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Simulation of self-pulsing in Kerr-nonlinear coupled ring resonators

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ABSTRACT

Nonlinear resonant structures consisting of coupled ring resonators can be modeled by difference-differential equations that take into account non-instantaneous Kerr response and the effect of loss. We present a simple and efficient numerical formalism for solution of the system and calculation of the time evolution. The technique is demonstrated by investigating the dynamical behavior of the coupled structure with two rings, namely, focusing on self-pulsing solutions. The influence of both, loss and non-instantaneous Kerr response, is also presented.

1. Introduction

A number of interesting dynamical phenomena can occur in optical systems with nonlinear feedback. Such systems, typically nonlinear cavities, can, within a certain range of input optical power, exhibit optical bistability (or even multistability), a phenomenon which is expected to play an important role in data processing applications. With a further increase of power, the systems may exhibit generation of optical pulses from continuous wave input (self-pulsing) and chaos. The nonlinearity is often provided by the Kerr effect, in which the intensity of propagating light alters the refractive index of the medium.

The chaotic behavior is related to the instability that was first investigated by Ikeda [1] in a single ring resonator. Under suitable conditions, namely for the finite relaxation time of the nonlinear response, the Ikeda instability can lead to self-pulsing (SP) [2]. These predictions were experimentally confirmed with a hybrid optically bistable device [3]. Similar effects were found in a nonlinear Fabry–Perot resonators [4] and distributed feedback structures [5,6]. The impact of the relaxation time was recently studied in Ref. [7]

Other nonlinear mechanisms can also induce the mentioned effects. For various types of optical microcavities, e.g., silicon microring resonators, the nonlinear effect is not only provided by the Kerr effect but also by the free carrier absorption (FCA) and free carrier dispersion (FCD) through the two-photon absorption (TPA) effect [8–13].

It is well-known that nonlinear effects are enhanced in coupled-cavity systems [14,15]. Coupled cavities with instantaneous Kerr response exhibit rich dynamics and offer more control over nonlinear switching, SP and chaos [16–20]. In particular, SP and chaos were observed in systems with two and three coupled microcavities [18]. For two-cavity systems, SP can be explained as a result of beating of modes and bistable switching [19]. However, SP can also be related to gap solitons [18]. For long microring chains, spontaneous generation of gap solitons from cw input was studied in Ref. [21].

In this paper, we focus on simulation of coupled ring resonators. Compared with the publications regarding coupled-cavity systems, we take into account non-instantaneous Kerr response. We also introduce the effect of loss which has not been considered in such systems so far.

Often, the simulation of nonlinear cavities is performed by coupled mode theory in time [18,19,7,22]. The technique can treat different kinds of cavities in the same manner. However, its validity is limited to the case of weak coupling [23]. Here we describe an alternative approach that can be used for ring resonators. We generalize the map presented in Ref. [2] and obtain a system of difference-differential equations. We formulate an efficient numerical technique for the solution of the system. The paper is organized as follows: In Section 2, we introduce the model and theoretical formulation. In Section 3, we demonstrate the technique by presenting the dynamical behavior of a system with two rings. The effect of loss as well as the effect of instantaneous and non-instantaneous Kerr response is also presented. Finally, in Section 4 we conclude the paper.

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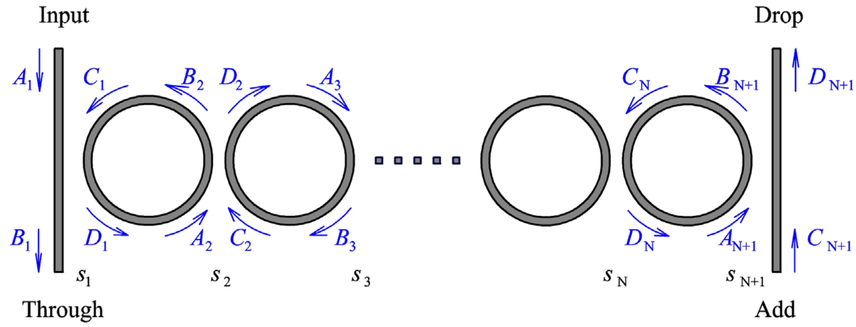

Fig. 1. Coupled ring resonators with four ports labeled. Each coupler is described by the parameter s_j. A_j, B_j, C_j and D_j represent mode amplitudes. Arrows indicate propagation of the modes.

2. Model and theoretical formulation

Consider a structure consisting of N coupled ring resonators side coupled with two waveguides as shown in Fig. 1. All the resonators are identical and made from the same single mode waveguide as the two waveguides. A_j, B_j, C_j and D_j represent the time-dependent (slowly varying) mode amplitudes at different positions in the rings or waveguides. The amplitudes are scaled to dimensionless form, as will be explained below. The presented model is valid for unidirectional propagation of modes, i.e. the structure is excited at the input and/or add port. In the subsequent numerical calculations (Section 3), however, we will always suppose excitation only at the input port, thus $C_{N+1}=0$.

Similarly as in [24,25], we assume that the coupling is lossless and localized at a single point. Then, by using the notation in Fig. 1, the interaction in each coupler (in time t) is given by

$$B_j(t)=r_jA_j(t)+is_jC_j(t), \tag{1}$$

$$D_j(t)=is_jA_j(t)+r_jC_j(t). \tag{2}$$

Here, is_j and $r_j=\sqrt{1-s_j^2}$, $(j=1,2,\ldots,N+1)$, are respectively the coupling and transmission coefficients which describe each coupler [24,25].

The structure exhibits Kerr nonlinearity, i.e., in the stationary state, the nonlinear change of the effective mode index at a certain position in the ring (or waveguide) is given by the relation of type $n_2|A_j'|^2$, where n_2 is the effective nonlinear Kerr-index (it is assumed to be constant in all rings/waveguides) and A_j' is the physical amplitude of the mode at this position. However, instead of A_j' we use the dimensionless amplitude A_j defined by the relation $A_j=(2\pi n_2 L_{\rm eff}/\lambda)^{1/2}A_j'$, where $L_{\rm eff}=[1-\exp(-\alpha L)]/\alpha$ is the effective length, α is the waveguide loss coefficient (it includes all possible linear loss mechanisms, such as material absorption or scattering), L is the half of the ring circumference, and λ is the wavelength of the light in vacuum. The same scaling applies for the amplitudes B_j, C_j and D_j. In accordance with these definitions, we define powers at the input, through, and drop ports by the relations $P_{\rm in}=(L/L_{\rm eff})|A_1|^2$, $P_{\rm t}=(L/L_{\rm eff})|B_1|^2$, and $P_{\rm d}=(L/L_{\rm eff})|D_{N+1}|^2$, respectively. In this way, the powers are directly related to the physical amplitudes and can also be used as a measure of nonlinearity strength.

Optical pulse propagation in a nonlinear dispersive media is well-described by Maxwell's equations [26,27] taking into consideration nonlinear polarization and applying the slowly varying-envelope approximation. In the presence of Kerr-nonlinearity and waveguide loss, the relations between mode amplitudes in Fig. 1 are

$$C_j(t)=B_{j+1}(t-\tau)\exp\left[-\frac{\alpha L}{2}+i\phi+i\beta_{j+1}(t-\tau)\right], \tag{3}$$

$$A_{j+1}(t)=D_j(t-\tau)\exp\left[-\frac{\alpha L}{2}+i\phi+i\delta_j(t-\tau)\right]. \tag{4}$$

Here, $1\le j\le N$, $\tau=n_gL/c$ is the group delay corresponding to propagation of the pulse over distance L, n_g is the mode group index and c is the velocity of light. Note that, the free spectral range, FSR, is related with the group delay by the relation $\tau\,\mathrm{FSR}=1/2$. $\phi=2\pi n_{\rm eff}L/\lambda$ is the linear phase shift acquired over distance L, $n_{\rm eff}$ is the linear effective mode index. The shift can be expressed as $\phi=\pi(m+\Delta f/\mathrm{FSR})$, where m is an arbitrary positive integer and Δf is the frequency detuning from resonance. In the following analysis, we will always assume even m (adaption of the formulation for odd m is obvious) and thus ϕ in Eqs. (3) and (4) can be replaced by $\pi\,\Delta f/\mathrm{FSR}$.

Nonlinear phase shifts β_j and δ_j are given by the response of the medium. Here, we assume the Debye relaxation [1,2,28]

$$T_{\rm R}\frac{d\beta_j(t)}{dt}+\beta_j(t)=|B_j(t)|^2, \tag{5}$$

$$T_{\rm R}\frac{d\delta_j(t)}{dt}+\delta_j(t)=|D_j(t)|^2, \tag{6}$$

where $T_{\rm R}$ is the medium relaxation time.

The above system of the difference-differential equations (Eqs. (1)–(6)), which is a generalization of the Ikeda equations for single ring [2,3], fully describes the time evolution of the amplitudes A_j, B_j, C_j, D_j and nonlinear phase shifts β_j, δ_j from given initial conditions.

The system can be readily solved in the approximation of instantaneous response. In this case, we consider the limit $T_{\rm R}\ll\tau$ and assume the solutions of Eqs. (5) and (6) in the form $\beta_j(t)=|B_j(t)|^2$ and $\delta_j(t)=|D_j(t)|^2$. Consequently, the whole system is reduced to a system of difference equations which appear, e.g., in Ref. [21].

For obtaining of the steady-state solutions (time independent solutions) of Eqs. (1)–(6), we assume no signal at the add port, $C_{N+1}=0$, and arbitrarily choose the amplitude at the drop port D_{N+1}. Then, the other amplitudes are calculated step by step with using Eqs. (1)–(4), finally the amplitudes A_1 at the input and B_1 at the through port are found. Note that in the steady state, solutions of Eqs. (5) and (6) are formally the same as in the approximation of instantaneous response.

Stability of the steady-state solutions was investigated by using the linear stability analysis. To this aim, we assumed the approximation of instantaneous response and calculated the eigenvalues of the Jacobian. The given solution is stable if and only if the absolute value of any eigenvalue is less than 1 [3].

In the general case of non-instantaneous response, we need an efficient technique for the numerical integration of Eqs. (5) and (6). To this aim, we write solution of Eq. (5) in the form

$$\beta_j(t)=|B_j(t)|^2+\exp\left(-\frac{t}{T_{\rm R}}\right)\left[\beta_j(0)-|B_j(0)|^2\right]-\int_0^t\exp\left(\frac{t'-t}{T_{\rm R}}\right)\frac{d|B_j(t')|^2}{dt'}\,dt'. \tag{7}$$