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Stationary phase analysis of generalized cubic phase mask wavefront coding

Ming Liu [a], Liquan Dong [a,*], Yuejin Zhao [a], Mei Hui [a], Wei Jia [a,b]

[a] School of Optoelectronics, Beijing Institute of Technology, Beijing 100081, China
[b] Typonteq Co., Ltd. & Typonteq Lab, Room 1203, Building 15, Anhui Dongli, Chaoyang District, Beijing 100101, China

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ABSTRACT

The modified generalized cubic phase mask (GCPM) has recently been applied in wavefront coding systems including infrared imaging and microscopy. In this paper, the stationary phase method is employed to analyze the GCPM characteristics. The SPA of the modulation transfer function (MTF) under misfocus aberration is derived for a wavefront coding system with a GCPM. The approximation corresponds with the Fast Fourier Transform (FFT) approach. On the basis of this approximation, we compare the characteristics of GCPM and cubic phase masks (CPM). A GCPM design approach based on stationary phase approximation is presented which helps to determine the initial parameter of phase mask, significantly decreasing the computational time required for numerical simulation.

1. Introduction

Wavefront coding technology has been extensively studied since Dowski and Cathey introduced the wavefront coding concept in 1995 [1,2]. However, most of the reported studies focus on one-dimensional conditions [3–7]. For two-dimensional analyses, researchers usually use original cubic phase mask (CPM) because it can be easily decomposed into a one-dimensional problem [8–12]. There are some other studies that report numerical [13] and experimental [14] analysis of the two-dimensional conditions encountered in non-CPM wavefront coding. So far, no analysis has been conducted for non-CPM two-dimensional wavefront coding.

A novel phase mask, generalized cubic phase mask (GCPM), based on the Zernike polynomial has been proposed in 2003 [8] and applied in infrared imaging [15,16] and optical microscopy [17,18]. For a GCPM system, its point spread function is free of lateral shifts and its modulation transfer function (MTF) exhibits superior off-axis performance. However, a GCPM cannot be decomposed into a one-dimensional problem, and has been analyzed numerically only so far [19].

Though the MTF of GCPM can be analyzed by Fast Fourier Transform (FFT) analysis, the analytical approximation is still vital in understanding the rules of MTF changing. As in the case of CPM, the approximation of CPM MTF in one dimension is firstly given in the paper of Dowski and Cathey.

In this paper, a stationary phase method is used to obtain an approximation of MTF of GCPM wavefront coding system under misfocus aberration. The approximation provides insights into the imaging characteristics of wavefront coding with GCPMs. The boundary of GCPM MTF is analyzed using the approximation, and then the MTF characteristics of GCPM and CPM are compared. A design approach based on the approximation is also presented.

2. Theory

The mathematical function of the GCPM is

$$\phi(x,y)=\beta(x^3+y^3)-3\beta(x^2y+xy^2) \tag{1}$$

where β determines the strength of the phase mask. For simplicity, a variable substitution for a circular aperture is made to rotate the phase mask 45° clockwise around its center. The GCPM mathematical function is then rewritten as

$$\phi(x,y)=\gamma(3x^2y-y^3) \tag{2}$$

where γ satisfies

$$\gamma=\sqrt{2}\beta \tag{3}$$

Fig. 1 shows the phase map of Eq. (2) in circular aperture. It is very similar to the GCPM phase maps in [15,16]. The only difference between them is the placement direction. The optical transfer function (OTF) as a function of misfocus is given as

$$H(u,v,\psi)=\frac{1}{S}\int\int P(x+u,y+v)\exp\left\{i\psi\left[(x+u)^2+(y+v)^2\right]\right\} \times P(x-u,y-v)\exp\left\{i\psi\left[(x-u)^2+(y-v)^2\right]\right\}dx\,dy \tag{4}$$

where u and v are the normalized spatial frequency (NSF), and P denotes the phase function expressed as in Eq. (5). S is the

* Corresponding author. Tel.: +86 10 689 12559.
E-mail address: kylind@bit.edu.cn (L. Dong).

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aperture area. which is π for a unit circular aperture.

$$P(x,y) = p(x,y)\exp(i\phi(x,y)) \tag{5}$$

where p is the aperture function. For a unit circular aperture, p is expressed as follows:

$$p(x,y) = \begin{cases} 1 & x^2+y^2 \le 1 \\ 0 & \text{else} \end{cases} \tag{6}$$

ψ in Eq. (4) is the misfocus parameter expressed as follows:

$$\psi = \frac{\pi L^2}{4\lambda}\left(\frac{1}{f} - \frac{1}{d_0} - \frac{1}{d_i}\right) \tag{7}$$

where L is the width of the aperture, f is the focal length, d_0 denotes the object distance from the first principal plane of the lens, d_i represents the distance of the image-capture plane from the second principal plane of the lens, and λ is the wavelength of light.

Using the stationary phase approach described in [20], we obtained the stationary phase approximation (SPA) of the GCMP MTF as follows. Where the details of this approximation are provided in Appendix A:

$$\begin{aligned} |H(u,v,\psi)| \approx & \frac{1}{6\gamma\sqrt{u^2+v^2}} p\left(-\frac{2\psi uv}{3\gamma(u^2+v^2)}+u, -\frac{\psi}{3\gamma}\left(\frac{u^2-v^2}{u^2+v^2}\right)+v\right) \\ & \times p\left(-\frac{2\psi uv}{3\gamma(u^2+v^2)}-u, -\frac{\psi}{3\gamma}\left(\frac{u^2-v^2}{u^2+v^2}\right)-v\right) \end{aligned} \tag{8}$$

As indicated in Eq. (8), ψ affects only the non-zero area, and not the shape of the approximation.Setting $\psi=0$, we obtain

$$|H(u,v,0)| \approx \frac{1}{6\gamma\sqrt{u^2+v^2}} p(u,v)p(-u,-v) \tag{9}$$

As shown in Eq. (9), MTF is non-zero in a circular area centered on (0,0), with a radius of 1. MTF equals 0 outside the circular area. If $\psi \neq 0$, the borderline equation of the non-zero MTF area is

$$\begin{aligned} & p\left(-\frac{2\psi uv}{3\gamma(u^2+v^2)}+u, -\frac{\psi}{3\gamma}\left(\frac{u^2-v^2}{u^2+v^2}\right)+v\right) \\ & p\left(-\frac{2\psi uv}{3\gamma(u^2+v^2)}-u, -\frac{\psi}{3\gamma}\left(\frac{u^2-v^2}{u^2+v^2}\right)-v\right) = 1 \end{aligned} \tag{10}$$

It is noted that Eq. (10), the borderline equation, is difficult to be resolved directly; we first made a numerical resolution and obtained Fig. 2, which shows that the non-zero MTF area is a hexagon with six symmetry axes. The intersections of the non-zero area border and the symmetrical axes generally define the non-zero area.

R_i and R_o are the inner and outer radius, respectively, of the non-zero hexagon (Fig. 3). Under the condition $u=0$ and $v=0$, solving Eq. (10) yields R_i and R_o respectively.

If $u=0$, the non-zero MTF borderline equation is written as

$$p(0,\psi/3\gamma+v)p(0,\psi/3\gamma-v) = 1 \tag{11}$$

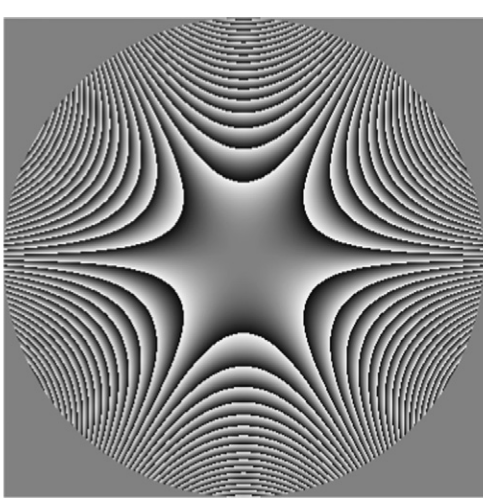

Fig. 1. Phase wrapped from $-\pi$ to π for GCPM design computed within a circular pupil aperture ($\gamma=100$, aperture radius$=1$).

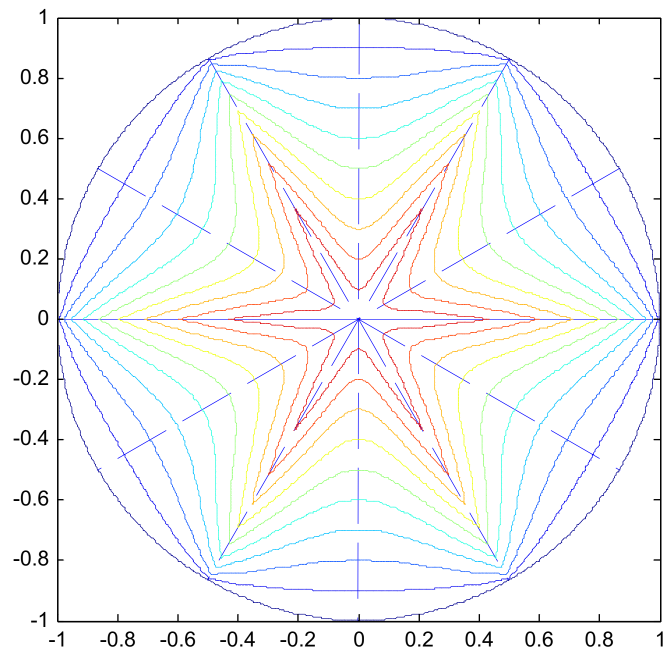

Fig. 2. Non-zero area boarder of SPA of GCPM MTF; $\gamma=20$, aperture radius$=1$, $\psi=0, 6, 12, \ldots, 60$ (from outer circle to inner hexagon), with six axes of symmetry (dashed line).

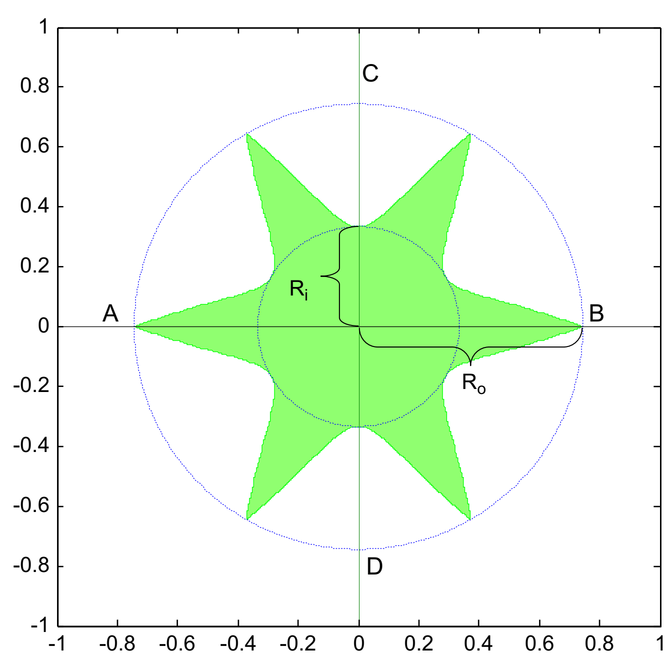

Fig. 3. Non-zero area of GCPM MPA of MTF (gray color); $\gamma=20$, aperture radius$=1$, $\psi=40$, with internal radius R_i and external radius R_o.