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Analysis of mode dispersion for abnormal total reflection from an isotropic medium to an arbitrarily oriented uniaxial crystal

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ABSTRACT

Mode dispersion is used to study the necessary conditions for the existence of abnormal total reflection, for wave propagating from a rare isotropic medium to a dense anisotropic medium with optical axis oriented arbitrarily in three-dimension space. The conditions can be put down to three inequalities of the relations among relative permittivity parameters in two media and the direction of optical axis of a uniaxial crystal. Based on the different geometrical characteristics of mode dispersion in isotropic and anisotropic media as well as the numerical simulations of spatial field, the physical essence of this abnormal total reflection is revealed, which means that the ray index in incident medium is still larger than that in refracted medium, although the refractive index in incident medium is smaller than that in refracted medium.

1. Introduction

Within the past decade, an increasing number of intriguing phenomena and applications associated with anisotropic material and anisotropic plasmonic structures [1,2] have been reported. Several representative examples include abnormal total reflection [3], non-symmetrical reflection [4], surface plasmons and related hybridized wave [5,6], nonlinear optical effect and mode recombination of surface plasmon polaritons [7–9], the negative Brewster angle [10], omnidirectional total reflection [11], total transmission [12] caused by anisotropic metamaterials, anisotropic optical effect on photonic crystals and on particles [13,14], optical constant determination of an anisotropic thin film [15,16], etc.

Among these phenomena, abnormal total reflection is rather unique, which means that the refractive wave from a rare isotropic medium to a dense anisotropic medium is in the dense medium when incident angle is equal to the critical angle of total reflection. The abnormal total reflection can diversify the ways to generate evanescent wave, which is essential for microscopes, sensors and imaging in lithography [17,18]. There are already several theoretical studies reported on the abnormal total reflection. When the optical axis is in the incident plane, Lin and Wu [19] first analyzed the phenomenon using elliptical surface of refractive index for extraordinary waves and boundary conditions of wave vectors. Later, Jen and Cheng [20] discussed the phenomenon by a distribution diagram and the boundary conditions of wave vectors. They got conditions of this abnormal total reflection and showed different forms of expression, respectively. Meantime, for the case that the optical axis coincides with an axis of a Cartesian-coordinate system, Castillo and Ballinas [21] studied it by establishing a relation between eikonal and Poynting vector in an anisotropic medium. However, there is still no literature available about the general situation of abnormal total reflection when the optical axis is oriented arbitrarily in three-dimension space.

In our paper, by taking advantage of simple mode dispersion, the conditions of abnormal total reflection are analyzed theoretically and numerically when the optical axis is oriented arbitrarily in three-dimension space. This new and extended feasible region is explored for abnormal total reflection from a rare isotropic medium to a dense anisotropic medium. Furthermore, our results reveal the physical substance of the abnormal total reflection between refractive index and ray index of two media.

2. Theory and analysis

In an infinite uniaxial crystal with optical axis $\hat{c}$ (unit vector) oriented arbitrarily, the corresponding relative permittivity tensor $\tilde{\varepsilon}$ is expressed by dyadic notation [22]

$$\tilde{\varepsilon} = \varepsilon_{\perp}\tilde{I} + (\varepsilon_{\perp} - \varepsilon_{\parallel})\hat{c}\hat{c} \quad (1)$$

where $\varepsilon_{\perp}$ and $\varepsilon_{\parallel}$ are the eigenvalues of $\tilde{\varepsilon}$, $\tilde{I}$ is the identity dyadic. For TM plane wave propagating in this uniaxial crystal, we can obtain its dispersion equation

$$\vec{k}_1 \cdot \tilde{\varepsilon} \cdot \vec{k}_1 = k_0^2 \varepsilon_{\perp}\varepsilon_{\parallel} \quad (2)$$

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of which k_0 denotes the free-space wave vector. Considering the general case that the directions of wave vector $\vec{k}_1$ and $\hat{c}$ are different, we decomposed $\vec{k}_1 = \vec{k}_\parallel + \vec{k}_\perp$ into its components parallel $\vec{k}_\parallel$ and perpendicular $\vec{k}_\perp$ to the optical axis, so Eq. (2) can be rewritten as

$$\frac{\vec{k}_\parallel^2}{k_0^2\varepsilon_\perp} + \frac{\vec{k}_\perp^2}{k_0^2\varepsilon_\parallel} = 1 \tag{3}$$

which displays an ellipsoid of revolution about the $\vec{k}_\parallel$ axis in three-dimensional $\vec{k}_1$ space. Similarly, for isotropic dielectric, with a dispersion equation

$$\vec{k}_2^2 = k_0^2\varepsilon \tag{4}$$

where ε is the relative permittivity of isotropic dielectric. Eq. (4) displays a sphere surface in three-dimensional $\vec{k}_2$ space.

In this paper, we discuss the propagation of wave in lossless nonmagnetic homogeneous material. In order to analyze and elucidate the existence conditions of abnormal total reflection in each and every case when a uniaxial crystal with optical axis is oriented arbitrarily, we show the geometry and orientation of the optical axis in Cartesian coordinates with Fig. 1. The direction of the optical axis can be expressed as $\hat{c} = \hat{x} \sin\theta \cos\varphi + \hat{y} \cos\theta + \hat{z} \sin\theta \sin\varphi$, where θ denotes the angle between the optical axis and the positive direction of y axis, and φ denotes the angle between the projection of the optical axis in the xoz plane and the positive direction of x-axis. The discussions below will be focused on the region of $0 \le \theta \le \pi/2$, because the analysis is similar for other regions. Without loss of generality, we set the xoy plane as the plane of incidence. The incident wave is propagating upward from an isotropic medium to a uniaxial crystal at an incident angle θ_i, relative to the normal to the interface of two media, and $0 \le \theta_i \le \pi/2$. Thus, Eqs. (2) and (4) can be rewritten as two binary quadratic equations

$$k_{1x}^2 A_{11} + k_{1x}k_{1y}A_{12} + k_{1y}^2 A_{22} = k_0^2\varepsilon_\perp\varepsilon_\parallel \tag{5}$$

$$k_{2x}^2 + k_{2y}^2 = k_0^2\varepsilon \tag{6}$$

where

$$A_{11} = \varepsilon_\perp + (\varepsilon_\parallel - \varepsilon_\perp)\sin^2\theta\cos^2\varphi \tag{7}$$

$$A_{12} = (\varepsilon_\parallel - \varepsilon_\perp)\sin\theta\cos\varphi\cos\theta \tag{8}$$

$$A_{22} = \varepsilon_\perp + (\varepsilon_\parallel - \varepsilon_\perp)\cos^2\theta \tag{9}$$

In Fig. 2, the dispersion Eqs. (5) and (6) are shown in reciprocal space, respectively. The blue circle and red ellipse represent mode dispersion curves in the media of incidence and refraction, respectively. The parameters we used in Fig. 2 are $\varepsilon = 3.0^2$, $\varepsilon_\perp = 3.8^2$, $\varepsilon_\parallel = 1.2^2$, $\theta = 47^\circ$, and $\varphi = 10^\circ$, respectively. The incident angle is

$$\theta_i = \arctan\left(\frac{b}{\sqrt{\varepsilon - b^2}}\right) \tag{10}$$

where

$$b = \sqrt{\frac{A_{11}\varepsilon_\parallel\varepsilon_\perp}{A_{11}A_{22} - A_{12}^2}} \tag{11}$$

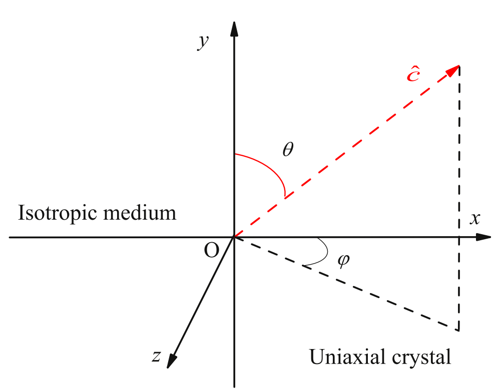

Fig. 1. (Color online) Schematic of the structure and the direction of the optical axis.

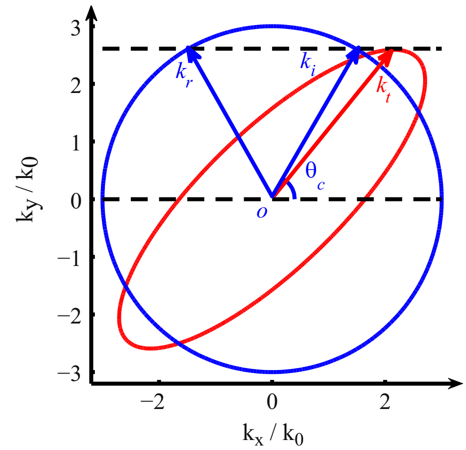

Fig. 2. Reciprocal space map for the isotropic medium and the uniaxial crystal. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The incident wave vectors $\vec{k}_i$, reflection wave vectors $\vec{k}_r$, and refraction wave vectors $\vec{k}_t$, must obey the continuous boundary conditions in the interface. In Fig. 2, the top dashed line indicates the case of abnormal total reflection, in which the relationship between corresponding wave vectors is $\vec{k}_t > \vec{k}_i$, proving that the wave propagates from a rare medium to a dense medium. The total internal reflection will occur when $\theta_i > \theta_c$, where θ_c stands for the incident angle of abnormal total reflection. Therefore, the conditions for abnormal total reflection should be satisfied simultaneously: (a) the value of ε, in between $\varepsilon_\perp$ and $\varepsilon_\parallel$, (b) the following two inequalities of the relations between ε, $\varepsilon_\perp$, $\varepsilon_\parallel$ and θ, φ:

$$\sin^2\theta\cos^2\varphi + \frac{\varepsilon}{\varepsilon_\parallel}\sin^2\theta\sin^2\varphi + \frac{(\varepsilon - \varepsilon_\perp)}{\Delta} > 0 \tag{12}$$

$$\sin^4\theta\cos^4\varphi + \frac{\varepsilon}{4\varepsilon_\parallel}\sin^4\theta\sin^2 2\varphi + \frac{1}{4}\sin^2 2\theta\cos^2\varphi - \frac{2\varepsilon_\perp - \varepsilon}{\Delta}\sin^2\theta\cos^2\varphi - \frac{\varepsilon\varepsilon_\perp}{\varepsilon_\parallel\Delta}\sin^2\theta\sin^2\varphi + \frac{\varepsilon_\perp(\varepsilon_\perp - \varepsilon)}{\Delta^2} > 0 \tag{13}$$

where $\Delta = \varepsilon_\perp - \varepsilon_\parallel$, and we only study the case of $\varepsilon_\perp > \varepsilon_\parallel$ in this paper, because the case of $\varepsilon_\perp < \varepsilon_\parallel$ is similar in analysis. Making use of these inequalities, we can obtain Fig. 3 to show the orientation of optical axis as functions of ε, $\varepsilon_\perp$, $\varepsilon_\parallel$ for abnormal total reflection to occur. A common feature of Fig. 3(a)–(d) is that the feasible range of θ becomes small with the value of φ being increased. When increasing Δ by increasing $\varepsilon_\perp$ and fixing $\varepsilon_\parallel$, ε, the feasible range of θ is enlarged and moves to the region of larger angles. When increasing Δ by decreasing $\varepsilon_\parallel$ but increasing $\varepsilon_\perp$ and fixing ε, the feasible range of θ is enlarged through extending both sides of the angle. No matter how Δ changes, the feasible range $\varphi \in [0, 0.45\pi]$ hardly changes if ε is kept unchanged by comparing the features of Fig. 3(a)–(c). When $\varepsilon_\parallel$ and $\varepsilon_\perp$ are kept unvaried but ε is increased, the feasible range of θ becomes large and moves to the region of