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# Focusing and imaging properties of a dense Gaussian apodized photon sieve

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ABSTRACT

Imaging and focusing properties of a dense Gaussian-based photon sieve were examined, in detail. This special distribution causes overlapping among pinholes of photon sieve in a few innermost zones so some of the zones are being fully cleared. However, from the focusing property point of view, the central spot size is reduced as well as the transmission of photon sieve is increased, accordingly visibility is enhanced. Theoretical and simulation works clarify that an increase in the number of the cleared zones not only improves the transmission but also causes the main lobe to become narrower at the expense of increasing of sidelobes intensity. Examination of imaging behavior of dense Gaussian photon sieve also proved that there is an optimized number of clear zone for which visibility is enhanced.


## 1. Introduction

Focusing and imaging of soft x-ray have many applications in physical and life sciences, such as high resolution x-ray microscopy, spectroscopy, and lithography. Because of strong absorption of solid materials in the x-ray spectral region, it is difficult to use a traditional refractive lens to focus soft x-rays. On one hand, traditional Fresnel zone plates (TFZP) can overcome these shortcomings. But, on the other hand its resolution is limited by width of the outermost zone [1]. To address this issue photon sieve (PS) was introduced and developed for focusing and imaging of x-rays with high resolution capabilities [2]. A PS is basically made from a great number of pinholes which are arranged over the clear zones of a TFZP. As a mathematical description, an individual far field model and the non-paraxial model for focusing high numerical aperture PS were proposed by Cao et al. [3,4]. Since the invention of PS, different pinhole distributions were introduced and imposed on PS to increase its resolution [5–8]. In all these proposed models of distributions, holes overlapping is avoided and considered as a destructive factor. Whereas, it was recently shown that overlapping acts not only as a destructive element but also serves as an effective feature to optimize the optical performance of PS [9]. Overlapping may be generated by introducing a novel distribution model so called dense Gaussian distribution in which number of the holes in each Fresnel zone is being raised so pinholes overlapping occur among holes as a result some innermost zones are cleared. In addition, transmission of the PS constructed by this novel distribution is effectively increased owing to having a larger number of the holes than the other known model of PS.

Although, focusing properties of the model have been compared with other known models of diffractive lens, however its optical properties have not been examined in detail. Hence, in this work we are going to develop and study the proposed apodized photon sieve in more detail, in order to get more insight into its focusing and imaging behavior. Besides, the role and impact of the number of the cleared zones on the point spread function and visibility of dense Gaussian photon sieve (DGPS) are examined theoretically and experimentally. It is observed that by increasing the number of the zones comprising fully overlapped pinholes, sidelobes are growing up so that in a limiting case they are joined to the principal main lobe and create wider central spot. Consequently, there is an optimum case in which the optical efficiency is optimized well.

## 2. Theory and Simulation

PS is composed of a lot of circular pinholes arranged in the radial direction such that both the center of locations and diameters correspond to the clear zones of the TFZP, to ensure that the diffracted field from each pinhole interferes constructively in the focal plane. Therefore, diffraction from PS can be expressed by the superimposing of diffracted wave fields from these pinholes. To this end, it is enough to get an expression for diffraction from an arbitrary hole located in the PS plane, and the expression could be easily generalized to the other pinholes. Finally, adding up all of the diffracted fields gives the total complex amplitude of

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the diffracted field [9]. For example, the diffracted field of a single pinhole can be expressed as

$$U \propto \frac{d}{w} J_1\left(\frac{\pi}{2}\frac{d}{w}\right), \tag{1}$$

where $d/w$ is the ratio of the diameter of the pinhole to the width of the local Fresnel zone, $J_1$is the first-order Bessel function of the first kind. A plot of this function is shown in Fig. 1. Obviously, the maximum absolute values of $U$ appear when $d/w$ is approximately equal to 1.5, 3.5, 5.5, etc. On the other hand, when $U>0$, light passing through the holes is making a positive contribution to the focused light, and when $U<0$, the transmitted light is acting to reduce the focused intensity.

However, a more general way is using convolution between the object pupil function and the free space response function. The amplitude of diffracted plane wave may be described by the Fresnel–Kirchhoff diffraction integral:

$$U(x,y)=\frac{e^{ikz}}{i\lambda z}\int M(x',y')e^{(ik/2z)[(x-x')^2+(y-y')^2]} \tag{2}$$

Within the Fresnel approximation and using the convolution theorem one may write (see for example [10]):

$$U(x,y,z)=\frac{e^{ikz}}{i\lambda z}\left[M(x,y)\otimes\exp\left(\frac{ik}{2z}(x^2+y^2)\right)\right] \tag{3}$$

where $U(x,y)$ is the amplitude of diffracted field at distance $z$ from an object, $M(x',y')$ is the pupil function as it is shown in Fig. 2 and $\otimes$ denotes the convolution operator. As well as, $k=2\pi/\lambda$ and $\lambda$ is the wavelength of the incident light. Eq. (3) may be written as a Fourier transform:

$$U(x,y,z)=FT^{-1}\{FT[M(x,y)]FT[e^{(ik/2z)(x^2+y^2)}]\}, \tag{4}$$

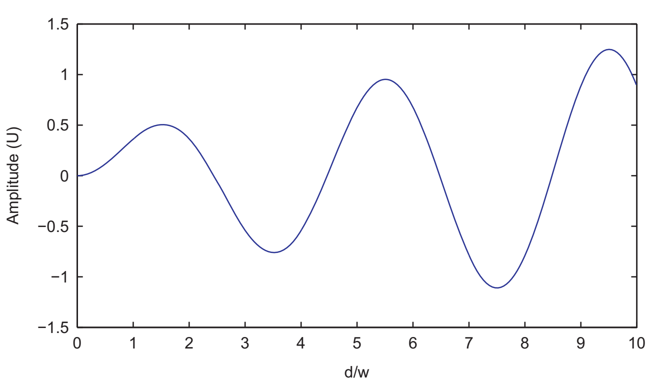


**Fig. 1.** Diffracted field of a single pinhole illustrated as the function of the ratio of the diameter of the pinhole to the width of the local Fresnel zone ($d/w$).

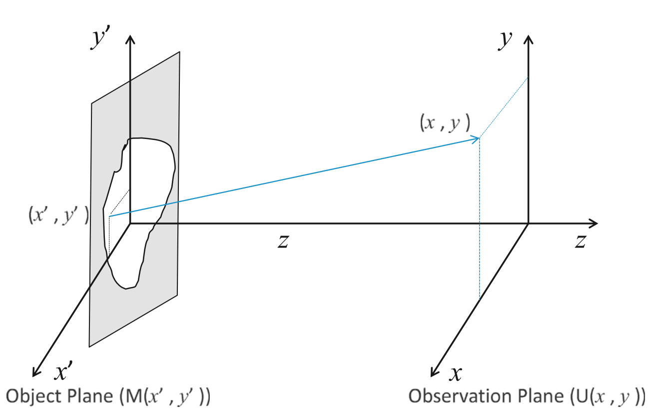


**Fig. 2.** Geometry used for calculating the diffraction pattern $U(x,y)$ at an observation plane produced by a pupil function $M(x',y')$.

in which some unimportant constants have been dropped and $FT$ denotes the Fourier transform. In this work, a PS is apodized by distributing a large number of pinholes over the Fresnel zones using the dense Gaussian function. Typically Gaussian distribution may be mathematically defined by the following expression:

$$W_n=n_0\exp\left(-\frac{(r_n-r_0)^2}{\alpha^2}\right), \tag{5}$$

where $r_n$ is the radius of the $n$th zone, $(n_0,r_0,\alpha^2)$ are parameters which determine the desired distribution. Dense distribution is a distribution in which a large number of pinholes are produced in each zone so that the holes are completely overlapped to form a clear zone that may be generated by choosing suitable parameters of Eq. (5). In general, the pupil function of a PS may be written as [11]

$$M(x',y')=\begin{cases}1 & (x-x_{mn})^2+(y-y_{mn})^2\le r_m^2,\\ 0 & \text{other,}\end{cases} \tag{6}$$

where $x_{mn}$, $y_{mn}$ are the center coordinates of the $n$th pinhole in the $m$th zone of the photon sieve (PS), so that $m=1,2,\ldots,M$, ($M$ is the number of the zones) and $n=1,2,\ldots,N$ ($N$ is the number of the pinholes in a given zone $n$ that comes from $W_n$ using Eq. (5)). A few photon sieves based on dense Gaussian (DGPS) with different Gaussian parameters at a wavelength of 632.8 nm (500 mm focal length, 15 mm pupil diameter, 44 total Fresnel zones) were considered and designated.

Fig. 3(a)–(c) shows typically generated DGPS with different clear zones. They were arranged from the lowest clear zones (one clear zone) to the highest ones (about 12 clear zones) and constructed using the structural parameters $(n_0,r_0,\alpha^2)$=(150, 44, 1000), (450, 44, 1000) and (650, 44, 1000) based on Eq. (5). Some simulations were also performed to evaluate transverse intensity distribution at the focal plane of four different DPGS. These DGPS have a different number of clear zones from the lowest number of clear zones (just one clear zone) to the highest ones (about 42 clear zones) indicated by (a)–(d) in Fig. 4. Results reveal that the more the clear zones (the zones comprise fully overlapped pinholes), the narrower the main lobe whereas the more the strength the more the sidelobes appear. It is obvious that any increase in the number of cleared zones enhances geometrical symmetry of the photon sieve, therefore sidelobes become stronger. In Fig. 4 (d) sidelobes have been joined to the main one and construct a very wide focal spot which confirms our assertion about growing up sidelobes owing to the increasing of the number of the clear zones. Although, it should be noticed that the quality of the focal spot is being completely different from the other samples, by considering the fact that intensity distribution is being noticeably uniform across the spot. For a comparison a transverse intensity profile marked with letter 'e' was also sketched for a traditional Fresnel zone plate (TFZP) with the same specifications as the DGPS has. As it is clear, it produces the widest focal spot in comparison to the other profiles. As a consequence, the proposed model may have high optical efficiency if only a few innermost zones form clear zones.

## 3. Experimental verifications

To verify the feasibility of the simulation works, some DGPS with the same features used in simulation works i.e. $(n_0,r_0,\alpha^2)$= (150, 44, 1000), (650, 44, 1000), (1000, 44, 1000), and (1200, 44, 1000) were selected which are corresponding to 1, 12, 30, and 42 clear zones, respectively. These samples are designed to work in the optical region at 632.8 nm wavelength (He–Ne laser) with radius and focal length 15 mm and 500 mm, respectively. They