Optics Communications 298–299 (2013) 13–17

Contents lists available at SciVerse ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Diffraction of waves by an impedance half-plane

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ARTICLE INFO

Article history:
Received 16 November 2012
Received in revised form
11 December 2012
Accepted 10 February 2013
Available online 6 March 2013

Keywords:
Impedance boundary condition
Diffraction theory
Reflection coefficient

ABSTRACT

The solution of the diffraction problem of waves by an impedance half-plane that satisfies the impedance boundary condition is obtained for the first time in the literature. A reflection coefficient which is the function of the angular and spatial variables in the polar coordinates is defined and its exact expression is obtained with the aid of the impedance boundary condition. The resultant diffraction field is compared with the solution of Malyuzhinets numerically and their differences are stressed. The structures of the total scattered, geometrical optics and diffracted waves are also investigated.

1. Introduction

The diffraction process of waves by a lossy half-plane is an important canonical problem, because it puts forward the effect of the surface impedance on the diffracted waves. This problem has application areas for the wave propagation in urban areas, terrains, containing hills and forests, etc. [1–3]. It is also considered in the interaction problems of electromagnetic waves with aircrafts and ships whose outer surfaces are covered by lossy dielectrics [4,5]. The impedance boundary condition is used to model perfectly conducting surfaces, covered by a thin dielectric layer [6]. The scattering problem of waves by a lossy half-plane was first investigated by Raman and Krishnan [7]. They multiplied the reflected diffracted field with a constant reflection coefficient. However their field expressions were far from satisfying for the impedance boundary condition. The solution of the problem was put forward by Senior in 1952 with the method of Wiener–Hopf factorization [8]. Eight years later, Malyuzhinets solved the impedance wedge problem, which can also be reduced to the half-plane, by using the plane wave spectrum integral [9]. These two solutions have been known as the exact solutions since 2009 [10–12]. In this year, we showed that the solutions of Malyuzhinets and Senior were not satisfying the impedance boundary condition [13,14]. Furthermore, the diffracted field expression of Senior was not compensating for the discontinuities of the geometrical optics (GO) waves at the transition regions [15,16]. Thus there is no exact solution of the diffraction problem of waves by an impedance half-plane in the literature.

This paper aims to fill this gap and provide a solution of the diffraction problem by an impedance half-plane that satisfies the impedance boundary condition. With this aim, we will take into account the diffracted waves by a perfectly conducting half-plane. The reflected diffracted wave will be multiplied by a reflection coefficient, which is the function of the polar coordinates (ρ,ϕ) in the two dimensional problem. The reason of this approach relies on the fact that the amplitude of the diffracted field is dependent on the GO wave's amplitude [17]. The unknown coefficients of the reflection coefficient will be determined by using the impedance boundary condition and the limiting values of the GO field's reflection coefficients for $\sin\theta\to\infty$ (soft surface) and $\sin\theta\to 0$ (hard surface). $\sin\theta$ is equal to Z_0/Z where Z_0 and Z are the impedances of the free space and half-plane respectively. The total field (normal derivative of the total field) is equal to zero on a soft (hard) surface. The resultant diffracted field expressions will be compared with the solution of Malyuzhinets numerically.

A time factor of $\exp(j\omega t)$ is suppressed throughout the paper. j is $\sqrt{-1}$. ω is the angular frequency and t shows time.

2. Definition of the problem

A half-plane, located at $y=0$ and $x>0$, is taken into account. The geometry is given in Fig. 1. A scalar plane wave of $u_0\exp[jk\rho\cos(\phi-\phi_0)]$ is illuminating the half-screen. u_0 is the complex amplitude and ϕ_0 the angle of incidence. k is the wave-number. The half-screen has equal face impedances, shown by Z, on the upper and lower surfaces at $\phi=0$ and $\phi=2\pi$ respectively.

The impedance boundary condition can be written as

$$u\big|_S = \mp\frac{1}{jk\rho\sin\theta}\frac{\partial u}{\partial\phi}\Big|_S \quad (1)$$

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0030-4018/$ - see front matter
http://dx.doi.org/10.1016/j.optcom.2013.02.035

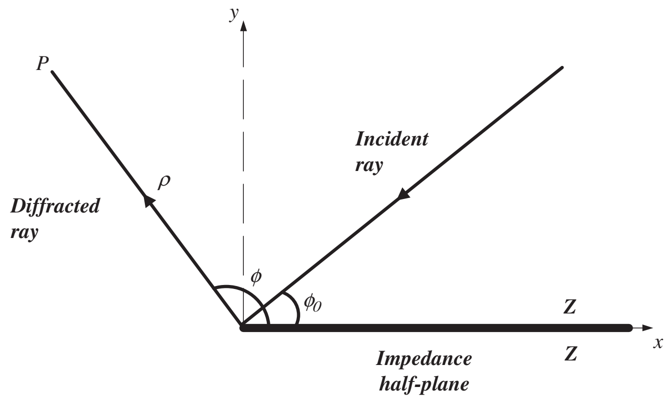

Fig. 1. Geometry of the diffraction of waves by a half-plane with equal face impedances.

on the surface of the half-plane [9]. u represents the total field. The $+$ and $-$ signs are valid for $\phi=0$ and $\phi=2\pi$. S is the surface of the half-plane. The aim of the problem is the determination of the scattered fields by the half-plane, which can be written as

$$u = u_{is} + u_{rs} \tag{2}$$

for which u_{is} and u_{rs} are the incident and reflected scattered waves respectively. A scattered field is composed of the GO and diffracted waves. The GO wave propagates unaffected from the discontinuity of the scatterer and has the same structure with the incident field. The reflected and transmitted GO waves can be evaluated by using the laws of GO. However they are not continuous and do not spread all over the space, because of the discontinuity of the scatterer's surface. The diffracted field is radiated by the edge discontinuity of the scatterer and has a shift of 180° on the transition region, which shows the location where the GO wave suddenly goes to zero. This phase shift of the diffracted wave compensates the discontinuity of the GO field at the transition region and its sum, which is the scattered wave, is continuous everywhere in space. This physical explanation of the GO and diffracted fields is based on the quantitative ideas of Young [18] and Rubinowicz [19]. The GO fields are known for this problem and can be expressed as

$$u_{GO} = u_{iGO} + u_{rGO} \tag{3}$$

where the incident and reflected GO fields of u_{iGO} and u_{rGO} can be defined by

$$u_{iGO} = u_0 e_- U(-\xi_-) \tag{4}$$

and

$$u_{rGO} = R u_0 e_+ U(-\xi_+) \tag{5}$$

respectively. u_{iGO} has the same structure as the incident field, but it is discontinuous at the shadow boundary. It can be seen from Fig. 1 that the incident wave that hits the semi-infinite aperture, at $x<0$, directly passes to the plane of $y<0$ till the edge point of $x=0$. After that point the incident wave cannot pass to the $y<0$ plane. This sharp transition creates a discontinuity in the incident GO wave and the location of the discontinuity is determined with the shadow boundary at $\phi=\pi+\phi_0$. In a similar way, the incident rays that hit the impedance screen, at $x>0$, will reflect by an amplitude change determined with the coefficient R. This reflection forms the reflected GO wave u_{rGO}. Since, the half-plane is limited by the edge point at $x=0$, the reflected GO waves exist in the region, bounded by the reflection boundary, which is located at $\phi=\pi-\phi_0$. $\xi_\pm$ has the expression of $-\sqrt{2k\rho}\cos[(\phi\pm\phi_0)/2]$ [20]. $e_\pm$ is $\exp[jk\rho\cos(\phi\pm\phi_0)]$. R is the reflection coefficient that can be represented by the equation

$$R = \frac{\sin\phi_0 - \sin\theta}{\sin\phi_0 + \sin\theta}. \tag{6}$$

$U(x)$ is the unit step function, which is equal to one for $x>0$ and zero otherwise. It determines the locations of the reflection and shadow boundaries where the reflected and incident GO waves have discontinuity. First of all we will show the defect of the solutions of Senior and Malyuzhinets. Their diffracted field expressions can be written as

$$u_d = \sigma(\phi)\frac{\exp(-jk\rho)}{\sqrt{k\rho}} \tag{7}$$

where σ is only a function of ϕ. The relation of

$$\sigma'(0) = jk\rho\sin\theta\sigma(0) \tag{8}$$

can be obtained when Eq. (7) is used in Eq. (1). A similar relation can be obtained for $\phi=2\pi$. The symbol $'$ represents the differentiation according to ϕ. Eq. (8) can only be satisfied if σ is also a function of ρ besides ϕ. As a result, the solutions of Senior and Malyuzhinets do not satisfy the impedance boundary condition, given by Eq. (1). Thus they are not the rigorous solutions of the impedance half-plane problem.

In this study, we will accept the total diffracted field as

$$u_d = -\frac{\exp(-j\pi/4)}{2\sqrt{2\pi}}\left[\frac{1}{\cos[(\phi-\phi)_0/2]} + \frac{\Gamma(\rho,\phi)}{\cos[(\phi+\phi)_0/2]}\right]\frac{\exp(-jk\rho)}{\sqrt{k\rho}} \tag{9}$$

in order to obtain a solution that satisfies Eq. (1). Γ is the reflection coefficient of the reflected diffracted field and its ρ dependence will not be shown further. As mentioned above, the diffracted field is radiated by the edge discontinuity located at $\rho=0$ and $z\in(-\infty,\infty)$. It is equivalent to the field, excited by a line source, located at the same coordinates. For this reason, the ρ dependence of the diffracted field is given by a cylindrical wave, originating from $\rho=0$ and $z\in(-\infty,\infty)$. The amplitude of this wave alters with respect to ϕ and is dependent on the angle of incidence ϕ_0. The diffracted wave changes its sign (has a phase shift of 180°) at the reflection and shadow boundaries, located at $\phi=\pi-\phi_0$ and $\phi=\pi+\phi_0$ respectively. Thus it compensates for the discontinuities of the GO waves at these transition regions. Note that the diffracted field expression, in Eq. (14), approaches infinity at the transition zones. The uniform expressions, which are finite in these regions, will be obtained later on. The sum of u_d and u_{GO} gives the total scattered field u. Γ can be determined by some relations with the reflection coefficient R, which approaches -1 and 1 for $\sin\theta\to\infty$ and $\sin\theta\to 0$ respectively. The same conditions must also be satisfied by Γ. Furthermore the condition of

$$\Gamma(\pi-\phi_0) = R \tag{10}$$

must also be satisfied on the reflection boundary, since the reflected diffracted field compensates for the discontinuity of the reflected GO wave here. Γ is a function of ϕ, because it must satisfy the impedance boundary condition at two ϕ-dependent coordinates at $\phi=0$ and $\phi=2\pi$. The general structure of R can be used to construct Γ. As shown in [20,21], the term of $\sin\phi_0$ can be represented by $\cos[(\phi-\phi_0)/2]$ for the reflected diffracted wave, because the cosine term becomes equal to $\sin\phi_0$ at the transition region, at $\phi=\pi-\phi_0$. Thus we can write the equation of

$$\Gamma(\phi) = \frac{\cos[(\phi-\phi_0)/2] - \sin\theta}{\cos[(\phi-\phi_0)/2] + \sin\theta} \tag{11}$$

for Γ. Note that Eq. (11) satisfies Eq. (10) and is equal to 1 and -1 at the limits of $\sin\theta\to 0$ and $\sin\theta\to\infty$ respectively. However it does not satisfy the boundary conditions, given by Eq. (1), because Γ must include two different coefficients which will be determined