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Average transmittance in non-Kolmogorov turbulence

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ABSTRACT

Average transmittance in non-Kolmogorov turbulence is evaluated. Our recently published equivalent structure constant formulation is employed in our numerical evaluations. At the fixed propagation distance and wavelength, and at the corresponding equivalent structure constant, as the power law exponent of the non-Kolmogorov spectrum increases, the on-axis transmittance is found to decrease. At the same power law exponent of the non-Kolmogorov spectrum, the off-axis transmittance is obtained to be smaller than the on-axis transmittance. Off-axis transmittance variation versus the power law exponent shows that similar to the on-axis case, increase in the power law exponent eventually causes the off-axis transmittance to decrease, however this decrease occurs at larger power law exponent for larger off-axis distance.

1. Introduction

Transmittance is an important parameter that provides how much of the transmitted power is transported to the receiver. Having knowledge about the windows for successful operation within the whole optical wavelength range under different atmospheric influences is crucial in the design of atmospheric optical communication, imaging, ranging and lidar systems. Molecular and aerosol effects on the transmittance are being investigated for a long time utilizing the MODTRAN (MODerate resolution atmospheric TRANsmittance and radiance) and HITRAN (HIgh resolution TRANsmission) codes [1–4]. In an effort to introduce the atmospheric turbulence effects on the transmittance, we have previously reported transmittance evaluations under partially coherent [5], different intensity profile incidences [6] and for Dense Wavelength Division Multiplexing system applications [7]. In these studies we have found the transmittances when the turbulence assumes Kolmogorov spectrum. However, it is known that under certain atmospheric conditions, turbulence follows non-Kolmogorov spectrum [8,9]. Various propagation parameters are lately being investigated by many researchers when turbulence has non-Kolmogorov spectrum [10–18]. In this respect, we have recently formulated the equivalence between the structure constants in non-Kolmogorov and Kolmogorov turbulences [19].

In the current paper, we formulate and evaluate the average transmittance in non-Kolmogorov turbulence and examine the effects of the power law exponent of the non-Kolmogorov turbulence spectrum on the average transmittance. Potential application of the tool we have developed will be used in the design of atmospheric optics links that will be operational in any kind of non-Kolmogorov turbulent atmosphere.

2. Average transmittance in non-Kolmogorov turbulence

The average transmittance is defined as

$$\langle \tau(\mathbf{p}) \rangle = \langle I(\mathbf{p}) \rangle / I^{v}(\mathbf{p}), \tag{1}$$

where $\langle I(\mathbf{p}) \rangle$ is the average received intensity at the receiver transverse coordinate $\mathbf{p}=(p_x, p_y)$ in turbulence and $I^v(\mathbf{p})$ is the received intensity at the same point $\mathbf{p}$ in vacuum, i.e., in the absence of turbulence. Employing the extended Huygens–Fresnel principle [20], $\langle I(\mathbf{p}) \rangle$ is found as

$$\langle I(\mathbf{p}) \rangle = \left(\frac{|A|}{\lambda L}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\mathbf{s}_1 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\mathbf{s}_2 \exp\left[-\frac{1}{2\alpha_s^2}(|\mathbf{s}_1|^2 + |\mathbf{s}_2|^2)\right] \exp\left[\frac{ik}{2L}\left(\left|\mathbf{s}_1-\mathbf{p}\right|^2 - \left|\mathbf{s}_2-\mathbf{p}\right|^2\right)\right] \langle \exp[\psi(\mathbf{s}_1,\mathbf{p}) + \psi^*(\mathbf{s}_2,\mathbf{p})] \rangle, \tag{2}$$

where L is the propagation distance, A is the amplitude of the source field, $i^2=-1$, λ is the wavelength, $\mathbf{s}_1=(s_{1x}, s_{1y})$ and $\mathbf{s}_2=(s_{2x}, s_{2y})$ are the source transverse coordinates, α_s is referred as the source size, $k=2\pi/\lambda$ is the wavenumber, $\psi(\mathbf{s},\mathbf{p})$ is the complex random phase for spherical wave in turbulence, $\langle\rangle$ represents the ensemble average over the medium statistics. The ensemble average term that represents the non-Kolmogorov turbulence

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effects is given by [18]

$$\langle \exp[\psi(\mathbf{s}_1,\mathbf{p})+\psi^*(\mathbf{s}_2,\mathbf{p})]\rangle=\exp\left[-\left(\frac{|\mathbf{s}_1-\mathbf{s}_2|}{\rho_0}\right)^{\alpha-2}\right], \quad (3)$$

where α is defined as the power law exponent of the non-Kolmogorov spectrum, ρ_0 is the coherence length for spherical wave in non-Kolmogorov turbulence which is dependent on α and is given as [18]

$$\rho_0=\left[\frac{2^{-\alpha}\alpha\Gamma(\alpha-1)\Gamma(-\alpha/2)\sin[(\alpha-3)\pi/2]}{(\alpha-1)\Gamma(\alpha/2)}k^2\tilde{C}_n^2L\right]^{(-1/(\alpha-2))}, \quad (4)$$

where Γ is the gamma function and $\tilde{C}_n^2$ denotes the structure constant for non-Kolmogorov spectrum. We note that in the limit of Kolmogorov spectrum, i.e., when $\alpha=11/3$, Eq. (4) correctly reduces to $\rho_0=(0.545k^2C_n^2L)^{-3/5}$ which is the coherence length of spherical wave in Kolmogorov turbulence. In [19], we have formulated the equivalent structure constant $\tilde{C}_n^2$ in non-Kolmogorov turbulence in terms of the structure constant C_n^2 in Kolmogorov turbulence. For completeness, this relation is repeated below by using [19]

$$\tilde{C}_n^2=-\frac{0.5\Gamma(\alpha)(2\pi)^{-11/6+\alpha/2}(\lambda L)^{11/6-\alpha/2}}{\Gamma(1-\alpha/2)[\Gamma(\alpha/2)]^2\Gamma(\alpha-1)\cos(\alpha\pi/2)\sin(\pi\alpha/4)}C_n^2. \quad (5)$$

Note that the negative sign is missing in [19]. Substituting Eq. (3) in Eq. (2),

$$\langle I(\mathbf{p})\rangle=\left(\frac{|A|}{\lambda L}\right)^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d^2\mathbf{s}_1\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d^2\mathbf{s}_2\exp\left[-\frac{1}{2\alpha_s^2}(|\mathbf{s}_1|^2+|\mathbf{s}_2|^2)\right]\exp\left[\frac{ik}{2L}(|\mathbf{s}_1-\mathbf{p}|^2-|\mathbf{s}_2-\mathbf{p}|^2)\right]\exp\left[-\left(\frac{|\mathbf{s}_1-\mathbf{s}_2|}{\rho_0}\right)^{\alpha-2}\right], \quad (6)$$

The received intensity in vacuum is found from Eq. (6) by taking $\rho_0=\infty$ which reads as

$$I^v(\mathbf{p})=\left(\frac{|A|}{\lambda L}\right)^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d^2\mathbf{s}_1\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d^2\mathbf{s}_2\exp\left[-\frac{1}{2\alpha_s^2}(|\mathbf{s}_1|^2+|\mathbf{s}_2|^2)\right]\exp\left[\frac{ik}{2L}\left(\left|\mathbf{s}_1-\mathbf{p}\right|^2-\left|\mathbf{s}_2-\mathbf{p}\right|^2\right)\right], \quad (7)$$

Applying Eq. 3.323.2 of [21] four times, Eq. (7) is found to be

$$I^v(\mathbf{p})=\left(\frac{|A|}{\lambda L}\right)^2\frac{(2\pi L\alpha_s^2)^2}{(L^2+k^2\alpha_s^4)}\exp\left(-\frac{k^2\alpha_s^2}{L^2+k^2\alpha_s^4}\left|\mathbf{p}\right|^2\right), \quad (8)$$

Evaluating Eq. (8) at $\mathbf{p}=0$, inserting Eqs. (6) and (8) into Eq. (1), the average transmittance in non-Kolmogorov turbulence is obtained as

$$\langle\tau(\mathbf{p})\rangle=\frac{(L^2+k^2\alpha_s^4)}{(2\pi L\alpha_s^2)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d^2\mathbf{s}_1\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d^2\mathbf{s}_2\exp\left[-\frac{1}{2\alpha_s^2}\left(\left|\mathbf{s}_1\right|^2+\left|\mathbf{s}_2\right|^2\right)\right]\exp\left[\frac{ik}{2L}\left(\left|\mathbf{s}_1-\mathbf{p}\right|^2-|\mathbf{s}_2-\mathbf{p}|^2\right)\right]\exp\left[-\left(\frac{|\mathbf{s}_1-\mathbf{s}_2|}{\rho_0}\right)^{\alpha-2}\right], \quad (9)$$

Eq. (9), together with Eqs. (4) and (5) represents our main result in this paper. We are not able to find an analytical solution to the integrals in Eq. (9). Thus, we have evaluated Eq. (9) numerically and have shown the results in Section 3. Under an applied point of view, Eq. (9) compares the influence of the turbulent atmosphere in an optical free space link, only in terms of the average intensity of the optical radiation. There are a lot of works devoted to this task [13,15,18], even taking into account more complex effects such as polarization [12]. In these works, the average intensity is evaluated by using a fixed structure constant for each realization of the power law exponent of the non-Kolmogorov spectrum, α. However, for a fair comparison of the average intensity and thus the average transmittance, structure constant variations at each power law exponent needs to be incorporated. We have recently reported in [19] one such correspondence through the equivalence in the structure constants at different power law exponents of the non-Kolmogorov turbulence, which is given by Eq. (5). The novelty of results

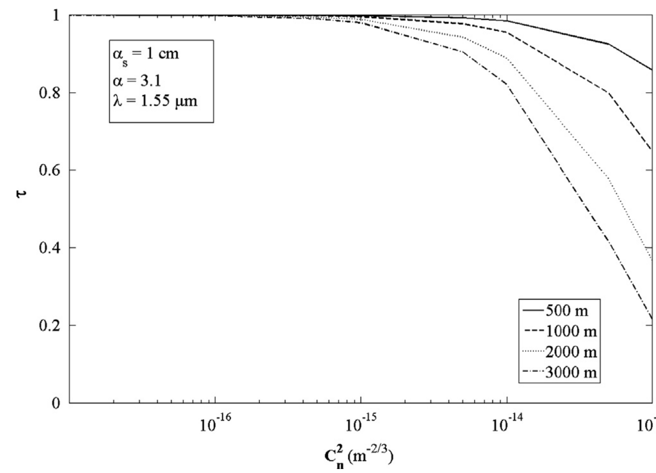

Fig. 1. Transmittance versus the structure constant at $\alpha=3.1$.

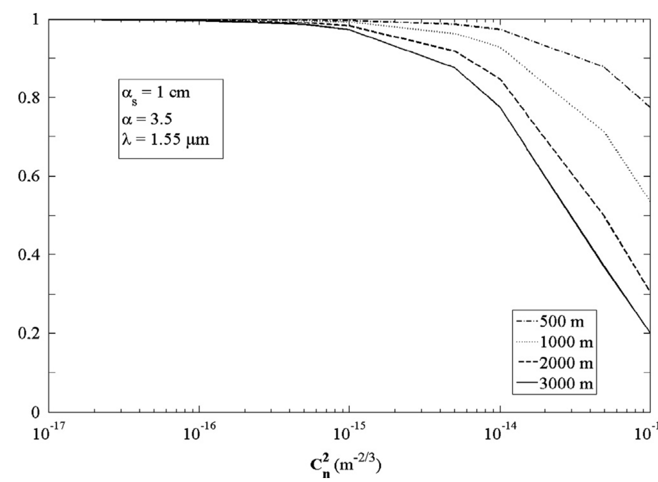

Fig. 2. Transmittance versus the structure constant at $\alpha=3.5$.

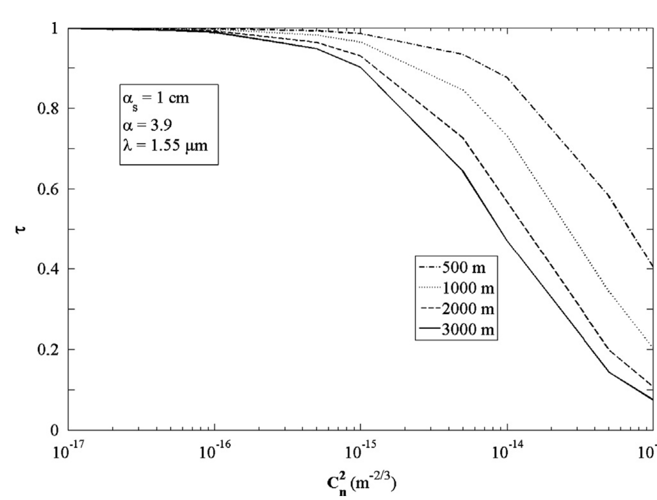

Fig. 3. Transmittance versus the structure constant at $\alpha=3.9$.