho$. Then the energy content (weight) of each of the spiral harmonics of any field distribution is determined by $P_{m|\ell} = C_{\ell m}/\sum_{-\infty}^{\infty} C_{\ell q}$.

It is indicated that the input power of mode ℓ spreads into neighboring OAM modes, i.e., the spreading of spiral spectrum occurs in the turbulence. The weight $P_{m|\ell}$, corresponding to the detection probability of OAM state m at measurement plane, is associated with the ensemble average over turbulent medium, implicated in the term $|a_{\ell m}(\rho,z)|^2$

$$\begin{aligned} |a_{\ell m}(\rho,z)|^2 = &\frac{1}{2\pi}\int_0^{2\pi}\int_0^{2\pi} u_\ell(\rho,\phi_1,z)\exp(-im\phi_1) \\ &\times u_\ell^*(\rho,\phi_2,z)\exp(im\phi_2)\,d\phi_1\,d\phi_2, \end{aligned} \tag{6}$$

where $*$ denotes complex conjugate. Substituting Eqs. (3) and (4) into Eq. (6) yields the following expression

$$\begin{aligned} |a_{\ell m}(\rho,z)|^2 = &\frac{1}{\pi^2}\left(\frac{w_0}{w}\right)^2\exp\left(-\frac{2\rho^2}{w^2}\right)\exp\left(-\frac{k_r^2 z w_0^2}{2R(z)}\right) \\ &\times I_\ell\left(\frac{z_R k_r\rho}{z - iz_R}\right) I_\ell\left(\frac{z_R k_r\rho}{z + iz_R}\right) \\ &\times\int_0^{2\pi}\int_0^{2\pi}\langle\exp[\psi(\rho,\phi_1,z)+\psi^*(\rho,\phi_2,z)]\rangle \\ &\times\exp[i(\ell-m)(\phi_1-\phi_2)]\,d\phi_1\,d\phi_2. \end{aligned} \tag{7}$$

The ensemble average term $\langle\exp[\psi(\rho,\phi_1,z)+\psi^*(\rho,\phi_2,z)]\rangle$ can be expressed as [9,24,25]

$$\begin{aligned} &\langle\exp[\psi(\boldsymbol{\rho}_1,z)+\psi^*(\boldsymbol{\rho}_2,z)]\rangle \\ &= \exp\left[-\frac{1}{3}\pi^2 k^2 z|\boldsymbol{\rho}_1-\boldsymbol{\rho}_2|^2\int_0^\infty \kappa^3\Phi_n(\kappa,\alpha)\,d\kappa\right], \end{aligned} \tag{8}$$

where $\boldsymbol{\rho}_1 = (\rho\cos\phi_1, \rho\sin\phi_1)$, $\boldsymbol{\rho}_2 = (\rho\cos\phi_2, \rho\sin\phi_2)$, the function $\Phi_n(\kappa,\alpha)$ denotes the spatial power spectrum of the refractive-index fluctuations of the atmospheric turbulence, in which κ is the magnitude of two-dimensional spatial frequency and α is the power-law exponent. To include both inner- and outer-scale effects, a non-Kolmogorov spectrum is used to model the atmospheric turbulence [24–26]

$$\Phi_n(\kappa,\alpha) = A(\alpha)\tilde{C}_n^2\frac{\exp[-\kappa^2/\kappa_m^2]}{(\kappa^2+\kappa_0^2)^{\alpha/2}}, \quad 0 \le \kappa < \infty, \quad 3 < \alpha < 4 \tag{9}$$

where $\kappa_0 = 2\pi/L_0$ and $\kappa_m = c(\alpha)/l_0$ with L_0 and l_0 being the outer and inner-scale parameters and $c(\alpha) = [\Gamma(5-\alpha/2)A(\alpha)2\pi/3]^{1/(\alpha-5)}$, $\tilde{C}_n^2$ is the generalized structure parameter with units $m^{3-\alpha}$ and $A(\alpha) = \Gamma(\alpha-1)\cos(a\pi/2)/4\pi^2$ with Γ being the gamma function. It is worth mentioned that the exponent parameter α varies from 3 to 5 in the general model of non-Kolmogorov power spectrum. However, considering the wave structure function as stated in [27], the range of α is further restricted to the range $3 < \alpha < 4$ and as a result the $\alpha > 4$ range is not of interest in our paper. Meanwhile, it is common that the exponent parameter takes this interval if the inner and outer scales are included [24–26]. Note that the spectrum reduces to the conventional Kolmogorov spectrum when $\alpha = 11/3$, $A(11/3) = 0.033$, $L_0 = \infty$, $l_0 = 0$, and $\tilde{C}_n^2 = C_n^2$.

After the term $T(\alpha) = \int_0^\infty \kappa^3\Phi_n(\kappa,\alpha)\,d\kappa$ has been defined, the expression of $T(\alpha)$ can be written as [9]

$$T(\alpha) = \frac{A(\alpha)\tilde{C}_n^2}{2(\alpha-2)}\left[\kappa_m^{2-\alpha}\beta\exp\left(\frac{\kappa_0^2}{\kappa_m^2}\right)\Gamma\left(2-\alpha/2, \frac{\kappa_0^2}{\kappa_m^2}\right) - 2\kappa_0^{4-\alpha}\right], \tag{10}$$

where $\beta = 2\kappa_0^2 - 2\kappa_m^2 + \alpha\kappa_m^2$. Substituting Eqs. (8) and (10) into Eq. (7) yields

$$\begin{aligned} |a_{\ell m}(\rho,z)|^2 = &4\left(\frac{w_0}{w}\right)^2\exp\left(-\frac{2\rho^2}{w^2}\right)\exp\left(-\frac{k_r^2 z w_0^2}{2R(z)}\right) \\ &\times I_\ell\left(\frac{z_R k_r\rho}{z - iz_R}\right) I_\ell\left(\frac{z_R k_r\rho}{z + iz_R}\right) \\ &\times\exp\left(-\frac{2}{3}\pi^2 k^2\rho^2 zT(\alpha)\right) I_{m-\ell}\left(\frac{2}{3}\pi^2 k^2\rho^2 zT(\alpha)\right), \end{aligned} \tag{11}$$

Therefore, the weight of each OAM state of BG beam at the z plane after propagating in non-Kolmogorov turbulence can be