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Effect of Kerr nonlinearity on the transverse localization of light in 1D array of optical waveguides with off-diagonal disorder

M. Khazaei Nezhad [a], M. Golshani [a], A.R. Bahrampour [a,*], S.M. Mahdavi [a,b,**]

[a] Department of Physics, Sharif University of Technology, Tehran, Iran
[b] Institute for Nanoscience and Nanotechnology, Sharif University of Technology, Tehran, Iran

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ABSTRACT

In this paper a simulation of the transverse localization of light in 1D array of optical waveguides in the presence of off-diagonal disorder is presented. Effects of self-focusing and self-defocusing Kerr nonlinearity on the transverse localization of surface and bulk modes of the disordered waveguides array are taken into consideration. The simulation shows that in the off-diagonal disordered array at low nonlinear parameters, the transverse localization of light becomes more than that of the corresponding diagonal disordered array. However by increasing the nonlinear parameters the diagonal disordered array is localized more than the associated off-diagonal disordered array for both surface and bulk modes. It is also found that the surface modes become more localized than the bulk modes by increasing the nonlinear parameter. The calculated effective beam width versus propagation distance for off-diagonal disordered arrays confirms these results.

1. Introduction

Since the first paper on the localization of the electron wave function, published by Anderson about half century ago [1], this area has still survived for theoreticians and experimentalists. This phenomenon is due to wave interference in disordered systems, so it is natural to expect that it could be applied in any wave system such as condensed matter systems, elastic and optical systems [2–8]. The Anderson localization of photons can be visualized easily; hence much theoretical and experimental work has been done on the optical Anderson localization [7–11]. One of the interesting topics in light localization is the transverse localization of light which was predicted in 1989 for the first time [12]. About two decades later, this phenomenon was observed experimentally in disordered photonic lattice systems [13]. After this experiment, disordered photonic lattice systems have attracted increasing attention as a ground to study the transverse localization of light [14–23]. One of the most experimentally realizable systems for studying transverse localization is a 1D array of optical waveguides [14,23]. This array can be built by methods such as optical induced techniques in photo refractive materials, laser writing methods or common lithographic methods [13,18–22,14].

In the presence of Kerr effects, Maxwell equations can be reduced to nonlinear Schrödinger equations [13–17,21–24]. To study the effect of Kerr nonlinearity on the transverse localization of light in disordered photonic lattices, the system of nonlinear Schrödinger equations is solved numerically and results are verified experimentally [13–17,21–23]. The impact of the nonlinear Kerr effect on the transverse localization in the diagonal disordered array of waveguides has been investigated theoretically and experimentally [14,23]. To study the effect of positive Kerr effect on the bulk and surface modes in the diagonal disordered waveguides array, nonlinear Schrödinger equations have also been solved numerically [23].

In this work, the 1D array of optical waveguides is considered. Disorder is introduced by changing the coupling coefficient randomly between each of the waveguides. The effect of self-focusing and self-defocusing Kerr nonlinearity on the transverse localization of light in this off-diagonal disordered waveguides array is studied. The results are compared to those of diagonal disordered array in the presence of positive and negative Kerr nonlinear effects.

The paper is organized in four sections. Section 2 contains the theoretical models for diagonal and off-diagonal disordered arrays. Discussion on the numerical simulation results is presented in Section 3 and Section 4 is devoted to conclusions.

2. Theoretical models

Fig. 1 shows the structure of a disordered one dimensional array of single mode-optical waveguides. For the single-mode

* Corresponding author.
** Corresponding author at: Department of Physics, Sharif University of Technology, Tehran, Iran.
E-mail addresses: bahrampour@sharif.edu (A.R. Bahrampour), mahdavi@sharif.edu (S.M. Mahdavi).

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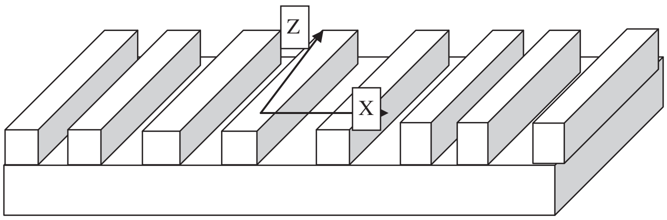

Fig. 1. Schematic of an array of 1D optical waveguides.

operation of optical waveguides, the waveguide width is determined by the light wavelength, while the coupling coefficients between waveguides are determined by the separation distance between adjacent waveguides [25]. At the entrance plane, light is injected in one of the waveguides of the disordered array and can be coupled to the neighboring waveguides by the tunneling effect. The slowly varying envelope approximation (SVEA) method is employed to write the propagation equation in the presence of nonlinear Kerr effect in an array of 1D waveguides [26]. In the tight-binding approximation governing equations have the simple form [14,23,26,27]

$$i\frac{dE_n}{dz}+K_nE_n+C_{n,n+1}E_{n+1}+C_{n,n-1}E_{n-1}+\gamma|E_n|^2E_n=0\ ,\quad n=1,2,\ldots,N \tag{1}$$

Here E_n is the amplitude of electric field in waveguide n. N is the total number of waveguides. K_n the propagation constant of light in site n, depends on the refractive index and the width of the corresponding waveguide. $C_{n,m}$ is the coupling coefficient between waveguides n and m. The coupling coefficients can be calculated by coupled mode theory, which depends on the separation distance and refractive index of material between waveguides [25]. The physical dimension of coupling coefficient is the inverse of length. $\gamma=n_2\omega/cA_{eff}$ is the Kerr coefficient [1/Wm], where A_{eff} is the effective area of the fundamental modes. c is the speed of light in free space and n_2 is the nonlinear refractive index [m^2/V]. The nonlinear Kerr coefficient can be positive or negative corresponding to the self-focusing and self-defocusing behavior respectively [23].

Diagonal disorder can be introduced in this system using randomized propagation constants of each waveguide. It is possible by randomized refractive index or the width of each waveguide. The off-diagonal disorder can be introduced by randomizing the coupling coefficient between the nearest neighbor waveguides. Once refractive index is randomized, the diagonal disorder will unavoidably be introduced. The regular part of propagation constants and coupling coefficients are defined by K_0 and C respectively.

In the presence of diagonal and off-diagonal disorders, the following normalized variables are employed [23]:

$$K_n=K_0+C\varepsilon_n, C_{n,n\pm1}=C(1+\varepsilon_{n,n\pm1});\quad s = Cz;\quad U_n=\frac{E_ne^{iK_0s/C}}{\sqrt{P}};$$

$$\chi=\frac{\gamma P}{C}, \tag{2}$$

where P is the power of incident light, and ε_n and $\varepsilon_{n,n\pm1}$ are the normalized random parts of the propagation constants and coupling coefficients respectively. The propagation equation (1) versus the normalized variables can be rewritten as follows:

$$i\frac{dU_n}{ds}+\varepsilon_nU_n+(1+\varepsilon_{n,n-1})U_{n-1}+(1+\varepsilon_{n,n+1})U_{n+1}+\chi|U_n|^2U_n=0, \tag{3}$$

where $n=1,2,\ldots,N$. In diagonal disordered arrays $\varepsilon_{n,n\pm1}$ are zero and ε_n is a random variable distributed on the $[-\Delta_d,\Delta_d]$ interval uniformly, while for off-diagonal disordered arrays $\varepsilon_{n,n\pm1}$ s are distributed uniformly on the interval $[-\Delta_o,\Delta_o]$ and ε_n s are zero. Δ_d and Δ_o are called the strength of diagonal and off-diagonal disordered arrays respectively.

To compare the diagonal and off-diagonal disorder effects on the transverse localization, it is assumed that both of the disorder strength for diagonal and odd-diagonal disordered arrays have the same value ($\Delta_d=\Delta_o=\Delta$). 1 and N waveguides are defined as the surface or edge of the array of optical waveguides and modes of propagation in the one and N waveguides are called the surface modes.

In order to define the initial value for the governing equations it is assumed that light is injected to one of the waveguides i.e. $U_n(s=0)=\delta_{n,n_0}$. When n_0 is set to 1 or N, the surface modes are excited, while for other values ($n_0\neq1,N$) the bulk modes can be excited [23].

The transverse localization length and participation rate are defined as transverse localization measures. If light is localized in the transverse direction, we expect that the light intensity decays exponentially in the transverse direction, with the decay constant being equal to the inverse localization length.

The participation rate (PR) is defined as follows [15–17,23]:

$$PR(s)=\left\langle\frac{\left(\sum_{n=1}^{N}|U_n(s)|^2\right)^2}{\sum_{n=1}^{N}|U_n(s)|^4}\right\rangle \tag{4}$$

Due to the statistical nature of disordered systems the average is taken over realizations of disordered waveguide arrays. In a completely transverse localized system ($U_n(s)\simeq\delta_{n,m}$) the PR of the system is approximately equal to one ($PR\simeq1$), while in a completely extended system the energy is uniformly distributed on the N waveguides, that is the intensity in each waveguide is proportional to the inverse of the number of waveguides and the normalized field intensity is equal to the inverse of the square root of the number of waveguides (delocalized,$U_n(s)\simeq1/\sqrt{N}$). In this case, the participation rate approaches the number of waveguides ($PR\simeq N$). The effective beam width is defined as the square root of the participation rate $\left(w_{eff}=\sqrt{PR}\right)$ [15–17].

To study the evolution of surface and bulk modes of diagonal or off-diagonal disordered waveguide arrays in the presence of focusing or defocusing nonlinearities, the transverse localization length and the effective beam width for different physical parameters are calculated.

3. Results and discussion

For numerical simulation, a 1D array of $N=200$ coupled waveguides with boundary conditions $U_0(s)=U_{N+1}(s)=0$ is taken into consideration. To study the effects of the off-diagonal disorder on the waveguide arrays' behavior, the coupling coefficients random variables are uniformly chosen on the interval $[-\Delta,\ \Delta]$. To obtain the normalized field amplitudes, U_n, the system of governing Eq. (3) is solved by the Runge–Kutta–Fehlberg method [28]. To investigate the statistical behavior of the disordered systems, a number of realizations (nr) of disordered arrays are generated and the field intensity distribution for each of the disordered systems is calculated. Then the statistical calculations are done on the number of different realizations.

3.1. Inverse localization length

For calculating the inverse localization length, after solving Eq. (3) and calculating the evolution of U_n up to the output position s_0, the output data are fitted with the exponential