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Spectrum changes of rectangular array beams through turbulent atmosphere

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A B S T R A C T

According to the extended Huygens–Fresnel principle, the propagation formulas of the spectrum for correlated and uncorrelated superposition rectangular array Gaussian Schell-model (RAGSM) beams propagating in a turbulent atmosphere have been derived. The results show that for the case of $M>1$ and $N>1$, the on-axis relative spectral shift of the RAGSM beam presents the blue-shift alternating with the red-shift with increasing propagation distance, and when the propagation distance is large enough the spectral shifts in free space will only exhibit blue-shift, whereas the spectral shifts will tend to 0 in turbulence due to both the diffraction of atmospheric turbulence and the correlation of the source. The off-axis relative spectral shift for the correlated and uncorrelated RAGSM beams exhibit rapid transitions of the spectrum upon off-axis distance in turbulence. In addition, the spectral shifts also depend on the characteristic parameters of turbulence, the beam parameters and the array parameters.

1. Introduction

With the rapid development of laser technology, more and more attention have been paid to laser array beams due to their wide applications, such as high-power system, inertial confinement fusion, high-energy weapons, and so on [1–4]. So far, various laser array beams including rectangular array beams have been developed to achieve high-power output laser beam and many researches have been carried out for the propagation characteristics of such laser array beams in free space or through a paraxial optical system have been studied extensively [5–9]. For instance, the phase locking of radial array CO_2 laser has been investigated theoretically by Chen et al. [10] and experimentally by Yelden et al. [11].

For many decades the propagation of laser beams in a turbulent atmosphere has been extensively investigated due to their useful applications in remote sensing, atmospheric optical communications, and track systems [12–16]. In 1986, Wolf pointed that the spectrum of light which is emitted from a spatially partially coherent source with a wide spectral bandwidth undergoes spectral shift referred to as correlation-induced even if through free space [17,18]. Later on, it was found that the spectrum of the partially coherent beam in the diffracted field will be changed as well referred to as diffraction-induced [19,20]. In fact, turbulent atmosphere is a diffracted field. So, partially coherent beams in turbulence have spectral shift by the correlation of source and the diffraction of turbulence. The present paper is focused on studying the spectral properties of correlated and correlated superposition RAGSM beams both in free space and in turbulence, and discussing the effects of the characteristic parameters of the turbulence, the beam parameters and the array parameters on spectral changes in detail.

2. Spectrum of correlated and uncorrelated superposition RAGSM beams propagating through turbulence

A rectangular laser array beam is now considered which consists of $M\times N$ beamlets positioned at the propagation distance $z=0$ and the beamlets are the same Gaussian Schell-model (GSM) beams. The beamlet number in x-direction and y-direction are M and N, the separation distances are x_0 and y_0, respectively. For the sake of simplicity, M and N are considered as odd numbers, but the extension to even number is straightforward. The schematic drawing of rectangular laser array beams has been shown in Fig. 1 [8].

Assume that GSM beamlets are perfect replicas of each other, and based on the theory of partially coherent light and the consideration of the quasi-monochromatic scalar field, the cross-spectral density function (CDF) of the correlated superposition RAGSM beams at the plane $z=0$ can be written as [21],

$$W_0(x_1',x_2',y_1',y_2',0)=\sum_{m_1=-m}^{m}\sum_{m_2=-m}^{m}\sum_{n_1=-n}^{n}\sum_{n_2=-n}^{n}$$

$$\times\exp\left[-\frac{(x_1'-m_1x_0)^2+(x_2'-m_2x)_0^2}{w_0^2}\right.$$

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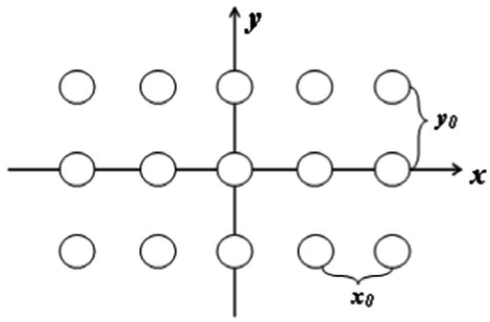

Fig. 1. Schematic drawing of a rectangular laser array beam.

$$-\frac{(y_1'-n_1y_0)^2+(y_2'-n_2y_0)^2}{w_0^2}\Bigg]$$
$$\times\exp\left\{-\frac{\left[(x_1'-m_1x_0)-(x_2'-m_2x_0)\right]^2}{2\sigma_0^2(\omega)}-\frac{\left[(y_1'-n_1y_0)-(y_2'-n_2y_0)\right]^2}{2\sigma_0^2(\omega)}\right\} \tag{1}$$

where w_0 is the waist width of array beamlets, $m=(M-1)/2$, $n=(N-1)/2$; (x_1', y_1') and (x_2', y_2') are the transversal coordinates of two points at the $z=0$ plane; $\sigma_0(\omega)$ is the spatial correlation length; For the case of $x_0=0$ and $y_0=0$ or the beam with $M=1$ and $N=1$, Eq. (1) represents the CDF of a GSM beam.The CDF of the uncorrelated superposition RAGSM beams at the plane $z=0$ can be expressed as [21],

$$W_0(x_1',x_2',y_1',y_2',0)=\sum_{m_1=-m}^{m}\sum_{n_1=-n}^{n}\times\exp\left[-\frac{(x_1'-mx_0)^2+(x_2'-mx_0)^2}{w_0^2}-\frac{(y_1'-ny_0)^2+(y_2'-ny_0)^2}{w_0^2}\right]\times\exp\left[-\frac{(x_1'-x_2')^2+(y_1'-y_2')^2}{2\sigma_0^2(\omega)}\right]. \tag{2}$$

Suppose that the source and the medium statistics are independent, and the turbulent atmosphere is statistically homogenous and isotropic. Thus, by using the extended Huygens–Fresnel principle, the CDF of the RAGSM beam in turbulence can be described as [13]

$$W(x_1,x_2,y_1,y_2,z,\omega)=\frac{1}{(\lambda z)^2}\int\int dx_1'dx_2'\int\int dy_1'dy_2'W_0(x_1',x_2',y_1',y_2',0,\omega)\times\exp\left\{-\frac{ik}{2z}\left[(x_1'^2+y_1'^2)-(x_2'^2+y_2'^2)-2(x_1'x_1+y_1'y_1)+2(x_2'x_2+y_2'y_2)+(x_1^2+y_1^2)-(x_2^2+y_2^2)\right]\right\}\langle\exp[\psi(x_1',y_1')+\psi^*(x_2',y_1')]\rangle_f \tag{3}$$

where z denotes the propagation distance, λ is the wave length, $k=\omega/c$ is the optical wave number, and c is velocity of light. The angle bracket with the subscript f denotes averaging over field ensemble and can be expressed as

$$\langle\exp[\psi(x_1',y_1')+\psi^*(x_2',y_2')]\rangle_f=\exp\left\{\frac{\left[(x_1'^2+y_1'^2)-2(x_1'x_2'+y_1'y_2')+(x_2'^2+y_2'^2)\right]}{\rho_0^2}\right\} \tag{4}$$

ρ_0 represents the coherent length of a spherical wave propagating in the turbulent medium, i.e.,

$$\rho_0=(0.545C_n^2k^2z)^{-3/5} \tag{5}$$

where C_n^2 represents the refractive index structure constant of the turbulence. After some tedious integral calculations, the analytical formulas for the spectral intensity of the correlated superposition RAGSM beam at the plane z=const in turbulence turns out to be

$$S=\frac{S^0(\omega)}{(\lambda z)^2}\sum_{m_1=-m}^{m}\sum_{m_2=-m}^{m}\sum_{n_1=-n}^{n}\sum_{n_2=-n}^{n}\frac{pp_xpp_y\pi^2}{A_1A_2}\exp\left(\frac{B_{1x}^2+B_{1y}^2}{4A_1}+\frac{B_{2x}^2+B_{2y}^2}{4A_2}\right) \tag{6}$$

Assume that the source spectrum $S^0(\omega)$ is of the Lorentzian line with central frequency ω_0, namely

$$S^{(0)}(\omega)=S_0\frac{\Gamma^2}{(\omega-\omega_0)^2+\Gamma^2} \tag{7}$$

where S_0 is a constant, and Γ is the half-width at half-maximum of the initial spectrum. From Eq. (7) $S^0(\omega)$ only depends on the Γ, ω and ω_0, and independent of the parameters of the turbulence, beam parameters and the array parameters.

And the other parameters in Eq. (6) can be expressed as

$$p=\left(-\frac{1}{w_0^2}-\frac{1}{2\sigma_0^2(\omega)}\right),\quad G=\frac{1}{\sigma_0^2(\omega)}+2u,\quad pp_x=\exp[pm_1^2x_0^2+pm_2^2x_0^2]\exp\left[\frac{m_1m_2x_0^2}{\sigma_0^2(\omega)}\right]$$
$$u=\frac{1}{\rho_0^2},\quad pp_y=\exp[pn_1^2y_0^2+pn_2^2y_0^2]\exp\left[\frac{n_1n_2y_0^2}{\sigma_0^2(\omega)}\right]$$
$$B_{1x}=-2pm_1x_0-\frac{m_2x_0}{\sigma_0^2(\omega)}+\frac{ik}{z}x,\quad B_{1y}=-2pn_1y_0-\frac{n_2y_0}{\sigma_0^2(\omega)}+\frac{ik}{z}y,$$
$$A_1=-p+\frac{ik}{2z}+u,\quad B_{2x}=\frac{B_{1x}G}{2A_1}-2pm_2x_0-\frac{m_1x_0}{\sigma_0^2(\omega)}-\frac{ik}{z}x,$$
$$A_2=-\frac{G^2}{4A_1}-p-\frac{ik}{2z}+u,\quad B_{2y}=\frac{B_{1y}G}{2A_1}-2pn_2y_0-\frac{n_1y_0}{\sigma_0^2(\omega)}-\frac{ik}{z}y\,. \tag{8}$$

For the uncorrelated superposition, the intensity of RAGSM beams through turbulence at the output plane can be expressed as

$$<I(\mathbf{r},z)>\;=\frac{\pi^2}{(\lambda z)^2}\sum_{m_1=-m}^{m}\sum_{n_1=-n}^{n}\frac{1}{A_1A_2}\exp\left(-\frac{2m_1^2x_0^2+2n_1^2y_0^2}{w_0^2}\right)\exp\left(\frac{B_{1x}^2+B_{1y}^2}{4A_1}+\frac{B_{2x}^2+B_{2y}^2}{4A_2}\right) \tag{9}$$

$$A_1=\frac{1}{w_0^2}+\frac{1}{2\sigma_0^2(\omega)}+\frac{ik}{2z}+u,\quad B_{1x}=\frac{2m_1x_0}{w_0^2}+\frac{ikx}{z},\quad G=\frac{1}{\sigma_0^2(\omega)}+2u$$
$$A_2=-\frac{G^2}{4A_1}+\frac{1}{w_0^2}+\frac{1}{2\sigma_0^2(\omega)}-\frac{ik}{2z}+u,\quad B_{2x}=\frac{B_{1x}G}{2A_1}+\frac{2m_1x_0}{w_0^2}-\frac{ikx}{z}$$
$$B_{1y}=\frac{2n_1y_0}{w_0^2}+\frac{iky}{z},\quad B_{2y}=\frac{B_{1y}G}{2A_1}+\frac{2n_1y_0}{w_0^2}-\frac{iky}{z} \tag{10}$$

if letting $x=y=0$ in Eqs. (8) and (10), the on-axis spectral intensity of the RAGSM beam through turbulence can be obtained.

For simplicity, the normalized spectrum $S(\omega)$ and the relative spectral shift $\delta\omega/\omega_0$ can be specified as follows:

$$S(\omega)=\frac{S(r,z,\omega)}{S_{\max}(r,z,\omega)} \tag{11}$$

$$\frac{\delta\omega}{\omega_0}=\frac{\omega_{\max}-\omega_0}{\omega_0} \tag{12}$$

where $S_{\max}(x, y, z, \omega_{\max})$ and $\omega_{\max}$ denote maximum values for spectrum intensity S and the corresponding frequency ω at the point (x,y,z), respectively.